THE RATIO

BETWEEN

DIAMETER AND CIRCUMFERENCE IN A CIRCLE DEMONSTRATED BY ANGLES,

AND

EUCLID'S THEOREM, PROPOSITION 32, BOOK 1,

PROVED

TO BE FALLACIOUS.

By JAMES SMITH, Esq.,

MEMBER OF THE MERSEY DOCKS AND HARBOUR BOARD, AND EX-CHAIRMAN OF THE LIVERPOOL LOCAL MARINE BOARD.

Liverpool:
EDWARD HOWELL, CHURCH STREET.
LONDON: SIMPKIN, MARSHALL & CO., STATIONERS' HALL COURT.
1870.

Address to the Reader.

THAT Euclid's Theorem, Prop. 32, Book 1, is fallacious, is a statement so startling to pretenders to geometrical knowledge, that it at once affrights them from their propriety, and "*upsets*" their equilibrium; and they rush to the conclusion that any man who could have the audacity to make such an assertion, can be fitted only for a lunatic asylum; and that with such a person "*discussion would be quite useless.*" In this theorem, Euclid leaves it to be *inferred*, that each of the angles in an equilateral triangle is equal to two-thirds of a right angle. No doubt this is true, whatever numerical value we may put upon a right angle. But, if this fact were of universal application in pure Geometry, it would follow, that equal angles in connection with the same circle may be subtended by unequal arcs, which is absurd. You will see my proof of this if you read to the end of this Address.

In modern times, "*recognised*" Geometers are beginning to differ among themselves, as to "*whether Euclid's Elements is the proper text-book for teaching Geometry to beginners.*" Mr. J. M. Wilson, Fellow of St. John's College, Cambridge, Senior Wrangler in 1859, and now Mathematical Master of Rugby School, in the preface to his recently published "*Treatise on Elementary Geometry,*" observes:—"The lately published Report of the Schools' Inquiry Commission has given an immediate

importance to the question whether Euclid's Elements is the proper text-book for teaching Geometry to beginners. Euclid's recognised and acknowledged faults as a system of Geometry, and as a specimen of analysed reasoning, are of slight importance compared with others of greater magnitude. The real objections to Euclid as a text-book are his artificiality, the invariably syllogistic form of his reasoning, the length of his demonstrations, and his unsuggestiveness. As to the first, he aimed, not at unfolding Geometry as a science, but at shewing on how few axioms and postulates the whole could be made to depend: and he has thus sacrificed, to a great extent, simplicity and naturalness in his demonstrations, without any corresponding gain in grasp or cogency. The exclusion of hypothetical constructions may be mentioned as a self-imposed restriction, which has made the confused order of his first book necessary, without any compensating advantage. There is no real advantage in the arrangement of propositions by sequence, which Euclid maintains so strictly, for nothing can be gained by excluding *any* sound method of reasoning: and if a direct proof can be found of any theorem, which naturally arises out of the data of the theorem, it is to be preferred to a circuitous proof which depends on other theorems. Thus, if the Fifth Proposition can be proved independently, and on its own evidence, it is certain that the decisive bearing of the data on the conclusion will be better appreciated than it would be on Euclid's method."

I have never met with the writings of a Geometer, either of the present or any past age, in which there is a fault found with Euclid's first postulate, or, with his definition of a right angle. The definition of a right

angle, and Euclid's first postulate, are clearly within reach of the capacity of any school-boy.

Now, one object of Euclid in his compilation of the Elements of Plane Geometry, was to make every proposition depend upon some antecedent proposition or propositions; thus it was essential to this object, that he should commence with a problem. Not only so, but it was also essential to this object, that his first problem (Prop. 1, Book 1) should be based upon the simplest of the postulates, viz.: "*Let it be granted, that a straight line may be drawn from any one point to any other point.*" Thus, Euclid begins by showing us how to construct an equilateral triangle. There can be no doubt that Euclid knew, that an equilateral triangle is also equiangular, and he might at once have proceeded to prove this: but this would have been inconsistent with Euclid's object. Well, then, Prop. 2 rests for its proof on Prop. 1: and Prop. 3 rests for its proof on Propositions 1 and 2. This brings us to Proposition 4, Euclid's first theorem, with reference to which some modern Geometers find fault with Euclid's proof. I quote the following from the pen of a "*recognised Mathematician*" (Cooley) *:—"The demonstration of the Fourth Proposition, as it has been hitherto expressed, affords a remarkable instance of confused reasoning, arising from the employment of the same term in different senses. The learner finds it impossible to discriminate exactly the several steps in the reasoning; and though he may learn the demonstration by rote, he is by no means thoroughly convinced of its completeness. In the language of Simson, copied by Playfair, it is said:

* Euclid's Elements of Plane Geometry. Appendix B. By W. D. Cooley, A.B.

'If the triangle A B C be applied to the triangle D E F. so that the point A may be on D, and the straight line A B upon D E, the point B shall coincide with the point E, because A B is equal to D E ; and since A B coincides with D E, A C shall coincide with D F, because the angle B A C is equal to the angle E D F; wherefore, also, the point C shall coincide with the point F, because A C is equal to D F.' Now, the word '*coincide,*' which occurs here three times, is used, the second time of its occurrence, in a vague sense, evidently different from that in which it is to be understood in the other two instances. Since it is proved that A B coincides with D E, and the angle B A C is equal to the angle D E F, it follows, that A C and D F fall together ; but that they coincide must be inferred, as in the case of A B and D E, and from the joint circumstances of their falling together, meeting at one extremity, and being equal. If it were a just inference, as expressed above, that '*A C shall coincide with D F, because the angle B A C is equal to the angle D E F,*' then it would be needless to conclude that the points C and F coincide, '*because A C is equal to D F.*' When two lines coincide, their extremities must necessarily coincide also. The logical inaccuracy here pointed out has been derived from the Greek original, but it is not on that account the less necessary to correct it. To admit vague or equivocal expressions into geometrical reasoning, is to destroy at once its chief merit and essential characteristic."

The vagueness and "*confused reasoning*" in Euclid's demonstration of his first theorem, thus referred to by W. D. Cooley, might have been avoided, had Euclid made the Propositions 9, 10, 11, and 12, of his first

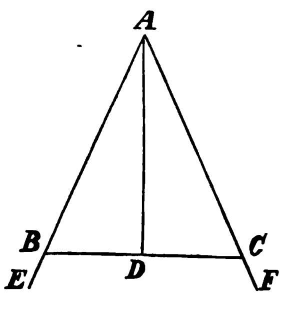
Book (which are all problems), follow Proposition 3: and then have made Prop. 13 his first theorem. In this case Prop. 4 might have been omitted altogether. *Geometrically*, one triangle can never be applied to another triangle. It is no doubt true, that with rule and compasses, we may construct two similar and equal triangles on paper: and with knife or scissors we may cut them out, and so separate and disunite them. If one of these triangles be applied to the other, that is, be laid upon the other, the angles of the one may be made to fall upon, or "*coincide*" with, the corresponding angles of the other; and the sides of the one may be made to fall upon, or "*coincide*" with, the corresponding sides of the other: and we infer—and rightly infer—that the triangles are similar in all respects, and are therefore equal in all respects. But, we might lay an angle or side of the one triangle on a non-corresponding angle or side of the other; and if so, (except in the case of an equilateral and equiangular triangle, in which all the angles and sides correspond,) the other angles and sides would not coincide. Should we, in this case, expect a school-boy to draw the inference, (although he may comprehend distinctly Euclid's first postulate and definition of a right angle,) that the two triangles are similar and equal? I think not!

We may teach a child the difference between a straight line and a crooked line, as readily as we may teach him the difference between the colour of two horses, or the difference between a sweet orange and a sour plum. What is an angle? Is it not a corner, made by the meeting of two non-parallel lines in a point? In geometry, do we not define an angle to be the space intercepted by two lines that intersect each other; or, as the space

comprised by two straight lines that meet in a point? Well, then, we may teach a child, at a very early age, the difference between an acute corner, an obtuse corner, and a right angled corner, by the corners in his nursery: but, it will depend upon how far a school-boy's education is advanced, and how far his reasoning powers are developed, whether he will *infer* that in equal triangles, the angles and sides of the one will "*coincide*" with the angles and sides of the other. This is an inference, and comes under the province of *reflection*, not of geometrical demonstration. No doubt, we can divide an equilateral triangle into two similar and equal right-angled triangles, and the bisecting line will be common to both triangles. We can conceive one of these triangles to be turned over and laid upon the other, and so coincide with it in all respects. But, can this be called a geometrical demonstration? Well, then, Euclid's proof of his first theorem is defective, inasmuch as the Proposition is incapable (strictly speaking) of geometrical demonstration.

This brings me to Euclid's Fifth Proposition of the first Book, which should have followed the 13th; or, still better, the following should have intervened:—

FIG. 1.



In the geometrical figure (Fig. 1). From the point A draw two straight lines, A B and A C, of equal length, but in different directions. Join B C, and produce AB and AC to the points E and F.

Euclid no where gives this simple problem, or a theorem founded on it; and yet, any school-boy who had mastered the seven first problems of Euclid—

all of which are extremely simple—would at once perceive that A D is perpendicular to B C, and, knowing the definition of a right angle, would perceive, that the angles on each side of A D are right angles: and having mastered Euclid's 13th Proposition, would—if possessed of moderate powers of reflection—also perceive that the angles A B C and C B E are together equal to two right angles; and similarly, that the angles A C B and B C F are together equal to two right angles: and even if these equations did not occur to a school-boy, he might be as readily taught them, and as easily convinced of their truth, as that two oranges are more than one.

If the foregoing Proposition had followed the 13th, and preceded Euclid's 5th Proposition, the apparent difficulties of the latter to beginners, would never have made their appearance. How many boys have been disgusted with, and driven from, the study of Geometry, from sheer inability to thoroughly catch the reasoning of Euclid, in the Fifth Proposition of his First Book?

Euclid never attempts to prove that the angles A B C and C B E, or, A C B and B C F, are together equal to two right angles, until he comes to the 27th Proposition of his First Book, by which—in connection with the four following Propositions—he furnishes the proof, by means of alternate angles and parallel lines.

By means of the foregoing very simple proposition, it may be demonstrated, even by pure Geometry, that the three interior angles of every right-angled triangle are together equal to two right angles. (No doubt, the proof can be greatly simplified, by putting a numerical value on a right angle.) Had this proposition occurred to Euclid, he would have escaped the gross blunder into which he has

fallen, in the 32nd Proposition of his First Book, in which he leaves it to be inferred that the three interior angles of *every* plane triangle are together equal to two right angles. Euclid nowhere proves this, nor could he, without the aid of Mathematics: and it was beyond the province of Euclid to introduce Mathematics into his Elements of Plane Geometry. Euclid dealt with Geometry as a distinct science, and thus, was limited to a comparison of line with line, and angle with angle, and it was not within the scope of his object, to put a numerical value either on a line or on an angle. In more modern times, Mathematicians have degraded the science of Geometry, and now broadly assert, that Geometry is not a distinct science, but a mere branch of Mathematics. A writer in the *Athenæum* says:—*Mathematics and logic are the two eyes of exact science.** If we are to speak of "*exact science*" as having two eyes, I maintain that these two eyes are *Geometry and Logic.*

Well, then, to furnish the proof that the three interior angles of any triangle are together equal to two right angles, we must put a mathematical value upon a right angle: it may be 90°, or it may be 60°, or it may be any other number of degrees we please; but having once fixed our value of a right angle, we must abide by it, in all our enquiries into the properties of triangles. By common consent, 90° is adopted as the measure of a right angle, and from the definition of a right angle and Euclid's 13th Proposition, it follows, that if within a circle we draw two diameters, the angles at the centre of the circle are together equal to four right angles.

Referring to the diagram (Fig. 1), because A D is

* See: Review of Wilson's Treatise on Elementary Geometry. *Athenæum*, July 18, 1868.

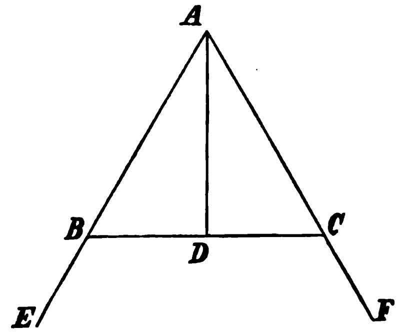
perpendicular to B C, by construction, it follows, that the isosceles triangle A B C is divided into two similar and equal right-angled triangles, A D B and A D C, by the line A D: and it also follows, from the definition of a right angle, that the angles A D B and A D C on each side of the line A D, are right angles.

Let the angle B A C be an angle of any number of degrees, say 60°. Then: $\frac{60^\circ}{2} = 30^\circ =$ the angles D A B and D A C; and A D B and A D C are right angles: and because the three angles of a right-angled triangle are together equal to two right angles, it follows, that the difference between the angles A D B and D A B, or, the difference between the angles A D C and D A C = the angles A B D and A C D. But the angle B A C is an angle of 60°, by hopothesis, and the angle B A C is bisected by the line A D; therefore, the angles D A B and D A C must be angles of 30°. Hence: a right angle — $30^\circ = 90^\circ - 30^\circ = 60^\circ =$ the angles A B D and A C D, and the angles at the base of the triangle A B C are equal; and it follows, that the three angles of the triangle A B C $= (60^\circ + 60^\circ + 60^\circ) = 180^\circ$, and are together equal to two right angles: and because the angles at the base are equal, it also follows, that the triangle A B C is an isosceles triangle.

The following definition (Euclid's 21st definition) is involved in the Fifth Proposition of Euclid's First Book: "*That which has two sides equal is called an isosceles triangle.*" Lexicographers and Mathematicians define an isosceles triangle as a triangle which has *only* two sides equal, and thus exclude from the category of isosceles triangles an equilateral and equiangular triangle. The true definition of an isosceles triangle is, a triangle of which the angles at the base are equal, and this is as

true of an equilateral and equiangular triangle (of which any side may be the base) as of any other isosceles triangle. Hence: Euclid's 21st definition is fallacious, and should be excluded from the list of definitions; or, Mathematicians should prove that a triangle is isosceles only, when two of the sides, and two only, are equal, and this is beyond the capacity of any Mathematician. Wilson proves my case in his 10th Theorem.

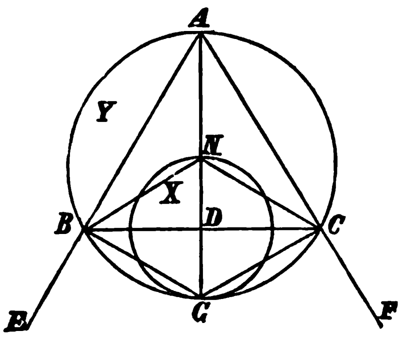
FIG. 2.



In the geometrical figure (Fig. 2), let A B C be an equilateral triangle, by construction. Bisect B C at D, join A D, and produce the sides A B and B C to the points E and F.

The reasoning to prove that the angles at the base of the triangle A B C are equal, and that the three interior angles of the triangle A B C are together equal to two right angles, would be precisely the same, whether we make Fig. 1 or Fig. 2 our datum.

FIG. 3.



In the geometrical figure (Fig. 3), let A B C be an equilateral triangle. by construction. Produce the sides A B and A C to the points E and F. From the points B and C draw straight lines perpendicular to A E and A F, to meet a straight line drawn

from the angle B A C at the point G. It is proved, by Euclid, Prop. 9, Book 1: that the angle B A C is bisected by the line A G: and it is proved by Euclid, Prop. 10, Book 1: that the line B C is bisected in D. Bisect A G at N, and with D as centre and D N as radius, describe the circle X, and about the equilateral triangle A B C, describe the circle Y. Join N B and N C.

Now, Euclid shews us (Prop. 5, Book 4) how to describe a circle about a given triangle, that is, about any triangle, whether equilateral or not. Is it not much easier to describe a circle about a triangle, when the triangle is equilateral, and therefore equiangular? We have simply to draw straight lines bisecting each angle of the triangle and its opposite side. These lines will intersect each other at a point, and this point will be the centre of a circumscribing circle. Euclid need not have waited till his fourth book for the proof of—how to circumscribe a circle about a given triangle. You, reader, may say that it was not Euclid's object to treat of the properties of a circle in his first and second books, and I may grant it; but Euclid could not construct his first three problems without circles, or the arcs of circles; and it will be obvious to every reflective Mathematician, that the definition of a circle is one of the first steps in, and essential to, the study of the Science of Geometry.

Referring to Figure 3, the point N is equi-distant from the angles of the equilateral triangle A B C, and it follows, that N B, N A, and N C are lines of equal length. It also follows, that the triangles A N B, B N C, and C N A are isosceles triangles and therefore have the angles at the base equal. Hence: The three angles A N B, B N C, and C N A at the centre of the circle, are

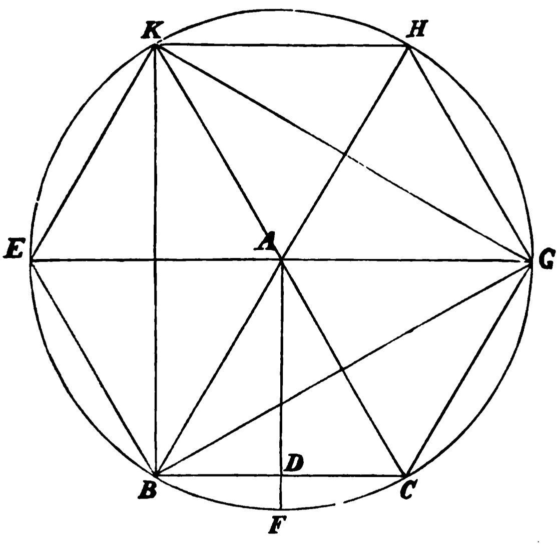
together equal to the sum of the six angles N A B, N B A, N B C, N C B, N C A, and N A C, at the circumference of the circle: and this equation = four right angles, whether the measure of a right angle be 90°, or 60°, or any other number of degrees.

Euclid proves (Prop. 20: Book 3) that the angle B N C at the centre of the circle Y, is double of the angle B A C at the circumference, standing on the same arc B G C. Hence: If the angles of the equilateral triangle A B C be angles of 60°, and therefore together equal to two right angles: the angles A N B, B N C, and C N A at the centre of the circle will be angles of 120°. These angles are subtended by arcs of 120°, that is, these angles are subtended by arcs equal to one-third part of the circumference of the circle, and the three angles A N B, B N C, and C N A, are therefore together equal to four right angles.

Now, referring to the diagram (Fig. 3), by hypothesis, let B A C be an angle of 90° Then: $\frac{\text{angle B A C}}{2} = \frac{90°}{2} =$ 45° = the angles D A B and D A C. But the difference between the angles D A B and A D B, or, the difference between the angles D A C and A D C = the difference between a right angle and the angle D A B or D A C; therefore, 90° — 45° = 45° = the angles A B C and A C B. This would make the angle D A B equal to the angle A B D, which is obviously contradicted by our "*eyesight.*" But, you, reader, may say:—The angle B A C is obviously less than a right angle, and must therefore be less than an angle of 90°. Well, then, by hypothesis, let B A C be an angle of 60°. Can an angle of 60° be subtended by an arc of 120°? Is not the arc

B G C an arc of 120°? My object is to suggest, not to exhaust, and I cannot help thinking that the foregoing facts will be suggestive to every "*reasoning geometrical investigator*" and *reflective mathematician*, that it is necessary to be careful and consistent in our application of Mathematics to the "*exact science*" of Geometry.

FIG. 4.



In the geometrical figure (Fig. 4), let A B C be an equilateral triangle, by construction. Bisect B C in D, and join A D. From the angle A in the triangle A B C, draw a straight line parallel to B C, and therefore at right angles to A D, to meet and terminate in the circumference of the circle at the point E. Produce A D to meet and terminate in the circumference of the circle at the point F. Produce E A, B A, and C A to meet and terminate in the circumference of the circle at the points

G, H, and K, and join K G, B G, and K B. Join B E, E K, K H, H G, and G C.

It is not my intention to write a Treatise on Elementary Geometry. My object will be served, if what I have already published, and what I am about to publish, shall be suggestive to future Geometers. Mr. J. M. Wilson, in his recently published "*Treatise on Elementary Geometry*," professes to remedy Euclid's defects, but his Treatise appears to me to be itself sadly defective. This work gives much that is suggestive in Geometry, and Mr. Wilson introduces new geometrical terms, which no one but a "*recognised*" mathematical authority would have ventured to do. Of some of these terms I shall avail myself, and I frankly admit, that Mr. Wilson's Treatise is a better text-book than Euclid's Elements for "*teaching Geometry to beginners.*"

Mr. Wilson, with reference to his 6th definition, observes: "An angle may be conceived as generated by the revolution of a line A B, starting from some initial position A C, and the angle is the '*quantity of turning*' required to make A C coincide with A B." Thus, refering to the diagram (Fig. 4), we may *conceive* a line starting from the initial position A C and revolving in the direction of F until it coincides with A B. "The '*quantity of turning*' required to make A C coincide with A B" is the angle B A C. We may also conceive a line starting from the initial position E G and revolving in the direction of K, until A G coincides with A F. The "*quantity of turning*" required to make A G coincide with A F is the angle F A G, and F A G is a right angle. Now, produce F A to meet and terminate in the circumference of the circle at a point N. Then: N F will repre-

sent a diameter of the circle, after revolving from the initial position E G round one-fourth part of the circumference of the circle. N F and E G will be diameters of the circle, and the angles made by them at the centre of the circle will be four right angles. It follows, that the angles at the centre of a circle contained by two diameters, whether these diameters are at right angles, or not at right angles, can never be either greater or less than four right angles. Hence: The angles at the centre of the circle contained by the diameters K C and H B, or, H B and E G, are together equal to four right angles.

The circumference of the circle (in Fig. 4) is divided into two semi-circles by the diameter K C or E G. Take the diameter K C. Now, we can conceive the semi-circle K H G C, on one side of K C, to be turned over, and laid on the semi-circle K E B C, on the other side of K C. The semi-circles will "*coincide.*" Not only so, but the three equilateral triangles K A H, H A G, and G A C, on one side of K C, will "*coincide*" in all respects with the three equilateral triangles K A E, E A B, and B A C, on the other side of K C: and similarly, if we take the diameter E G. These are either geometrical demonstrations or they are not. Mathematicians may take their choice. I care not on which their choice may fall.

Now, produce F A to meet and terminate in the circumference of the circle at a point N. Then: N F will be a diameter of the circle. Conceive the semi-circle on one side of N F to be turned over and laid on the semi-circle on the other side of N F. The semi-circles will "*coincide.*" Not only so, but the parallelogram H G C A will "*coincide*" with the parallelogram K E B A: but, the equilateral triangles A B C and A K H will coincide with nothing. It is no doubt true,

that the equilateral triangle A B C is divided by the line AD into two similar and equal right-angled triangles; and it is also true, that if the triangle A D C be turned over and laid on the triangle A D B, the triangles A D C and A D B will "*coincide*" in all respects. But, I maintain that this is defective as a geometrical demonstration; and much more is the 4th Proposition of Euclid's First Book defective as a geometrical demonstration. Although some "*recognised Mathematicians*" maintain that Euclid's demonstration of the 4th Proposition of his First Book is perfect; many modern Geometers, besides Cooley, have expressed their doubts on this point; and it appears to me, that no merely mechanical operation can ever be imported into a strictly geometrical demonstration. In Geometry a line is defined as having length without breadth; or as Mr. Wilson puts it, "a *line* has position, length and direction, but is not considered as having breadth or thickness." Hence, although we can *conceive* that two triangles may "*coincide*" in all respects, we can never *geometrically* apply one triangle to another.

Mr. Wilson in his "*Treatise on Elementary Geometry*" (Section 3, Plane Triangles) gives the following definition of an isosceles triangle:—"A triangle is called *isosceles* when two of its sides are equal." In Theorem 9 he proves, that "*an isosceles triangle has the angles at its base equal.*" In Theorem 10 he gives the converse of this proposition, and proves, that "*a triangle which has the angles at its base equal is isosceles.*" Well, then, is not an equilateral and equiangular triangle *isosceles* on Mr. Wilson's own shewing?

Now, in Fig. 4, A B C is the generating figure of the diagram, and is an equilateral and equiangular triangle,

by construction; and the three sides have different directions. The triangles A B E, A E K, A K H, A H G, and A G C, are similar and equal triangles, and are all similar and equal to the triangle A B C, by construction; and B A C, C A G, G A H, H A K, K A E, and E A B, the six angles of these triangles at the centre of the circle, are together equal to four right angles, neither more nor less. The triangle K B G is similar but not equal to the triangle A B C. The angles B K G and B A C are equal; that is to say, if we *conceive* the angle B A C to be applied to, or laid upon, the angle B K G, the side A B of the triangle A B C will fall on the side K B of the triangle K B G; and the side A C of the triangle A B C will fall on the side K G of the triangle K B G; and it follows, that the angles B K G and B A C are equal, and the triangles K B G and A B C similar triangles. But, the triangles K B G and A B C do not "*coincide*" in all respects; for, if we conceive an angle of one of these triangles to be applied to an angle of the other by "*superposition*"—and it is conceivable that any angle of the one may be applied to any angle of the other—then, if so applied, although the angles at the base of both triangles will be equal, is it not self-evident that the bases themselves will be unequal?

Now, equal angles at the centre of a circle are subtended by equal arcs: and in Fig. 4, B A C, C A G, G A H, H A K, K A E, and E A B, the six equal angles at the centre of the circle, are subtended by arcs equal to one-sixth part of the circumference of the circle; and having fixed 90° as the measure of a right angle, these angles are angles of 60°, and equal to ⅔ of a right angle: and the three angles of each of the triangles A B C, A C G,

A G H, A H K, A K E, and A E B are together equal to two right angles. But, two angles *only* of the triangles A B C, &c., touch the circumference of the circle, while all the angles of the triangle K B G touch the circumference of the circle: and because the angle B A C is *geometrically* equal to the angles of the equilateral and equiangular triangle K B G, it follows, that the angles B K G, K G B, and G B K are angles of 60°, and therefore equal to ⅔ of a right angle. But these angles are subtended by arcs equal to one-third part of the circumference of the circle. Hence: Equal angles may be subtended by unequal arcs, in the same circle, depending upon, whether the angles are angles at the centre, or angles at the circumference of the circle. You, reader, may say, this is apparently inconsisent with what you have stated in the first paragraph of this Address. Granted! Instead of the words "*which is absurd*," which appear in that paragraph, I might, and perhaps should have said, *which involve an absurdity.* What is the absurdity involved? It is this. If it were true, that Geometry is a mere branch of Mathematics, as Mathematicians say, they would be able to prove that angles of 60° at the circumference of a circle, and angles of 60° at the centre of the same circle, are subtended by equal arcs, and so, "*upset*" Euclid's Theorem: Prop 20: Book 3. Having fixed the value of a right angle at 90°, can an angle of 60° be subtended by an arc either greater or less than one-sixth part of the circumference of a circle? Can an angle of 30° be subtended by an arc either greater or less than one-twelfth part of the circumference of a circle?

Well, then, the angle E A B is an angle of 60° and equal to ⅔ of a right angle; and the angle B A F is an

angle of 30° and equal to $\frac{1}{3}$ of a right angle; and the angles E A B and B A F are together equal to the right angle E A F. Take the triangle E A B, bisect the angle A and its subtending chord E B, and produce the bisecting line to meet and terminate in the circumference of the circle; and similarly, take the triangle A G C, bisect the angle A and its subtending chord G C, and produce the bisecting line to meet and terminate in the circumference of the circle: and so of the remaining similar and equal equilateral triangles A H G, A K H, and A E K. In this way we should get twelve angles of 30° at the centre of the circle, and the sum of these twelve angles = four right angles = 360°. Would not these angles be subtended by arcs of 30°? Is not the sine of any arc half the chord of twice that arc? Who can dispute it? Hence: The sum of these twelve angles at the centre of the circle, is equal to the sum of the three angles K A B, B A G, and G A K at the centre of the circle, that is = 360°, or four right angles.

The triangles A B G, A B K, and A G K, are similar and equal triangles, and one angle of each is an angle at the centre of the circle. Take one of these triangles, say the triangle A B G. It is not necessary, for the purpose of my argument, to refer to more than one, since what is true of one, is true of all.

Now, I frankly admit that Euclid proves (Prop. 20, Book 3,) that in the triangle A B G, the angle B A G, at the centre of the circle, is the double of the angle B K G at the circumference, standing on the same arc; and *apparently* this makes an angle of 60° to be subtended by an arc of 120°. How is this to be explained? Nothing easier! With K as centre, and K B or K G as radius,

describe another circle. Then: the angle B K G will not only be subtended by the arc B C G, but will also be subtended by another arc, *smaller than,* and therefore within the arc B C G. In fact, it will be subtended by an arc equal to one sixth part of the circumference of a circle, of which K B and K G are radii. Hence: two circles are necessarily involved in the "*comparison of triangles*" and angles.

Mr. Wilson, in the preface to his "*Treatise on Elementary Geometry,* observes:—" Unsuggestiveness is a great fault in a text-book. Euclid places all his theorems and problems on a level, without giving prominence to the master-theorems, or clearly indicating the master-methods. He has not, nor could he be expected to have, the modern felicity of nomenclature. The very names of *superposition, locus, intersection of loci, projection, comparison of triangles,* do not occur in his treatise. Hence there already exists a wide gulf between the form in which Euclid is read, and that in which he is generally taught. Unquestionably the best teachers depart largely from his words, and even from his methods. That is, they use the work of Euclid, but they would teach better without it. And this is especially true of the application to problems. Everybody recollects, even if he have not the daily experience, how unavailable for problems a boy's knowledge of Euclid generally is. Yet this is the true test of geometrical knowledge; and problems and original work ought to occupy a much larger share of a boy's time than they do at present."

There are many things in Mr. Wilson's "*Treatise on Elementary Geometry*" that ought to have suggested to him new methods of examining the properties of angles

and triangles. Although Mr. Wilson finds fault with the unsuggestiveness of Euclid's Elements of Geometry, his own Treatise appears to have failed to be suggestive of anything to himself. The Editor of the *Athenæum* failed to perceive that the writer of the review on Mr. Wilson's work, which appeared in that Journal of July 18, 1868, is —*geometrically speaking*—" *blind as a bat.*"

After spending about three weeks at Harrogate, the day I returned home I received a letter, of which the following is a copy:—

11, LYNDHURST ROAD,
PECKHAM, SURREY, S.E.,
19*th October*, 1869.

DEAR SIR,

I beg to acknowledge the receipt of your brochure. I have been very ill since writing to you, otherwise I would certainly have written before to acknowledge your favour.

I am this evening perusing the "Whitworth" correspondence, and I do think he sufficiently and painstakingly refutes you.

I am, Sir,
Yours very truly,
R. F. GLAISTER.

JAMES SMITH, ESQ.

I had written so far when I received the following communications:—

R. F. GLAISTER, ESQ., *to* JAMES SMITH.

11, LYNDHURST ROAD,
PECKHAM, SURREY, S.E.,
Monday, *October* 25*th*, 1869.

DEAR SIR,

I send you on the five sheets within enclosed a proof

that π is 3·14159, &c., at any rate *not* $3\frac{1}{8}$. I most sincerely hope you will find it satisfactory, and that it will convince you that it is *your* π that is "*the mockery, delusion, and snare.*" It only requires a knowledge of Euclid's first six books to comprehend it.

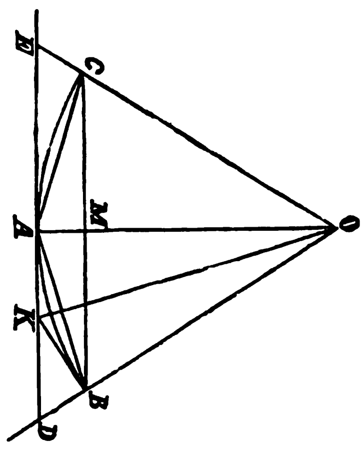
I am the more particular in writing this since my neglect to reply to your polite Letter, and accompanying present of a book, justly surprised you. I trust you will find the papers to be legible; but neither my sight nor hands are in a very good condition just yet.

With great admiration for your energy and courage, I am sure you are grievously wrong in your mathematics, and am much disposed to think you are one of the many who fruitlessly endeavour to make natural talent (valuable enough in its way) a complete substitute for study. It is obvious to any one who reads your publications, that you are almost totally ignorant of anything beyond Euclid. I say this with the utmost respect for your age and position, but flattery is not desirable in mathematical investigations. Professor Whitworth would not allow your Euclid. I completely endorse all he says as to π, series, decimals, &c.

I am, Sir,

Yours truly,

R. F. GLAISTER.



Let 2π represent the circumference of a circle whose radius is unity. We may always make the radius our unit. If it be an inch, then our circle will be 2π inches round; if it be a million miles, then our circle will be $2\pi \times 1000000 \times 5280 \times 12$ inches round, since there are 5280×12 inches in 1 mile. I am thus prolix in order to show you I am not at all taking away from the generality of my demonstration by making radius = unity. Our object is to

find π. I shall suppose the very terms sine, cosine, &c., to be unknown, and treat the problem as a purely geometrico-algebraical one. I think I shall make myself more clear to you by so doing, than by employing trigonometrical terms. Let B C, D E, be the sides respectively of polygons of n sides inscribed within, and circumscribed about the circle, whose radius is the line O A = O B = O C. Arc B A C = $\frac{2\pi}{n}$ that is to say, that if O A represent the unit of linear measurement, the arc B C will be $= \frac{2\pi}{n}$ such units. Supposing we had not made the connection that O A was to be *the* unit, we must put arc B C $= \frac{2\pi}{n} \cdot$ O A. Now, carefully observe—this is pure *linear* measurement. I have here nothing to do with *degrees* of arcs or of angles, nor shall I use them in my demonstration. Draw tangents to the circle at B and A, and complete the figure as shown in the diagram. Obviously, A K = K B, angle K O B = angle K O A. B A, A C, will be sides of polygon of 2 n sides *in*scribed; B K, K A, half-sides of polygon of 2 n sides *circum*scribed.

Let I_p and E_p mean the areas of the interior and exterior (the inscribed and circumscribed) polygons of p sides.

Now $I_n : I_{2n}$: : triangle O B C : 2 × triangle O B A
$\therefore$ O B M : O B A.
: : line O M : line O A.
I_2 : E_n : : triangle O B A : triangle O D A.
$\therefore$ line O B : line O D.
: : line O M : line O A.
Therefore $I_n : I_{2n} :: I_{2n} : E_n \therefore I_{2n} = \sqrt{I_n E_n}$.

Now distinctly and clearly keep in view the meaning of the last equation, it has nothing to do with actual geometrical space.

I means the *number* of squares (side = unity), contained in a certain polygon. I here use *number* in an extended sense, so as to include fractions. For instance, if a foot be taken as unity-lineal, a square whose side is a foot will be unity-superficial; a square whose side is an inch, will contain $\frac{1}{144}$ superficial units, $\frac{1}{144}$ being a number.

My last equation, therefore, in *words*, is—

The number I_{2n} = the square root of the number made by multiplying the number I_n by the number E_n, a purely arithmetical equation. For algebra is merely *literal* and *general* arithmetic, at least, so far as we shall here have occasion for algrebra.

Again by Euclid (Prop. VI. Book 3), A K : K D : : A O : O C : : MO : OB, and AK : A K + KD (= AD) :: M O : MO + O B; or, AK : A D : : M O : M O + O A (since O A = O B) : : $I_n : I_n + I_{2n}$.

Also $E_{2n} : E_n :: 2 \times$ triangle A O K : triangle A O D.

$$\therefore 2\,AK : AD :: 2\,I_n : I_n + I_{2n}$$

Whence $E_{2n} = \dfrac{2\,I_n\,E_n}{I_n + I_n}$

We have now the two formulas following—

$$(\alpha)\ I_{2n} = \sqrt{I_n\,E_n} : (\beta)\ E_{2n} = \frac{2\,I_{2n}^2}{I_n + I_{2n}}$$

bear in mind that (β) is a formula of numbers just like (α) as we explained before.

So having given the areas of a circumscribed and inscribed polygon of any number of sides, we can from (α) and β find the areas of those of double the number of sides.

Now $I_4 = 2$, I_4 being the number of superficial units in the inscribed square.

$E = 4$. $I_8 = 2\sqrt{2}$. $E_8 = 8(\sqrt{2} - 1)$. I shall not enter into the details of the calculation, but it will certainly be found that $I_{32} = 3{\cdot}1214$, &c., (*interminable decimals*) and $I_{64} = 3{\cdot}1365$, &c., (*interminable decimals*)—Now your value of π, *i.e.*, 3·125, makes the numerical measure of the area of the circle ($= \pi \times 1^2 = \pi \times 1 \times 1 = \pi$ (radius unity) $= 3{\cdot}125$ superficial units, viz., *less* than the measure of its inscribed polygon of 64 sides.

I shall be happy to explain this demonstration further to you should it strike you as too concise, but I think I have said enough. It is not my own, but a modification of one given by a "*recognised*" mathematician in 1837. If you continue the process, which is certainly laborious enough in all conscience, you will find at length,

$I_{16384} = 3{\cdot}1415925$, &c.

$E_{16384} = 3{\cdot}1415927$, &c.

Now the circle lies between these, true therefore to 6 places of decimals.

$$\pi = 3 \cdot 141592.$$

and so with time, life, strength and industry, you could find π to *billions* of decimals, but it is *incommensurable*, *interminable*, and nevertheless *finite* and *determinate*

I might have expanded this demonstration, and should so have done had I been writing to a mere beginner, but I think with your knowledge of algebra and geometry, it will be clear enough to you.*

Monday, *October 25th*, 1869.

11, LYNDHURST ROAD,
PECKHAM, SURREY, S.E.,
October 25th, 1869.

DEAR SIR,

In addition to the Letter and papers which you will receive contemporaneously with this, I subjoin the following remarks. I would recommend you to read the article "Quadrature of the Circle," *Penny Cyclopædia*; also "Quadrature of the Circle," page 250, *Penny Magazine*, Sept. 29th, 1832. I think then you will see it is not so easy as—

$$3 \cdot 125 \text{ or } \frac{3125}{1000}$$
$$\text{or } 3\tfrac{1}{8}$$
$$\text{or } \frac{25}{8}$$
$$\text{or } \frac{5^2}{2^3}$$

Here are a few infinite convergent series for π (my π, *the* π, the recognised π!) or its powers — $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}$ *ad infinitum*.

Or, $\pi = \sqrt{6}\ \sqrt{}$ (the series above) — $\frac{\pi}{8} = 1 + \frac{1}{3^2} \quad \frac{1}{5^2} + \frac{1}{7^2}$ &c., *ad infinitum*.

$$\frac{\pi}{4} = \left(\frac{1}{2} + \frac{1}{3}\right) - \frac{1}{3}\left(\frac{1}{2^3} + \frac{1}{3^3}\right) + \frac{1}{5}\left(\frac{1}{2^5} + \frac{1}{3^5}\right) - \frac{1}{7}\left(\frac{1}{2^7} + \frac{1}{3^7}\right) \text{ \&c.}$$

and now follow the *most* convergent of all, viz:—

* Mr. Glaister revised this Paper.

$$\frac{\pi}{4} = 4 \left\{ \frac{1}{5} - \frac{1}{3} \cdot \frac{1}{5^3} + \frac{1}{5} \; \frac{1}{5^5} - \frac{1}{7} \cdot \frac{1}{5^7} + \text{\&c, } \textit{ad infinitum} \right\}, \quad \ldots$$

$$\left\{ \frac{1}{239} - \frac{1}{3} \cdot \frac{1}{239^3} + \frac{1}{5} \cdot \frac{1}{239^5} \text{ \&c., } \textit{ad infinitum.} \right\}$$

and all these if summed up will give to 36 places of decimals (if you want so many!) *Orthodox* π =

3·141592653589793238462643383279502884.

π has been carried to 128 places of decimals. To demonstrate these series requires the aid of the Differential Calculus.

π can also be expressed as a fraction with an infinite numerator and denominator. There now! Or as a continued fraction. But enough of π.

I hope I have now atoned for my inevitable, though *prima facie* inexcusable silence.

Yours truly,

R. F. GLAISTER.

J. SMITH, ESQ.

LYNDHURST ROAD,
PECKHAM, SURREY, S.E.,
October 25th, 1869.

DEAR SIR,

I made a slight oversight, a mere verbal one, in my former letter of this date. I should have said, you made the *area* of the circle less than the *area* of the 64 polygon, and that 3·1365485 was the *area* to radius unity of such a polygon. The perimeter would be about 3·13, &c. You *do* however make the circumferenee of the circle less than that of a polygon of 18 sides, which is about 3·1256, &c.

In the limit, however, when the number of sides is infinite, the ultimate polygon in the circle is = to $\pi \times$ radius, or π.

But whilst the number of sides is finite, the area of the polygon is NOT = semi-perimeter × radius. The area of a polygon of n sides inscribed in a circle is $= \frac{n}{2} \cdot \text{Sin} \left(\frac{360^\circ}{n}\right)$ (radius unity) the perimeter $2\, n \text{ Sin} \left(\frac{180^\circ}{n}\right)$

I beg you to consider this as part of my Letter. I fell into the oversight from copying and modifying the proof given by another person. I dare say you would have seen this, but I prefer pointing out my own slip.

Yours truly,

R. F. GLAISTER.

J. SMITH, ESQ.

The following was my reply to these communications :—

BARKELEY HOUSE, SEAFORTH,
27th October, 1869.

DEAR SIR,

I am in receipt of your three Letters—all dated October 25th, 1869—and the elaborate Paper enclosed in one of them. I have carefully perused, and thoroughly understand these communications.

My π is $\frac{5^2}{2^3} = \frac{25}{8} = \frac{5^5}{1000} = 3{\cdot}125$, making 1 to 3·125 the ratio of diameter to circumference in every circle.

Hence :

$\pi\,(\pi^2)^2 = \pi^5$, and this equation = area of a circle when the radius is π^2.

Proof:

If r denote the radius of a circle, $2\,\pi\,(r)$ = circumference of the circle and $\pi\,(r^2)$ = area of the circle, whatever be the value of π : and $2\,\pi\,(\pi^2) = 6{\cdot}25 \times 9{\cdot}765625 = 61{\cdot}03515625$ = circumference of a circle, when radius $= \pi^2$. But, $\frac{\pi^2}{2} = \frac{3{\cdot}125^2}{2} = \frac{9{\cdot}765625}{2} = 4{\cdot}8828125$ = semi-radius of a circle, when radius $= \pi^2$: and circumference multiplied by semi-radius = area in every circle, whatever be the value of π. Hence: $61{\cdot}03515625 \times 4{\cdot}8828125 = 298{\cdot}023223876953125$ = area of the circle when radius $= \pi^2$; or in other words, $61{\cdot}03515625 \times 4{\cdot}8828125 = 3{\cdot}125 \times 9{\cdot}765625^2$ and this equation = area of a circle, when radius $= \pi^2 = 3{\cdot}125^2 = 9{\cdot}765625$.

I am preparing a work for the press, in which I shall give your communications, and my reply : and it may probably remain for the Mathematicians of another generation to decide, whether mine is the true π, or—as you term it—"*a mockery, delusion, and the snare.*"

You must not think me discourteous, if I do not reply to any more of your communications.

I am, dear Sir,
Yours truly,
JAMES SMITH.

R. F. GLAISTER, ESQ.

11, LYNDHURST ROAD,
PECKHAM, SURREY, S.E.,
October 27th, 1869.

MY DEAR SIR,

I, by this post, send you a revised proof of the true value of π. It is, in fact, the old proof put in a neater and more Euclidian form, and I think will please you. I wish and sincerely *hope* you may be induced in time to see the lamentable paralogisms into which you have fallen. The excessive complication of your diagrams totally precludes any one from completely keeping up with your geometrical demonstrations, step by step, but it is easy to see you are wrong.

After I find π, I then shew you how to find a sine without the aid of a series. Mind I do not say Professor Whitworth's series is *in*correct, far from it. I then finish with a few remarks on polygons.

I have several editions of "Trigonometry" of the Rev. Professor Hall, of King's College, London, 5th wrangler of 1824, a most experienced teacher, and whose pupils have taken high Cambridge honors; so I took a few leaves from the 2nd edition, 1836, and send them to you, they save my writing, and of course will be clearer to you. I fully endorse his demonstrations. But I think Whitworth has most completely shown that you make a certain arc = to its chord. See Figure, page 38 of your large book, Whitworth's Letter, bearing date November 28th, 1868.

I much regret the terms in which you speak of Augustus

De Morgan, who is, I think, since the death of Sir W. Rowan Hamilton, and the recent death of Count Libri, unquestionably the first mathematician in Europe.

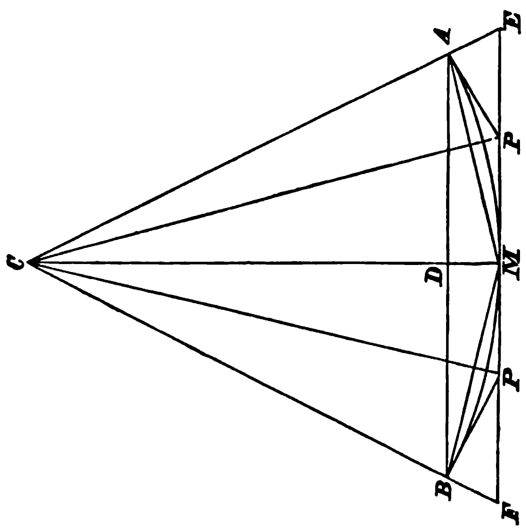
I am sorry for the style in which I first addressed you. I was not then aware of your age, not having read the 'Wilson' part of your book, or as Wilson says of himself, I would have adopted a more courteous tone. My object is as you say TRUTH, and I should be glad if I could convince you, you are grossly wrong in your Mathematics.

Yours truly,

JAMES SMITH, ESQ. R. F. GLAISTER.

P.S.—I think I have now said all I *can* say on π, and its kindred subjects.

Having given the area of a regular polygon inscribed in a given circle, and also the area of a similar polygon circumscribed about the said circle, to find the areas of regular inscribed and circumscribed polygons each of double the number of sides.



Let A B be the side of the inscribed polygon, and E F parallel to A B that of the similar polygon described about the circle. C the centre; if the chord and the tangents A P, B P be drawn, chord A M shall be the side of the

inscribed polygon of double the number of sides, and P P or 2 P M the side of the similar circumscribed polygon. Put A for area of polygon (side A B), B for area of circumscribed polygon, *a* for area of polygon, of which A M is a side, *b* for the area of the similar circumscribed polygon. Then A and B are by hypothesis known, and it is required to find *a* and *b*.

1. The triangles A C D, M C A, have a common vertex A, and are by Euclid, (Book VI., Prop. 1,) to one another as their bases C D, C M; besides these triangles are to one another as the polygons of which they form like parts; therefore $A : a :: CD : CM$. The triangles C A M, E M C, which have a common vertex, are to each other as bases C A, C E, also as polygons *a* and B, of which they the said triangles are like parts, $\therefore a : B :: CA : CE$. And $\because$ D A is parallel to M E, $CD : CM :: CA : CE \therefore A : a :: a : B \therefore a = \sqrt{A \times B}$.

2. The triangles C P M, C P E, which have the same altitude C M, are to one another as their bases (Euclid VI. 1), or as PM : PE. But as CP bisects the angle ECM, $PM : PE :: CM : CE$ (Euclid VI. 3), or $:: CD : CA$ or $:: A : a, \therefore CPM + CPE$ (which is C M E) $: CPM : A + a : A$ and $CME : 2CPM :: A + a : 2A$. But C M E and 2 C P M or C M P A are to one another as the polygons B and *b* of which they are like parts. $\therefore A + a : 2A :: B : b$. Therefore $b = \dfrac{2 \times A \times B}{A + a}$.

And these formulæ will give amongst other things (the radius being unity).

Area of inscribed polygon of 64 sides = 3·13654, &c.
Area of ....16384 sides = 3·1415925, &c.
..................................32768 sides = 3·1415126, &c.

the last line differing only at the 7th decimal place, or by about $\dfrac{1}{10,000,000}$ which, if the radius were 1000 miles long, would be about 6 inches!!! This may fairly be taken as approximate to π. Your principles would make the area 3·125, which *must*, if the preceding be correct, be erroneous; and where is the flaw in the above reasoning? It is that of the late Professor Wallace of Edinburgh, and is to be found

in the *Encyclopædia Britannica* (Edition 7th, page 457, vol. Xth) article "Geometry." I prefer its form to the form of demonstration I sent you the other day, it is substantially the same.

I will now proceed to find the general expression for areas and perimeters of regular polygons.

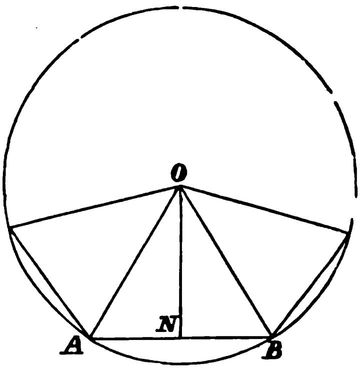
(If R and r be the radii of the circumscribed and inscribed circles, and a, b, c, be the sides, then—

$$\frac{a+b+}{2}\, r = \text{area} = \frac{a\,b\,c}{4\,\text{R}};$$

$$\therefore \text{R}\, r = \frac{a\,b\,c}{2\,(a+b+c)}.$$

Find the area of an equilateral polygon of (n) sides inscribed in a given circle.

A B one of the sides, O the centre of the circle. Draw O N perpendicular to A B, and join O A, O B; then, by drawing lines from O to the angular points of the polygon, it may be divided into as many equal triangles as the figure has sides, and the area of the polygon will equal the sum of these triangles.



$\therefore$ Polygon $= n$. triangle

$$\text{O A B} = \frac{n\,\text{A B.O N}}{2} = n\,\text{O N.A N, for A B} = 2\,\text{A N}.$$

Now the angles at the point O = 4 right angles = 2π;

$$\therefore \text{A O B} = \frac{1^{\text{th}}}{n} \text{ of the angles at O} = \frac{2\,\pi}{n};$$

$$\therefore \text{A O N} = \tfrac{1}{2} \text{ angle A O B} = \frac{\pi}{n}. \quad \text{Let A O = rad.} = r.$$

$$\text{Then, } \frac{\text{A N}}{\text{A O}} = \sin.\ \text{A O N, or A N} = r.\sin.\ \frac{\pi}{n}.$$

$$\frac{\text{O N}}{\text{A O}} = \cos.\ \text{A O N, or O N} = r.\cos.\ \frac{\pi}{n}.$$

$$\therefore \text{area} = n\,r^2.\sin.\ \frac{\pi}{n}.\cos.\ \frac{\pi}{n} = \frac{n\,r^2}{2}.\sin.\ \left(\frac{2\,\pi}{n}\right)."$$

The perimeter $= n \times AB = 2n\,AN = 2nr.\sin.\frac{\pi}{n}.$)

"To rad. = 1, area = perimeter $\times$ cos. $\frac{\pi}{n} \div 2$, so perimeter is always greater than area in numerical measure."

(Find the area of a regular octagon inscribed in a circle.

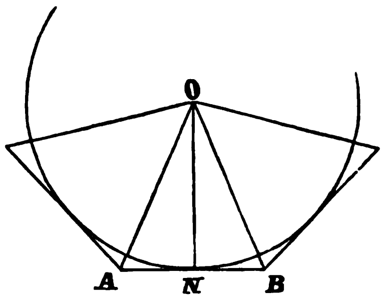
Here $n = 8$, $\frac{2}{n} = \frac{360}{8} = 45.$

Area $= \frac{8r^2}{2}.\sin.45 = 4r^2\frac{1}{\sqrt{2}} = 2\sqrt{2}.r^2.$

$\therefore$ area : (rad)^2 :: $2\sqrt{2} : 1 :: \sqrt{8} : 1.$

Find the area of a regular polygon of n sides described about a circle.

A B one of the sides, O N the radius, and therefore perpendicular to A B at N.



Then polygon $= n$. triangle O A B $= \frac{n.ON.AB}{2}$

$= n.AN.ON.$

Let $ON = r$, also angle $AOB = \frac{2\pi}{n}$, and angle $AON = \frac{\pi}{n}.$

Then $\frac{AN}{ON} = \tan.AON = \tan.\frac{\pi}{n};$

$\therefore AN = r.\tan.\frac{\pi}{n},$

and area $= nr^2.\tan.\frac{\pi}{n}.$

The perimeter $= n\,AB = 2nr.\tan.\frac{\pi}{n}.$)

"To rad. unity, perimeter = 2 area in numerical measure."

(Hence we may compare the areas of the inscribed and circumscribed polygons.

XXXV.

$$\frac{\text{Area of inscribed polygon}}{\text{Area of circumscribed polygon}} = \frac{\sin.\frac{\pi}{n}\,.\cos.\frac{\pi}{n}}{\tan.\frac{\pi}{n}} = \frac{\cos.^{2}\frac{\pi}{n}}{1}$$

Now as (n) increases, $\frac{\pi}{n}$ decreases; and when (n) becomes infinitely great, $\frac{\pi}{n}$ is infinitely small, and $\cos.\frac{\pi}{n} = 1$.

Therefore, when the number of sides is infinitely great, the inscribed and circumscribed polygons are equal. But the circle includes one of these areas, and is included by the other; when, therefore, the two limits become equal, it is also equal to either of them; or the area of a circle is equal to that of the inscribed polygon, whose sides are infinite in number.

Now the area of inscribed polygon $= \frac{n r^2}{2}\,.\sin.\frac{2\pi}{n} =$

$\pi r^2 \times \dfrac{\sin.\left(\frac{2\pi}{n}\right)}{\left(\frac{2\pi}{n}\right)}$; and when n is infinite, $\frac{2\pi}{n}$ and $\sin.\frac{2\pi}{n}$ are each $= 0$; we must, therefore, find the ratio between the sine and arc at the time they vanish.

But arc $>$ sine $<$ tangent, or A $>$ sin. A $<$ tan. A, or the arc lies between the sine and tangent.

$$\text{But } \frac{\sin.\text{A}}{\tan.\text{A}} = \frac{\cos.\text{A}}{1} = \frac{1}{1} \text{ if A} = 0.$$

or the sine and tangent are ultimately equal. Hence the arc which lies between them is ultimately equal to either of them.

$$\therefore \frac{\sin.\frac{2\pi}{n}}{\frac{2\pi}{n}} = 1, \text{ when } n = \text{infinity.}$$

Therefore the area of a polygon of an infinite number of sides inscribed in a circle $= \pi r^2$, which is therefore the area of the circle.

Hence, also, the perimeter of the circle $= 2nr\cdot\sin.\frac{\pi}{n} = 2\pi r.$

$\dfrac{\sin.\left(\frac{\pi}{n}\right)}{\left(\frac{\pi}{n}\right)} = 2\pi r$, when n is infinite.)

"See also De Morgan's Trigonometry, page 10, for an elementary mode of finding π."

"The sin. $1'$ differs but little from the arc of $1'$ in length, and the error arising from this assumption is less than the cube of the decimal which represents such arc."

(Now the length of the circumference of the circle, whose radius is unity, $= 2 \times (3\cdot14159)$;* therefore, since the number of minutes $= 360 \times 60$, the length of a minute $= 2 \times \dfrac{(3\cdot1415926)}{360 \times 60} = \dfrac{3\cdot1415926}{360 \times 30}$

$$= \frac{\cdot031415926}{108} = \cdot000290888, \&c.;$$

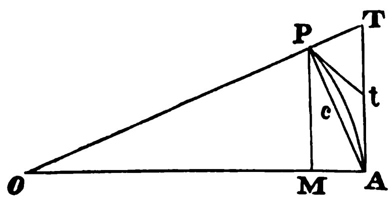
$\therefore$ Sin. $1' = \cdot000290888$, &c.

Cos. $1' = \sqrt{1 - \sin.^2 1'} = \cdot99999995$, &c.

The limit of the error committed in assuming the sin. $1'$ = the length of the arc of $1'$, may be thus found.

Lemma: The arc (assumed less than a quadrant) is less than the tangent, but greater than the sine.

Let A P be an arc $= (a)$, A*c*P its chord, AT a tangent, and P M the sine; P*t*, a tangent at P; therefore P*t* and A*t* are equal, and each is the tangent $\frac{a}{2}$. Euclid, Book III. 36.



Then, manifestly, the arc A P is $\angle$ At + Pt, but $\urcorner$ chord AcP; $\therefore \urcorner$ P M,

$$\text{or } a \angle 2 \tan. \frac{a}{2}, \urcorner \sin. a.$$

$$\text{But } \tan. a = \frac{2 \tan. \frac{a}{2}}{1 - \tan^2. \frac{a}{2}}; \therefore \urcorner 2 \tan. \frac{a}{2}$$

$$\therefore \text{arc } a \angle \tan. a. \urcorner \sin. a.$$

Since, by the lemma, we have seen that

$$\tan. a, \text{ or } \frac{\sin. a}{\cos. a} \urcorner a;$$

* This I have shown in my modification of Wallace's demonstration.

$\therefore$ sin. $a \mathrel{7} a$. cos. a, and $\therefore$ *à fortiori*, $\mathrel{7} a$. cos.^{2} a.
For cos. a is a fraction $\angle$ 1, and therefore diminishes the quantity into which it is multiplied ;

$\therefore$ sin. $a \mathrel{7} a - a$ sin.^{2} a, for cos. $^2a = 1 -$ sin. ^2a;

$\therefore a$. sin. $^2a \mathrel{7} a -$ sin. a.

But sin. $a \angle a$; therefore $a \times$ sin. $^2a \angle a^3$;

$\therefore$ *à fortiori*, $a^3 \mathrel{7} a -$ sin. a, or $a -$ sin. $a \mathrel{7} a^3$.

Hence the error is less than (·000290888)^3, or ·000000000008 nearly, where the first significant figure is the 12th from the decimal point.

The computation of sin. 1′ has proceeded upon the principle that the radius is unity ; but if it be wished to find its numerical value to a radius r, we have merely to multiply by a radius (r) the value of sin. 1′ calculated to a radius unity.

The sin. 1′ has been found by assuming that the length of the arc 1′ may be put equal to its sine ; the accuracy of the result may be tried by obtaining the value by a different process.

We have seen that sin. $5a = 5$ sin. $a - 20$ sin. $^3a + 16$ sin. ^5a, and sin. 15 is known $= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$. Assume therefore $5a = 15$, and therefore $a = 3$, and we shall be able to find sin. 3° ; but from sin. $3a = 3$ sin. $a - 4$ sin. ^3a we may calculate sin. $a =$ sin. 1° ; and again, by trisection, we find sin. 20′, then sin. 10′, and sin. 2′ and lastly sin. 1′.)

"The perimeter of polygons will be larger in the numerical magnitude (radius unity) than the areas. As area = perimeter $\times$ cos. $\dfrac{\pi}{n} \div 2$, and as co-sines and sines are always less than unity, literally, this means the number expressing the length of the perimeter (radius 1) multiplied by cos. $\dfrac{\pi}{n}$ (which is less than 1), and divided by 2 = the number which expresses the number of squares and parts of squares on the linear unit which is contained in the area of the polygon."

"Thus the perimeter of a polygon of 18 sides is 6·251, radius being unity, its area $\dfrac{6·251}{2} \times ·98 = 3·125 \times ·98 = 3·06250$.

" N.B.—There is a great chance in these things of confounding half-circumferences and circumferences in the calculations. I have, I think, confounded in this way before in my correspondence with you, so has Mr. Skeat whose Letter in your Treatise did not catch my eye till this morning."

Finally: Your π makes the circumference of a circle (radius = unity) 6·25, less than the circumference of the 18 sided polygon, and makes the area 3·125, or less than the area of the 64 sided inscribed polygon.

R. F. GLAISTER.

" P.S.—The pages sent herewith are taken from the last edition of Professor Hall's (of King's College, London), Trigonometry, 2nd Edition, page 36." *(See paragraphs in brackets.)**

11, LYNDHURST ROAD,
PECKHAM, SURREY,
October 28th, 1869.
9 a.m.

DEAR SIR,

Thanks for yours of the 27th. You will, when this reaches you, have received further communications from me.

I have *nothing more* to say on the matter. As for the " mathematicians of another generation,"—"*Magna est veritas, et prevalebit.*"

In justice to Mr. Skeat, I must say I find it is I who mistook his proof. He takes diameter = unity, I took radius = unity. Hence our apparent discrepancy. *Really* we agree.

I would cordially advise you to peruse " Trigonometry " in a valuable Cambridge text-book, " Goodwin's Elementary Course of Mathematics."

I remain, yours truly,
R. F. GLAISTER.

J. SMITH. ESQ.

* Mr. Glaister revised this Paper, and his Letters, of October 25th, 27th, and 28th.

11, Lyndhurst Road,
Peckham, S.E.,
October 28th, 1869.

My Dear Sir,

A few words for the last. You say that you may publish my communications to you. Be it so. But pray publish *all* the mathematical part including that I sent you yesterday, my revised proof as I may term it; publish *every*thing I have sent in fact.

I write this not as really doubting your honour and straightforwardness, which I am disposed to believe are very strong in you, but simply as a safeguard; as, years ago, a 'man of education,' I will not call him gentleman, with whom I had a correspondence on a non-quadrature subject, chose to pick the bits out of my letters which suited him, and suppress those which did not.*

Yours truly,

R. F. Glaister.

James Smith, Esq.

The reasoning in Mr. Glaister's elaborate Paper, although endorsed by such high mathematical authority, is based upon an implied but "*unproved premiss.*" This premiss is an *assumption*, not self-evident, but nevertheless *a lurking and fallacious assumption*, and until "*recognised Mathematicians*" come to make this discovery, so far as they are concerned, π will remain "*lurking in his den*," unseen and unknown.

Let D denote the diameter, and A the area of a circle. Let B denote the difference between A and the area of an inscribed square to the circle: and let C denote the difference between A and the area of a circum-

* Mr. Glaister's complaint is similar to that I bring against my opponents, that is to say, my opponents catch at an expression which they think suits them, like a drowning man catching at a straw, and pass over arguments that do not suit them, with silent contempt. Where is an opponent to be found that ever attempted to grapple with a theorem founded upon an indisputable geometrical problem?

scribing square to the circle, The "*unproved premiss*" is based upon the *fallacious*, but not self-evident, *assumption*, that B and C are of the same arithmetical value. This is not true.

Some compilers of Euclid's Elements of Plane Geometry, give his 19th definition in the following terms:—"*A polygon is a rectilineal figure having more than four sides.*" J. Radford Young—a living recognised Mathematician—gives Euclid's 19th definition in the following words:—"*Multilateral figures, or polygons, are enclosed by more than four straight lines*:" but immediately observes:—"*The term polygon, however, is often employed as a general name for rectilineal figures of all kinds, without regard to the number of sides; so that the rectilineal figures defined above* (that is, trilateral and quadrilateral figures) *may, without impropriety, be called polygons of three and four sides respectively.*"

Well, then, by hypothesis, D denotes the diameter, and A the area of a circle: B denotes the difference between A and the area of a regular polygon of four sides inscribed within the circle: and C denotes the difference between A and a regular polygon of four sides circumscribed about the circle. Now, if D be represented by unity $= 1$, our unit of length being 1, the difference between B and C $= \left(\frac{D}{4}\right)^2 = \cdot 28125 - \cdot 21875 = \cdot 0625$: and $\cdot 0625$ is the area of a square on the semi-radius of a circle of diameter unity. If D be represented by the digit or numeral 8, the difference between B and C $= \left(\frac{D}{4}\right)^2 = 18 - 14 = 4$: and 4 is the area of a square on the semi-radius of a circle, when the diameter is 8. Now, when $D = 1$, then, $4\,\pi\,(s\,r^2) = 12\frac{1}{2}\left(\frac{\pi}{50}\right)$, and this equa-

tion = area of a circle of diameter unity, whatever be the value of π: and because the property of one circle is the property of all circles, it follows, that $12\frac{1}{2}$ $(s\,r^2)$ = area in every circle. But there can but be one arithmetical quantity, which divided by 50 will give the quotient ·0625: and the fact is indisputable, that $\frac{3{\cdot}125}{50} = {\cdot}0625$. What, then, in the name of common sense can the arithmetical value of π be, but 3·125 ?

Hence: When $D = 1$, $\frac{D}{4} = {\cdot}25$: $\left(\frac{D}{4}\right)^2 = {\cdot}0625$; therefore, $4\,\pi\,({\cdot}0625) = 4\,\pi\left(\frac{3{\cdot}125}{50}\right)$, and this equation $= {\cdot}78125$ = area of a circle of diameter unity; and it follows, that $4\,\pi\,(4^2) = 8^2\,({\cdot}78125) = 50$: and this equation = area of a circle, when the diameter is 8. It also follows, that $1 - {\cdot}78125 = {\cdot}21875 = C$: and ${\cdot}78125 - \frac{1}{2} = 78125 - {\cdot}5 = {\cdot}28125 = B$, when the diameter of the circle is 1. If those who differ from me can't comprehend these truths, "*I can't help it* :" all I can say is, that to controvert them, is beyond the capacity of any Mathematician.

The following pages consist of Correspondence written since my work on "*The Geometry of the Circle*" was in print; and the greater part was in type prior to receiving Mr. Glaister's elaborate Paper of October 25, 1869, and his subsequent communications. It is my intention, never again to be inveigled into a written controversy on the long vexed question of π's arithmetical value: or make any further effort to convince mathematicians that Euclid's Theorem, Prop. 32: Book 1: is fallacious. "*Magna est veritas, et prevalebit.*"

I had written so far, and with it intended to conclude

my address to the reader, when I received a Note or Letter, of which the following is a copy :—

ST. JOHN'S COLLEGE, CAMBRIDGE,
30th October, 1869.

MY DEAR SIR,

You seem to be using my name very freely. Would it not be well to indicate me by an initial as you do the Vicar of Blythe?

Yours faithfully,
W. ALLEN WHITWORTH.

JAMES SMITH, ESQ.

I confess my inhability to comprehend the meaning of this communication, and must leave readers to put their own construction upon it.

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
Near Liverpool,
1st November, 1869.

CORRESPONDENCE.

CORRESPONDENCE.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

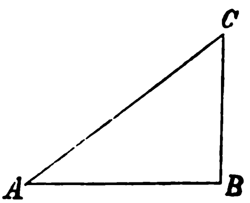
LANEAST, LAUNCESTON, CORNWALL
19th July, 1869.

MY DEAR SIR,

I beg to offer my thanks for the " Extracts "* just received. You repeat my assertion quite correctly in page 395, to wit, that $\pi = 3\frac{1}{8}$ would make the perimeter of a polygon *greater* than the circumference of its circumscribing circle.

But you err in page 398, as to my making any error or confusion as to the meaning of a *sine* (or any other trigonometrical function, though I employed only a sine.)

The sin. of an *angle* is a ratio: Thus, Sin. A $= \frac{CB}{AC}$; but if A C is given in *length*, the sine is also given in *length*. Thus: if A = 30°, sin. A = $\frac{1}{2}$ A C; and if A C be 1 inch, sin. A = $\frac{1}{2}$ inch, absolute length. No Mathematician, I should think, *could* make a blunder in so simple a thing as this.



I know you feel convinced that the circumference of a circle *must* bear an exact finite ratio to its diameter, but to my mind there

* When my work on *The Geometry of the Circle* was in the course of printing I threw off a short pamphlet. This pamphlet consisted of Appendix D, Appendix E, and the address to the reader, which appear in that work. The word extracts, refers to this pamphlet, a copy of which I sent to many Mathematicians, to whom I felt under no obligation to send a copy of the work itself.

is nothing to shew such necessity, any more than there is for the *diagonal* of a square to bear a finite relation to its *side*.

Yours very truly,

G. B. GIBBONS.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS, B.A.

BARKELEY HOUSE, SEAFORTH,
21st July, 1869.

MY DEAR SIR,

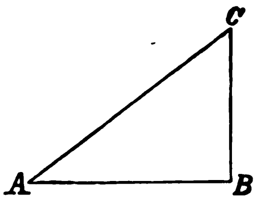
Accept my thanks for your favour of the 19th inst., which came to hand last evening.

You say: " *You repeat my assertion quite correctly in page* 395, *to wit, that* $\pi = 3\frac{1}{8}$ *would make the perimeter of a* (regular) *polygon greater* than the circumference of its circumscribing circle." I thank you sincerely for this admission.

You then observe: " But you err in page 398, as to my making any error or confusion as to the meaning of a *sine* (or any other trigonometrical function, though I employed only a sine.")

You then say: " The sine of an *angle* is a ratio: Thus, Sin. A $= \frac{CB}{AC}$; but if A C is given in *length*, the sine is also given in length. Thus: If A = 30°, sin. A $= \frac{1}{2}$ AC; and i A C be 1 inch, sin. A = $\frac{1}{2}$ inch, absolute length. No Mathematician, I should think, *could* make a blunder in so simple a thing as this."

FIG. 1.



Now, my dear Sir, I frankly admit your premisses and your conclusion. But, the sine of an angle is not the angle itself, but its arithmetical value; and the logarithm of that value is the lòg-sin. of the angle. " No Mathematician, I should think, *could* make a blunder" as to such simple truths as these.*

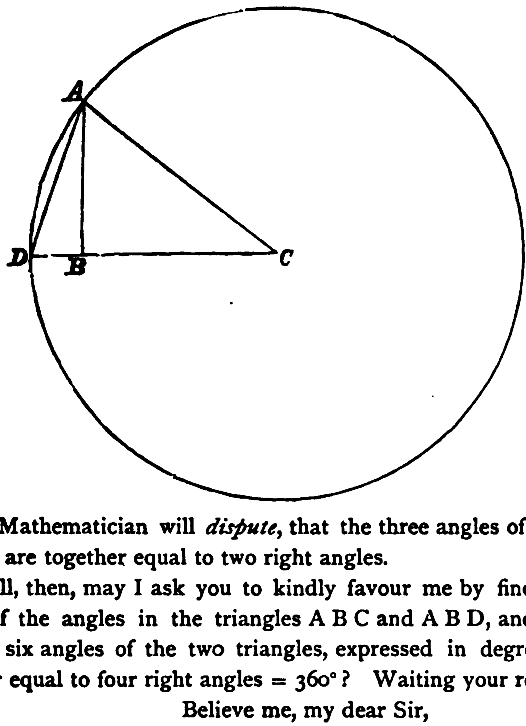
* An arc has its sine as well as an angle: and the sine of any arc is half he chord of twice that arc. Hence: The sine of a right angle = unity: and the sine of an angle of 30° $= \frac{\text{unity.}}{2.}$

In the geometrical figure (Fig. 2) I first construct the right-angled triangle A B C, making A B and B C the sides that include the right angle in the ratio of 3 to 4, and with C as centre, and C A as interval, describe the circle. I then produce C B to meet and terminate in the circumference of the circle at the point D, and join A D, and so construct the isosceles triangle D A C.

It follows, that A B : A C : : 3 : 5.

A B : B D : : 3 : 1.

FIG. 2. A B : A D : : 3 : $\sqrt{10}$.



No Mathematician will *dispute*, that the three angles of a plane triangle are together equal to two right angles.

Well, then, may I ask you to kindly favour me by finding the values of the angles in the triangles A B C and A B D, and prove that the six angles of the two triangles, expressed in degrees, are together equal to four right angles = 360°? Waiting your reply,

Believe me, my dear Sir,

Very truly yours,

JAMES SMITH.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

LANEAST, LAUNCESTON,
22nd July, 1869.

MY DEAR SIR,

Though it is going over the old ground—trodden by a couple of hundred letters that have passed between us—I will do as you wish, touching the triangles.

We have no reason to expect an *exact* expression for either of the acute angles, because our division of the circle into degrees, minutes, &c., is purely arbitrary, and we have no more reason to expect C to come out in an exact number of *minutes*, than in an exact number of *degrees*.

The French, for instance, use a different graduation from what we employ; by *their* notation we should not generally determine the angle in any *exact* number of "grades."

Again: Our values of the *sine* are generally expressed in interminable decimals; but if the value of this function be put down in a long series of decimals, we may get a corresponding degree of *nearness* to the true value of the angle.

$$\text{Sin. } C = \frac{3}{5} = \cdot600000\ldots \text{ to six decimals.}$$

$$\text{Sin. } 36^\circ\, 52' = \cdot599955$$

$$45 = \text{difference.}$$

$$\text{So that } C = 36^\circ\, 52' + x.$$

$$BAC = 53^\circ\ \ 8' - x.$$

$$90^\circ\ 0'$$

$$\text{Similarly, Sin. } DAB = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} = \frac{3\cdot162277}{10}$$

$$= \cdot316227\ldots$$

$$\text{Sin. } 18^\circ\, 26' = \cdot316201$$

$$26 = \text{difference.}$$

$$\text{So that } DAB = 18^\circ\, 26' + y$$

$$ADB = 71^\circ\, 34' - y$$

$$90^\circ\ 0'$$

And the four angles you require would be

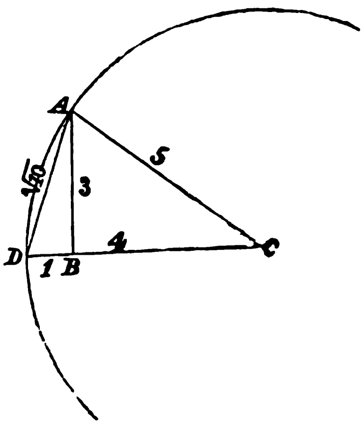
$$\begin{aligned} C &= 36^\circ\ 52' + x. \\ B\,A\,C &= 53^\circ\ \ 8' - x. \\ D\,A\,B &= 18^\circ\ 26' + y. \\ A\,D\,B &= 71^\circ\ 34' - y. \quad * \end{aligned}$$

(The lines for $B\,A\,C$ and $D\,A\,B$ are joined by a brace marked *.)

$180^\circ\ 0' + 2$ right angles at B $= 360^\circ$.

Noticing the equality of these *

$71^\circ\ 34' + y - x = 71^\circ\ 34' - y$; or $x = 2\,y$.



The digits 3 and 4 in the diagram, are data.

To find (x) nearly:

Sin. $36^\circ\ 52' = {\cdot}599955$
Sin. $36^\circ\ 53' = {\cdot}600188$

233 = difference for $1'$.

Hence: $C = 36^\circ\ 52'\ \frac{45}{233}$ nearly.

To find (y) nearly:

Sin. $18^\circ\ 26' = {\cdot}316201$
Sin. $18'\ 27' = {\cdot}316477$

276 = difference for $1'$.

$y = \frac{26}{276}$ and $D\,A\,B = 18^\circ\ 26'\ \frac{26}{276}$ nearly.

Converting both into decimals of a minute,

233) 45·0 (·193	276) 26·00 (·094
233	2414
2170	1160
2097	1104
·730	
699	

So that $\left.\begin{array}{r} C = 36^\circ\ 52{\cdot}193' \\ DAB = 18^\circ\ 26{\cdot}094' \end{array}\right\}$ $\begin{array}{l} \text{when } x = {\cdot}193, \\ \text{equals nearly } 2y = 2\,({\cdot}094). \end{array}$

And would be nearer to equality if we took 7 decimal places.

I never said or thought, that the sine of an angle was the angle itself; and never used them as synonymous. I avoided logarithms as needless. Specially after you said they were erroneous. But the result will be the same, whichever method we employ within the limit of exactness, that our number of decimals will give :—

$$\begin{array}{lll} \text{Sin. } \phi = \frac{3}{5} \cdot & \text{Log } 3 = & {\cdot}477121 \\ & \text{Log } 5 = & {\cdot}698970 \\ \hline & & 9{\cdot}778151 \\ \text{Log. Sin. } 36^\circ\ 52' & & 9{\cdot}778119, \text{ difference } 32 \\ \text{Log. Sin. } 36^\circ\ 53' & & 9{\cdot}778287, \text{ difference } 168. \end{array}$$

$$\text{Or } \phi = 36^\circ\ 52'\ \frac{32}{168} = 36^\circ\ 52{\cdot}194'.$$

I am aware this is all the "*old story*," but I have complied with your request.

Yours very truly,

J. B. GIBBONS.

JAMES SMITH, *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
Saturday Evening,
24th July, 1869.

MY DEAR SIR,

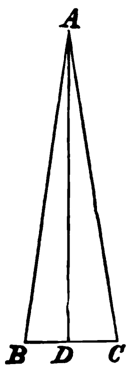
Your favour of the 22nd inst., but bearing the Launceston post-mark of yesterday, is just to hand.

You observe :—"*I never said or thought that the sine of an angle was the angle itself; and never used them as synonymous.*" You then say, "*I avoided logarithms as needless.*" Hence: I must avoid logarithms too for the present, but I shall also avoid tables for the present, which you do not.

You make the angle C = $36^\circ\ 52'\ \frac{45}{233}$ nearly. But, $\frac{45}{233}$ (60) = 12 nearly. Hence: You make the angle C = $36^\circ\ 52'\ 12''$ nearly. You make the angle D A B = $18^\circ\ 26'\ \frac{26}{276}$ nearly. But, $\frac{26}{276}$ (60) = 6 nearly. Hence: You make the angle D A B = $18^\circ\ 26'\ 6''$ nearly.

Now, my dear Sir, I tell you that the angle D A B = half the angle C, whatever be the value of the angle C, and your own argument and method of calculation proves it, "*nearly.*"

Since you decline to grapple with a proof by Logarithms, I must deal with you in some other way.



Let the geometrical figure in the margin represent an isosceles triangle A B C, of which the angle A and the side B C are bisected by the line A D, and let B A C denote an angle of 16° 16′. It is self-evident that the isosceles triangle A B C is divided into two triangles, by the line A D, and that the line A D is perpendicular to the line B C. It is also self-evident that the line A D is the perpendicular of, and common to, the triangles A D B and A D C, and because A D is perpendicular to B C, it follows of necessity, that A D B and A D C are similar and equal right-angled triangles.

Now, it is self-evident, that D B and D C are the geometrical sines of the angles D A B and D A C ; and upon this very simple geometrical figure, I found the following theorems, which I give you for solution :—

THEOREM 1.

Find the ratios of side to side in the triangles A D B and A D C.

THEOREM 2.

Find the trigonometrical values of the sine and co-sine in the right-angled triangle A D B or A D C.

THEOREM 3.

Let D B and D C the geometrical sines of the angles D A B and D A C = $\sqrt{\cdot 02}$. Find the length of the line A D, the geometrical co-sine of the angles D A B and D A C.

If you will kindly favour me with the solution of these theorems, I think I shall be able to convince you of the fallacy of the reasoning in your last communication. Waiting your reply,

Believe me, my dear Sir,

Very truly yours,

JAMES SMITH.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

LANEAST, LAUNCESTON,
26th July, 1869.

MY DEAR SIR,

I beg to offer you my best thanks for the handsome volume just received by post, "The Geometry of the Circle."*

I kept no copy usually of my letters to you, but many of them are preserved in your publications. In pages 364—5 of the present volume, I recognise my former remarks on the triangle you lately sent me for calculation.

This new volume contains in substance, everything I have offered, or could suggest, on the subject of π.

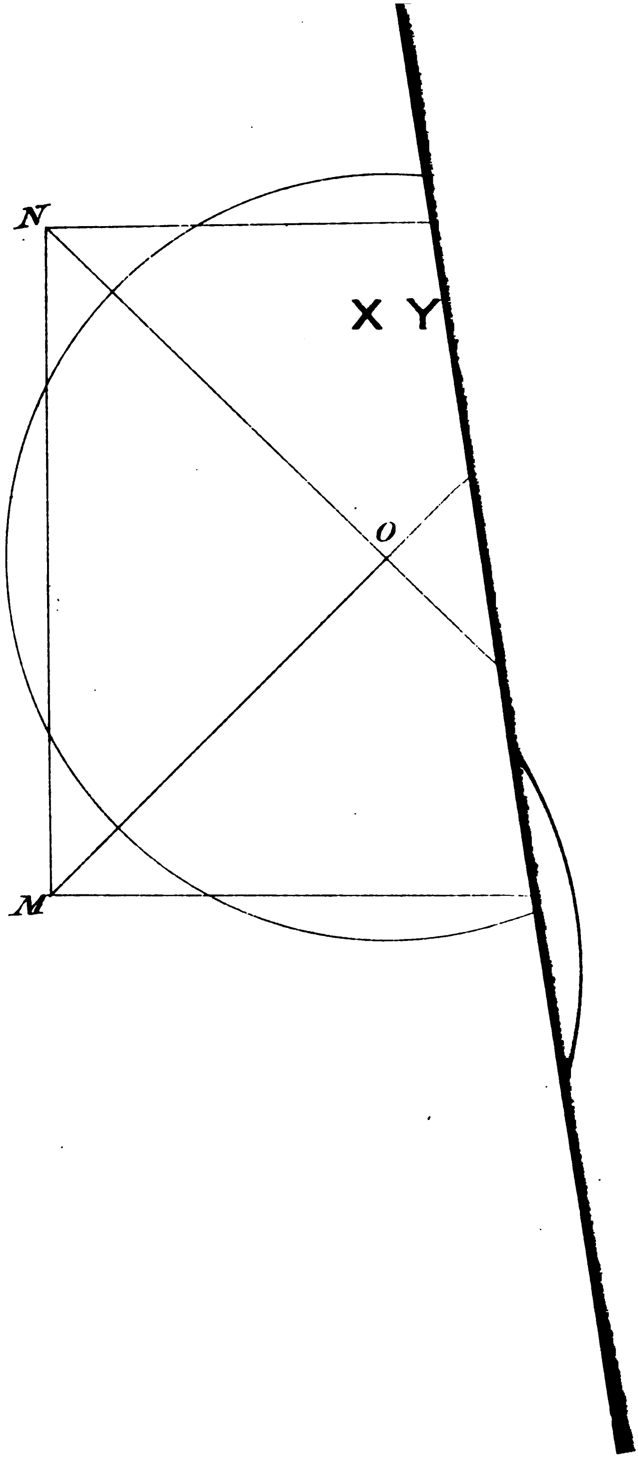
Believe me, dear Sir,

Yours very truly,

G. B. GIBBONS.

P.S.—I meditate a letter to the *Athenæum*, but perhaps the Editor is tired of the subject, and will not insert it. G. B. G.

* There is a critique on this work in the *Athenæum* of August 21, 1869. See Appendix A.


N
X Y
O
M

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
27th July, 1869.

MY DEAR SIR,

I might have had a reply to my Letter of the 24th inst. this afternoon, and as I am without an answer to that communication, I may fairly infer, that you find yourself incompetent to grapple with the theorems I gave you for solution.

The geometrical figure represented by the enclosed diagram (*see diagram A*) is a fac-simile of that in my Letter of the 21st inst., with certain additions; that is to say, the right-angled triangle A B C is the generating figure of the diagram. The isosceles triangle A D C is similar to the isosceles triangle A D C in my Letter of the 21st inst., and the angles D E F and P C G are equal angles.

CONSTRUCTION OF DIAGRAM A.

Let A B C represent a right-angled triangle, of which the sides A B and B C, which include the right angle B, are in the ratio of 3 to 4, by construction. With C as centre and C A as interval, describe the circle X. Produce C B to meet and terminate in the circumference of the circle X, at the point D, and join A D, and so construct the isosceles triangle D A C. With C as centre and C B as interval, describe the circle Y, and, with B as centre and B D as interval, describe the circle Z. Produce B A to a point E, making B E = (A B + B C), or, 7 (B D), and join E D and E F, and so construct the isosceles triangle E D F. From the angle A in the right-angled triangle A B C, draw a straight line parallel to the line E F, to meet and terminate in the circumference of the circle X at the point G, and join G C. Or, from the point C, the centre of the circle X, draw a straight line at right angles to A C, the radius of the circle X, to meet and terminate in the circumference of the circle X at the point G, and join A G. By either method of construction we obtain the same *geometrical* result. Produce A B to meet and terminate in the circumference of the circle X at the point P, and join P C. Produce

A C and G C to meet and terminate in the circumference of the circle X at the points K and L, and join A L, L K, and G K, and so construct the square A G K L, which, it is self-evident, is an inscribed square to the circle X. On E D, a side of the isosceles triangle E D F, describe the square E D M N, and draw the diagonals E M and N D. It is axiomatic, if not self-evident, that the diagonals of the square E D M N intersect and bisect each other at the point O, and it follows, that the point O is the centre of the square E D M N. With O as centre and B C, the base of the right-angled triangle A B C, the generating figure of the diagram, as radius, describe the circle X Y.

Upon this geometrical figure I found two theorems, which I give you for solution.

THEOREM 1.

Prove that the circles Y and X Y, and the squares A G K L and E D M N, are all exactly equal in superficial area. Or, if unequal in superficial area, pray prove it.

THEOREM 2.

Prove that the angles D E F and P C G are equal angles. Or, if these angles are unequal, pray prove it.

That the angles D E F and P C G are equal angles, may readily be demonstrated by PURE GEOMETRY. YOU, my dear Sir, *avoid Logarithms as needless:* but, I venture to tell you, that to demonstrate the equality of these angles mathematically, Logarithms are NEEDFUL, and that their equality can only be demonstrated MATHEMATICALLY by the aid of Logarithms.

Waiting the favour of your solution of these theorems,

Believe me, my dear Sir,

Very truly yours,

JAMES SMITH.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

LANEAST, LAUNCESTON,
27th July, 1869.

MY DEAR SIR,

Before receiving yours to-day, I had posted a letter thanking you for the book you kindly sent me.

I am sure you will agree with me, that it would be wasting time if we were to revive our long controversy, when I have nothing fresh to offer. I don't like to write perpetually the same things over again; nor would you wish to receive such repetitions.

There is a lapsus in your Letter. You say: angle D A B is half angle C—you mean half the angle B A C. This is a mere mistake of letters. Given B A C = 16° 16′.

THEOREM 1.

By Hulse's Tables:—

$$*\frac{BD}{AB} = \text{Sin. } 8^\circ\ 8' = \cdot 1414772.$$

$$\frac{AD}{AB} = \text{Cos. } 8^\circ\ 8' = \cdot 9899415.$$

$$\therefore \frac{BD}{AD} = \text{Tan. } 8^\circ\ 8' = \cdot 1429147.$$

The values being given to 7 decimals.

THEOREM 2.

If B A C = 16° 16′:

$$\left.\begin{array}{l}\text{Sin. } 8^\circ\ 8' = \cdot 1414772 \\ \text{Cos. } 8^\circ\ 8' = \cdot 9899415\end{array}\right\}\text{ to seven decimals.}$$

$$AD = \frac{BD}{\tan.\ \phi} \qquad r\phi = \tfrac{1}{2}(BAC).$$

THEOREM 3.

$$\text{Given } BD = \sqrt{\cdot 02} = \frac{\sqrt{\cdot 02}}{\tan.\ \phi}$$

So that ϕ cannot be determined unless some value is assigned to A D or A B.

*$\frac{BD}{AB}$ should be $\frac{BD}{AD}$, and, $\frac{AD}{AB}$ should be $\frac{AB}{AD}$. "This is a mere mistake of Letters," and Mr. Gibbons would no doubt have made the correction, if he had been afforded the opportunity of revising this Letter.

Suppose A B $= 1$ Sin. $\phi = \sqrt{\cdot 02}$

$\text{Cos.}^2 \phi = 1 - \text{Sin.}^2 \phi = 1 - \cdot 02 = \cdot 98$

$\text{Cos.}\ \phi = \sqrt{\cdot 98}.$

$= \cdot 9899495$ to 7 decimals.

$\text{Cos.}\ 8^\circ 7' = \cdot 9899826$

$331 =$ difference.

difference for $1' = 411$

So that $\phi = 8^\circ\ 7' \frac{331}{411}$

In No. 2 I suppose you did not mean that I should calculate, *independently*, Sin. 8° 8'... That labour would be too great, though the process is simply arithmetical. For in such calculations, you are aware, that each *sine* in the calculation is made to depend on the one preceding it, say by 1'.

Thus:

Sin. $2' =$ Sin. $1' + 6$ Sin. $1'$ $\text{Sin.}^2\ 30'' +$ s'nd.

Sin. $3' =$ Sin. $2' + \ldots$

Sin. $4 =$ Sin. $3' + \ldots$

So that the calculator must work up by minutes, till he reaches Sin. 8° 8'.

This is no objection, when a man is computing a whole set to form tables of sines; indeed it is the easiest way; but it would be very long work to do this for each sine we wanted to employ.

I will send you the complete formula, if you like, with a few of the calculations worked out. With a formula before me, the process is simple arithmetic; but we cannot find Sin. 8° 8', without finding all the sines to minutes, preceding this.

But the calculator has for his aid what are called "Formulæ of Verification," to check any mistake in his work, and here is one (out of several)—

Sin. A = Sin. (60° + A)—Sin. (60° — A.)

Suppose I want Sin. 8° 8', and find it by the Tables. ·1414772. I wish to verify this. Thus—

Sin. 8° 8' = Sin. 68° 8'—Sin. 51° 52'.

Now, by taking Sin. $68^\circ\ 8' = \cdot9280531$
$\quad$ Sin. $51^\circ\ 52' = \cdot7865759$

$\cdot1414772$ = difference.

which agrees with this value for Sin. $8^\circ\ 8'$, in the Tables.

Yours very truly,

G. B. GIBBONS.

For practical purposes, the measure of a right angle is fixed at 90°, and since the three angles of every right-angled triangle are together equal to two right angles, it follows, that in every right-angled triangle, (a right-angled isosceles triangle excepted), one of the acute angles must be greater, and the other less, than half a right angle = 45°.

Thus:

$$\text{Sin. } 53^\circ\ 8' = \tfrac{4}{5} = \cdot8.$$
$$\text{Sin. } 36^\circ\ 52' = \tfrac{3}{5} = \cdot6.$$

$\cdot2$ = difference.

$\frac{\cdot2}{10} = \cdot02$, and $\sqrt{\cdot02} = \cdot1414213... =$ sin. of an angle of 8° 8′. But, $7\ (\sqrt{\cdot02}) = \sqrt{\cdot98}$: and $\sqrt{\cdot02}^2 + \sqrt{\cdot98}^2 = (\cdot02 + \cdot98) =$ unity; and sin.^{2} + cos.^{2} = unity in every right-angled triangle. This makes $\sqrt{\cdot98} = \cdot9899495$, the cosine of an angle of 8° 8′.

Hence:

If the sides that contain the right angle in a right-

angled triangle, are in the ratio of 7 to 1, the angles of the triangle are angles of 90°, 81° 52′ and 8° 8′, and are together equal to two right angles. The similar and equal triangles E B D and E B F, in the diagram A, (page 9) are two of such triangles, by construction: and it follows, that the three angles of the isosceles triangle E D F are angles of 16° 16′, 81° 52′, and 81° 52′, and are together equal to two right angles. The three angles of the right-angled triangle A B C, the generating figure of the diagram A, are angles of 90°, 53° 8′, and 36° 52′, and are together equal to two right angles. The three angles of the right-angled triangle A B D are angles of 90°, 18° 26′ and 71° 34′, and are together equal to two right angles. The angles B A C and B A D are angles of 53° 8′ and 18° 26′, and are together equal to 71° 34′, making the sum of the angles B A C and B A D = the angle A D B, and demonstrates that the angles at the base of the isosceles triangle D A C are equal; and it follows, that the six angles of the two triangles A B C and A B D are together equal to four right angles = 360°: and it also follows, that the angles D E F and P C G are equal angles of 16° 16′. It is axiomatic, but not self-evident, that if the angle at the apex of an isosceles triangle be greater than an angle of 60°, a straight line cannot be drawn from an angle at the base perpendicular to its opposite side. The "*old ground—trodden by a couple of hundred Letters that have passed between*" Mr. Gibbons and me—ought to have convinced that gentlemen of these facts.

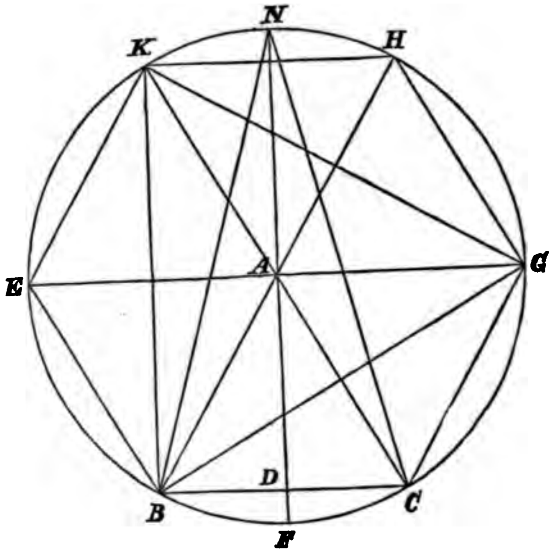
"*Recognised Mathematicians*" fancy they are "*reasoning geometrical investigators*," nevertheless, they fall into the blunder of making Geometry a mere branch of Mathematics, and then, attempt to make Geometry subservient

to *their* Mathematics. From a given point A draw two straight lines AB and AC at right angles, of equal length, and join BC. Then, BC will be a side of an inscribed square to a circle, of which AB and AC are radii, and ABC will be a right-angled isosceles triangle. The angles B and C will be equal, and each equal to half a right angle, whether we fix the measure of a right angle at 90°, 60°, or any other number of degrees. Construct the figure, and it will be self-evident to a "*reasoning geometrical investigator*," that we cannot draw straight lines from the angles B and C, perpendicular to their opposite sides AC and AB. How happens it that a Mathematician of Mr. Gibbons' reputation did not make these discoveries? Had he made them, should we have found him falling into the common error of making Geometry, NOT a distinct science, but a mere branch of Mathematics?

But further: With reference to the angle BAC, in the right-angled triangle ABC, in the diagram A (page 9), Mr. Gibbons, in his Letter of the 29th July, says:—"Given AB = 16° 16′." Having fixed the measure of a right angle at 90°, it is self-evident, since B is a right angle, that BAC must be greater than half a right angle, that is, greater than 45°. It is hardly conceivable that Mr. Gibbons could have fallen into so gross a blunder as that of conceiving—even hypothetically—the angle BAC to be an angle of 16° 16′.

The geometrical figure (Fig. 1) is a fac-simile of that on page xv. of the Address to the Reader, with the following additions:—Produce FA to meet and terminate in the circumference of the circle at the point N. Join NB and NC, and thus, construct the isosceles triangle NBC.

FIG. 1.



By Euclid: Prop. 20: Book 3: the angle B A C at the centre of the circle, is the double of the angle B N C at the circumference, standing on the same arc B F C: and Euclid's demonstrations of this theorem are irrefragable. Now, A B = A C = B C = radius of the circle, and Mathematicians admit that, "*we may always make the radius our unit.*"* Our unit of length being 1, we may make radius 1, and in fact, Mathematicians make radius of a circle 1, in all their attempts to find the ratio between radius and circumference in a circle. Well, then, having fixed 90° as the measure of a right angle, it follows, that the angle B A C, which is equal to the angle B A E, and therefore equal to two-thirds of the right angle E A F = 60°. Now, the angle B A C is bisected by the

* See Mr. R. F. Glaister's Paper in the Address to the Reader, page xxiv.

radius A F, and it follows, that D A B and D A C are equal angles of 30°, and are subtended by arcs equal to one-twelfth part of the circumference of the circle. But, the angle B N C at the circumference is equal to half the angle B A C at the centre of the circle, and it follows, that B N C is an angle of 30°. But, the angle B N C is bisected by the line N F, a diameter of the circle, and it follows, that F N B and F N C are equal angles of 15°. Now, it is self-evident that the line B D is the sine of, and common to, the angles D A B and D N B, and, because A D is perpendicular to B C, the angle A D C is a right angle, and equal to the angle A D B in the right-angled triangle A D B. But, the angles D A B and D N B are not equal angles, and I put the following question to Mathematicians :—Can unequal angles have a common sine *trigonometrically?* That unequal angles may have a common sine *geometrically* is self-evident, from a mere inspection of the geometrical figure represented by the diagram, Fig. 1. The fact is, Mathematicians fall into the gross blunder of making the sine of the angle D N B, that is, the sine of an angle of 15°, equal to the hypothenuse of a right-angled triangle, of which the lines B D and D F in the diagram, are the sides that include the right angle. To make this statement unmistakeably intelligible, draw the chord of the arc B F, that is, join B F. Mathematicians make the sine of the angle D N B equal to the straight line B F. Our "*recognised Mathematicians*" have much to learn, and I have no hesitation in telling them, that they will never make *their* Mathematics harmonize with pure Geometry.

If $A B = A C = 1$, then, $B D = \frac{1}{2} = \cdot 5$, *absolute length*, as Mr. Gibbons puts it in his Letter of the 19th July, and

"no Mathematician I should think *could* make a blunder in so simple a thing as this." Now, B D is the sine of the angle D A B; and D A B is an angle of 30°. But, B D is also the sine of the angle D N B: and D N B is an angle of 15°. The sine of an angle of 15° is given in our mathematical tables as ·2588190 to 7 decimals. The Mathematicians of a future generation will discover, that ·25 is the *trigonometrical* sine of the angle D N B. Well, then, the arc B F C is the double of the arc B F or F C: and the chord B C is the double of the line B D or D C: and when A B = A C = 1, then, 6 (B D + D C) = 6 times B C = 6 = the perimeter of the inscribed regular hexagon K E B C G H. Mathematicians may dispute, but they cannot controvert the *fact*, that the perimeter of an inscribed regular hexagon to a circle of radius 1 = 6. Now, 6 + $\frac{1}{24}$ (6) = 6 B C + $\frac{1}{2}$ B D, and this equation = 6·25 = the circumference of a circle of radius 1. If the diameter of a circle be represented by unity, then, $\frac{1}{2}$ = ·5 = radius. Add $\frac{1}{24}$ part = ·0208333 with 3 to infinity; then, 6 (·5 + ·0208333 with 3 to infinity) = 6 (·5208333, &c.,) = 3·124998, &c., (and by extending the decimals we may get 3·124 and as many nines as we please, the last decimal excepted, which must be one less than 9), which stands for 3·125, the circumference of a circle of diameter unity.

Having fixed 90° as the measure of a right angle, it follows, that the circumference of a circle = 360°. Now, if 25 equal isosceles triangles be inscribed in a circle, the angles at the centre = $\frac{360}{25}$ = 14·4, which, expressed in degrees = 14° 24′, and the circular measure of a right angle = $\frac{90° \times \pi}{180}$, and is equal to the semi-circumference of a circle of diameter unity, or the quadrant of a circle

of radius 1. Now, by hypothesis, let π (which denotes the circumference of a circle of diameter unity) $= 3\cdot1416$. Then: $\frac{90^\circ \times 3\cdot1416}{180} = \frac{3\cdot1416}{2} = 1\cdot5708 =$ semi-circumference of a circle of diameter unity, or a quadrant of a circle of radius 1, on the hypothesis that $\pi = 3\cdot1416$. The circular measure of an angle of $14^\circ\ 24' = \frac{14^\circ\ 24' \times \pi}{180} = \frac{864' \times 3\cdot1416}{180 \times 60}$ $= \frac{2714\cdot3424'}{10800} = \cdot251328 = \frac{8\pi}{100}$, on the hypothesis that $\pi =$ $3\cdot1416$. But, $\frac{1\cdot5708}{\cdot251328} = 6\cdot25$, and is not equal to the orthodox value of 2π, but equal to a value of 2π that makes 8 circumferences $= 25$ diameters in every circle. Hence: The circular measure of a right angle divided by $\frac{8\pi}{100}$ is a constant quantity $= 6\cdot25$, and is equal to the circumference of a circle of radius unity. One of my correspondents asserts that this is an "*unproved premiss.*" This is an assertion without a shadow of proof, and is simply untrue. For example: By hypothesis, let $\pi = 3\cdot2$: Then: $\frac{3\cdot2}{2} = 1\cdot6 =$ the quadrant of a circle of radius unity: and $\frac{90^\circ \times 3\cdot2}{180} = \frac{288}{180} = \frac{3\cdot2}{2} = 1\cdot6 =$ semi-circumference of a circle of diameter unity, on the hypothesis that $\pi = 3\cdot2$. The circular measure of an angle of $14^\circ\ 24'$ $= \frac{14^\circ\ 24'}{180} = \frac{864' \times 3\cdot2}{180 \times 60} = \frac{2764\cdot8'}{10800} = \cdot256 = \frac{8\ (\pi)}{100}$, on the hypothesis that $\pi = 3\cdot2$. But, we may hypothetically adopt $\pi = 3$, or $\pi = 4$, or $\pi =$ any arithmetical quantity intermediate between 3 and 4, and prove that the equation $\frac{\pi}{2} \div \frac{8\pi}{100}$ is a constant quantity $= 6\cdot25$. It is therefore simply absurd to assert that the circular measure of a right angle divided by $\frac{8\pi}{100}$ is not a

constant quantity, and therefore an "*unproved premiss;* since we can furnish the proof by means of any hypothetical value of π intermediate between 3 and 4, so that it be *finite* and *determinate.*

Now, referring to the Fig. 1. we may conceive the equilateral triangle A B C "*to expand continuously,*" still retaining its character as an equilateral triangle, till it arrives at the equilateral triangle K B G in magnitude. The idea of the continuous expansion of a geometrical figure is not mine: I get it from Mr. J. Radford Young, a living "*recognised Mathematician.*" I quote the following from his remarks on Euclid's fourth book :—*

"ON THE QUADRATURE OF THE CIRCLE.

"Intimately connected with the researches just adverted to, is the problem of the quadrature of the circle, which, like that of the trisection of an angle, has for ages occasioned the fruitless expenditure of much valuable time and thought. I shall endeavour to give you here some notion of the meaning and object of this celebrated problem, not only because it is a matter of such historical interest, that you ought to know something about it ; but because, moreover, in certain elementary writings on the subject, the thing is put before the student in an erroneous form ; and, consequently, a wrong impression as to the real character of the problem is conveyed.

"The problem of *Squaring the Circle,* as it is popularly called, has a twofold meaning—namely, the *geometrical* quadrature, and the *numerical* quadrature. In the first

* See: Elements of Plane Geometry, by J. Radford Young, in "Orr's Circle of the Sciences."

of these senses the problem is to construct a square that shall be equal in surface to a given circle ; in the second, the problem is to express the numerical measure of the surface of a circle when the measure or length of its diameter is given in numbers. The former of these is the more ancient form of the problem ; and all that can be fairly said of it is—as was said of the trisection of an angle—that the solution has never been effected ; a square equal to a circle has never yet been constructed. We have no grounds for affirming that this construction is *impossible*, for the equivalent square *exists*. You may readily satisfy yourself of this by the following reflections:—The square on the *diameter* of the circle would be too great, and the square on the chord from an extremity of the diameter, to cut off a fourth part of the circumference, would be too small, since the former square would be circumscribed about the circle, and the latter inscribed in it ; the circle therefore is in magnitude somewhere *between* the two. Conceive, now, the smaller of these two squares to expand continuously, still retaining its character as a square, till it arrives at the larger square in magnitude: then, as all intermediate magnitudes are thus reached and passed through, and as the circle is one of these intermediate magnitudes, it necessarily follows, that our expanding square must, at a particular stage of its progress, have exactly attained the magnitude of the circle ; so that if its progress could be arrested at that stage, or—to drop this idea of progression—if the individual square could be isolated and exhibited, the problem of the geometrical quadrature of the circle would be solved. It is plain, therefore, that there is nothing visionary or absurd in the search after this square, as if it were a thing

that had no existence; although some very able geometers have, strangely enough, condemned the enquiry on these grounds. The only sound reasons for abandoning the investigation are these two—namely—first, that the problem has been earnestly and laboriously attempted, by the profoundest geometers, for thousands of years, and they have been obliged to abandon it in despair; and secondly, that the successful solution of it would be of no theoretical or practical value if furnished. (*The second reason is an assertion without a shadow of proof, and moreover, is not true.*) As far as utility is concerned, the other form of the problem of the quadrature of the circle is by far the more important: that is, to discover the numerical measure of the surface of a circle from the measured length of its diameter being given. But, under this aspect of it, the accurate solution of the problem is really impracticable; it can be *proved* to be so; and the proof will be given in a subsequent part of the present mathematical course. (*This proof is not given, and cannot be given, for the assertion that "the accurate solution of the problem is really impracticable," is simply untrue.*) It is just as impracticable as it is to assign accurately the square root of 2; and, in fact, this square root does repeatedly enter into the approximative numerical process."

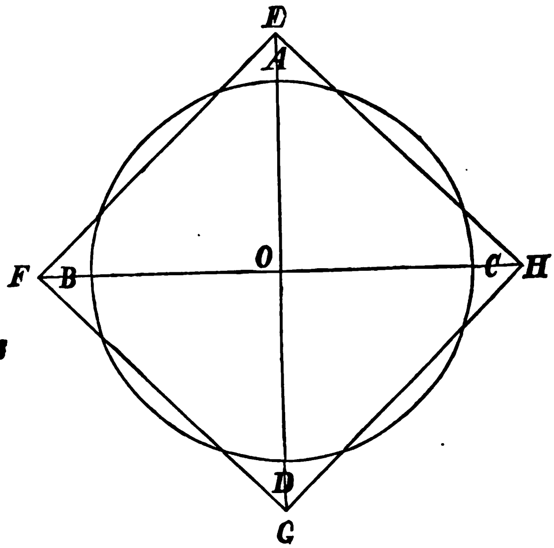
Now, referring to the figure on page 16 (Fig. 1), conceive the equilateral triangle A B C to "*expand continuously*," still retaining its character as an equilateral triangle, till it arrives at the equilateral triangle K B G in magnitude. What would follow? Would not all the angles of the triangle A B C then touch the circumference of the circle? Would not the triangle A B C be equal to

the triangle K B G? Would not the angle A of the triangle A B C rest on the angle N of the isosceles triangle N B C? Would not the angle B A C still be the double of the angle B N C? Would not the triangle A B C be similar in all respects to the triangle K B G? Would not the values of the angles in the triangle A B C have remained unchanged at every step in the course of its expansion? Well, then, the angle B A F = the angle N A K: and the angle E A B = the angle E A K. But the angles B A F and B A E are together equal to the right angle E A F; and the angle N A K and E A K are together equal to the right angle E A N: and because the angles B A F and N A K are subtended by arcs equal to one-twelfth part of the circumference of the circle, and are therefore equal to one-third of a right angle, it follows, that the angles E A B and E A K are equal to two-thirds of a right angle, whatever numerical value we may put upon a right angle. Let Mathematicians find a better proof, that the angles of an equilateral and equiangular triangle are equal to two-thirds of a right angle; but the unerring laws of the "*exact science*" of Geometry, will not admit of their arriving at this conclusion (which is unquestionably true) by mere inference, when a direct proof is possible.

We may describe a circle with any radius, and draw two diameters at right angles, and it follows from Euclid's definition of a right angle, that the angles at the centre of the circle are together equal to four right angles. We may also describe a regular hexagon, about it circumscribe a circle, and draw straight lines from the angles of the hexagon to the centre of the circle. Then, because a right angle at the centre of a circle is subtended by an arc equal to one-fourth part of the circumference: and

because the chord of an arc equal to one-sixth part of the circumference of a circle, is a side of a regular inscribed hexagon, and therefore equal to a side of an equilateral triangle, of which the sides are equal to the radius of the circle ; and because 6 : 4 : : 90 : $\frac{2}{3}$ (90), it follows, that the angle subtended by a side of a regular inscribed hexagon is equal to two-thirds of a right angle, no matter what numerical value we may put upon a right angle. No such proof as this is to be found in Euclid.

FIG. 2.



If a square equal in surface to a given circle can be constructed, isolated and exhibited, the numerical measure of the surface of both can be found, from the "*measured length*" of the diameter of the circle being, given: The "*numerical quadrature*" and the "*geometrical quadrature*" must stand or fall together: and it is simply absurd to suppose, that it is possible to find the

one, without at the same time discovering the other. The numerical measure of the surface of the square E F G H, in Fig. 2, is exactly equal to the numerical measure of the surface of the circle on which it stands. It is self-evident, that we cannot prove this equality by a mere inspection of the diagram, and although a person may accidentally stumble upon the construction of the figure, it is self-evident, he cannot prove that the square and circle are equal in superficial area, without the aid of numbers. We may fancy that the parts of the square outside the circle are equal to the parts of the circle outside the square: or we may fancy that the numerical surface of the square and circle are equal: but these fancies are but as the fancies of a child, who fancies that the sun and moon are about the same size. Well, then, although we may have constructed Fig. 2 by the "*rule of thumb*," it would be very absurd if we were to say, that we can prove that the square and circle are equal in superficial area, without the aid of numbers. I take the following Letter from the *Correspondent* (a London Journal, open to receive communications on all scientific subjects) of February 10, 1866:—

SIR,

I have not had a line to you on the subject of the famous quadrature, and I think I have now a good opportunity of seconding Mr. Smith.

In a little book, published in 1838, and called "*The Millwright's and Engineer's Pocket Director*," by John Bennett, there are some geometrical problems. Prop. XI. is:—" In a circle given, to provide a square of similar area." The solution is not lucidly explained, and I leave Mr. Smith to puzzle it out, or to refer to Bennett's larger work, "*Geometrical illustrations*."

The next problem, number XII., is the same, but after a more simple method, and, in Mr. Bennett's own words, is as follows:— "Divide the circle into four parts, and one of the angular divisions into four parts also, set off one more part, making it five, after which inscribe the square as shown in the diagram (Fig. 2)." Or, simplifying Bennett's words, the method is as follows:—Draw two diameters at right angles; divide each radius into four parts and produce it one part more, making five; join the four parts, or corners, so found, and the equal square will be formed.

Now, the above is identical with Mr. Smith's value of π, as may be shewn thus:—Let the radius be 4, then it will be produced to 5, so that the diagonal will be 10, and the area, being half the square of the diagonal, will be $\frac{1}{2} \times 100 = 50$. Now, the circle will have an area $= \pi\,(4^2) = \frac{25}{8} \times 16 = 50 =$ area of square.

Thus Bennett's quadrature agrees with Smith's, and the question now is, who is to have the honour and glory? Bennett is undoubtedly the first, but he does not specially mention $\pi = 3\frac{1}{8}$, but simply gives the geometrical quadrature. I expect we shall have a discussion among Mathematicians on this point similar to that regarding the priority of Newton or Liebnitz in the discovery of fluxions.

I trust Mr. Smith will attend to Mr. Garbett's Letter, and proceed to calculate his tables.* He has been informed by Captain Judkins that there is always a difference of some 40 miles in a ship's place, as found by observation on calculation, no doubt owing to incorrect tables, and no doubt, also, a fearful cause of wrecks.

Your obedient Servant,

D. J. C.

* This refers to the following quotation from a Letter signed E. L. Garbett, which appeared in the *Correspondent* of February 3, 1866:—

"Instead of disputing whether π *does* $= 3{\cdot}125$, you see I have taken for granted that it does, and the most important result I can deduce is this, Sir,—that any creature in the livery of human shape who knows and can prove it to be so, and fiddles while the ships are sinking, and instead of moving a little finger towards the production of the true tables, eats and drinks, and calls Astronomers names, must be such a "*natural born noble*" as heaven preserve us natural born blunderers, Sir, from ever becoming." It will be time enough for me to proceed to the construction of "*true tables*" when Mathematicians and Astronomers shall have *honestly* admitted that $\pi = 3{\cdot}125$.

J. S.

This Letter must be taken—as it was intended to be taken—ironically; and the last paragraph is positively untrue. In a Paper read at a Literary and Philosophical Society I stated, and had Captain Judkin's permission to do so, that between the latitudes of 40° and 50°, he could never make his "*ship's place*," as found by a lunar observation, agree within 10 to 15 miles with his ship's position as found by chronometer, even under the most favourable circumstances for taking his lunar observation: and on no other occasion have I ever made a public use of Captain Judkin's name.

Now, Bennett's construction of Fig. 2 is perfectly correct. Future Geometers may give "*the honour and glory*" of discovering the geometrical quadrature to Bennett, (be it so), which according to Mr. J. Radford Young is of "*no theoretical or practical value.*" What matters it, who gets "*the honour and glory*"? I have my reward in the consciousness of having used my best endeavours, to deliver geometrical and mathematical science from the absurdities that have hitherto shrouded them with reference to π's true value.

Well, then, would it not be very absurd if I were to say that either Bennett or myself constructed the figure by the "*rule of thumb*"? Would it not be equally absurd if I were to say that I can furnish the proof that the square and circle are equal in superficial area, without the aid of numbers? Mathematicians have never admitted Bennett's construction of the figure, and if they admit the construction to be correct, how can they prove our statement that the square and circle are equal in superficial area, to be untrue, either with or without the aid of numbers? It is self-evident that A D and B C are

diameters of the circle, by construction ; and it follows, that O A, O B, O D and O C are radii of the circle ; and it is admitted that "*we may make radius unity:*" but, even if this be not admitted, it cannot be denied, that Mathematicians *make radius unity* their starting point, in the search after the circumference of a circle of diameter unity ; or in other words, in their search for the ratio of diameter to circumference in a circle : and they assume the symbol π to denote the circumference of a circle of diameter unity. The higher branches of mathematics have done nothing towards finding π's true value, and so far as our "*recognised Mathematicians*" are concerned, "*π still lies lurking in his den.*" Mathematicians may assert that a square equal in surface to a given circle, cannot be constructed, "*isolated and exhibited,*" but this is a mere assertion without a shadow of proof, and, moreover, is not true. Well, then, there is one point upon which all Mathematicians must be agreed, and that is, that if A O, a radius of the circle in Fig 2, = unity, the length of B A C a semi-circumference of the circle will be equal to the arithmetical value of the symbol π.

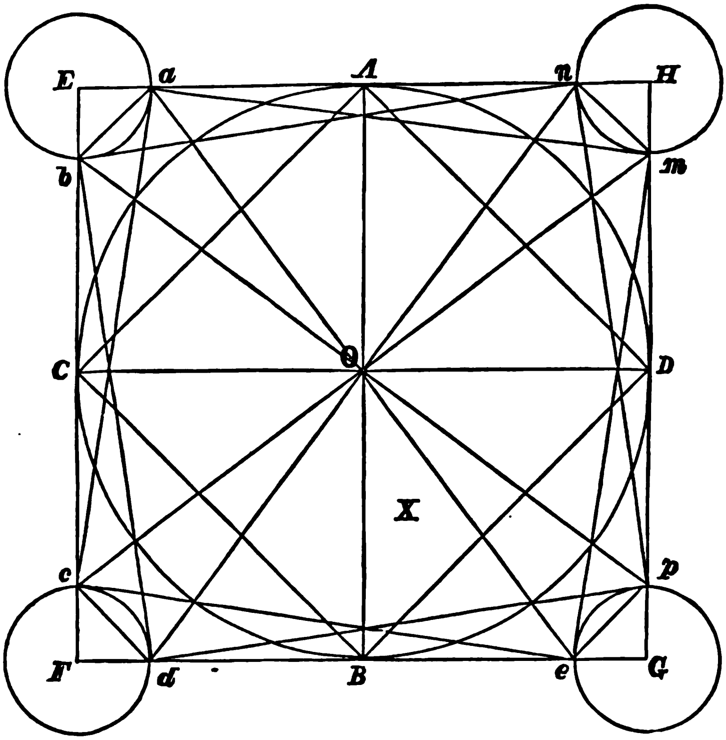
The geometrical figure represented by the diagram (Fig. 2), is constructed as follows:—Draw two straight lines at right angles, intersecting at the point O, and with O as centre, and any radius, describe the circle. It is self-evident, that A D and B C are diameters of the circle. Produce O A, O B, O D, and O C to the points E, F, G, and H, making A E, B F, D G, and O H equal to one fourth part of the radius of the circle, and join E F, F G, G H, and H E, and so construct the square E F G H, which is equal in superficial area to the circle.

Proof: Making radius unity, O A a radius of the circle $= 1$. $(OA + \frac{1}{4} OA) = (1 + \cdot25) = 1\cdot25 = OE$, and O F, O G, and O H = O E, by construction; and it follows, that $2 (OE) = 2(1\cdot25) = 2\cdot5 = EG$, a diagonal of the square E F G H. But, by Euclid, Prop. 47: Book I: $(OE^2 + OF^2) = (1\cdot25^2 + 1\cdot25^2) = (1\cdot5625 + 1\cdot5625) = 3\cdot125 = EF^2 =$ area of the square E F G H; therefore, $\sqrt{EF} = \sqrt{3\cdot125} = EF$, a side of the square EFGH. But, $3\cdot125\,(OA^2) = (3\cdot125 \times 1 \times 1) = 3\cdot125 =$ area of the square E F G H; and since $\pi r^2 =$ area in every circle, it follows, that $\pi (OA^2) =$ area of the circle. But, F G = E F, by construction; and by Euclid: Prop. 47: Book I.: $EF^2 + FG^2 = (\sqrt{3\cdot125^2} + \sqrt{3\cdot125^2}) = (3\cdot125 + 3\cdot125) = 6\cdot25 = EG^2$; therefore, $\sqrt{EG^2} = \sqrt{6\cdot25} = 2\cdot5 = EG$. Hence: The ratio between the diameter of the circle and diagonal of the square E F G H is as 2 to 2·5; and when Mathematicians can prove that this is not the true ratio between A D and E G, they will be able to prove that the square E F G H and the circle are not equal in superficial area—but not till then.

The geometrical figure represented by the diagram, Fig. 3, appears at first sight to be somewhat complicated, but on careful examination it will be found to be extremely simple. It is readily constructed in the following way:—

Draw two straight lines at right angles intersecting at O. With O as centre and any radius, describe the circle X, and about it circumscribe the square E F G H. With each of the angles of the square, E F G H as centre, and a demi-semi-radius of the circle X as interval, describe the four small circles. Join *a b*, *c d*, *e p*, *m n*, *n b*, *b d*, *d p*, *p n*, *a c*, *c e*, *e m*, *m a*, A C, C B, B D, and D A.

FIG. 3.



This remarkable geometrical figure is a study for Geometers, and I shall not go into all its properties; indeed, if I were to attempt to do so, I should have to go over much of the same ground that I have already "*trodden*" in my Letters of the 9th, and 16th November, 1868, to Mr. J. M. Wilson, Mathematical master of Rugby School, and those Letters are in print: but I may, without impropriety, direct the attention of Mathematicians to certain facts, which I have not brought out in those Letters.

The eight angles A O *a*, A O *n*, D O *m*, D O *p*, B O *e*, B O *d*, C O *c*, and C O *b*, at the centre of the circle X, are equal angles of 36° 52′: and the four angles *a* O *b*, *c* O *d*, *e* O *p*, and *m* O *n*, at the centre of the circle are equal angles of 16° 16′; and the twelve angles are together equal to four right angles; and having fixed 90° as the measure of a right angle = 360°. No good Geometer, even if his knowledge of Mathematics be somewhat limited—if the knowledge he has be rightly applied—can have the slightest difficulty in convincing himself of the numerical values of these angles. Now, if we take one of the squares on the radius of the circle X, say the square O B F C, and draw the diagonal O F, bisecting the angle O and its subtending chord *c d* at a point, say N, dividing the isosceles triangle O *c d* into two equal right-angled triangles O N *c* and O N *d*; it follows, that the angles N O *c* and N O *d*, will be angles of 8° 8′: but no man in the world can prove this without the aid of mathematics: not only so, but no man can prove the value of the angles at the centre of the circle, unless he know how to make a right use of mathematics. Had Mr. Gibbons known how to rightly apply Mathematics to Geometry, we should not have found him making the angles C O *c* and B O *d* angles of $36° \, 52' + x$, and the angles N O *c* and N O *d* angles of $8° \, 8' - y$.

Again: O B is a radius of the circle X: $\frac{3}{4}$ (O B) = B *d* = B *e*, by construction: and $\frac{1}{3}$ (B D) or $\frac{1}{3}$ (B *e*) = G *e* = F *d*, by construction. The sum of O B and B *e* = F *e*: and similarly, the sum of O B and B *d* = G *d*: the difference of O B and B *e* = G *e*, and similarly, the difference of O B and B *d* = F *d*. But, F *c* = F *d*, by construction, and F is a right angle;. therefore, C F *e* is a

right-angled triangle: and similarly, $Gp = Ge$, by construction, and G is a right angle; therefore, pGd is a right-angled triangle; therefore, the triangles cFe and pGd are similar and equal right-angled triangles: therefore, $(CF^2 + Fe^2) = (pG^2 + Gd^2)$, and this equation = area of the squares *c e m a*, and *p d b n*, standing on the circle X: The difference between cF^2 and $Fe^2 = (Fe^2 - cF^2)$, and the difference between pG^2 and $Gd^2 = (Gd^2 - pG^2)$; therefore, $(Fe^2 - cF^2) = (Gd^2 - pG^2)$, and this equation $= 6\,(OB \times \frac{1}{2}\,OB)$, and is therefore equal to the area of a regular inscribed dodecagon to the circle X. Let A denote the area of the dodecagon. To A, add $\frac{1}{24}$th part. Then: $(A + \frac{1}{24}A) = 3\frac{1}{8}\,(OB^2)$, = area of the squares *c e m a* and *p d b n*, and makes these squares and the circle X exactly equal in superficial area.

THEOREM I.

From a given area of the square A C B D inscribed in the circle X, find the area of the circle X and prove that it is equal to $3\frac{1}{8}(OB^2)$.

It is self-evident, that A C B D is an inscribed square, and E F G H a circumscribed square, to the circle X. Let A denote the area of the inscribed square: C denote the area of the circle: and B denote the area of the circumscribing square. Then, the following formula solves the theorem:—

$\{(A + \frac{1}{4}A) + \frac{1}{4}(A + \frac{1}{4}A)\} = C : \frac{C}{\frac{1}{4}(3\frac{1}{8})} = B : \frac{\sqrt{B}}{2} = OB$: and, $3\frac{1}{8}\,(OB^2) = C$.

Proof: Let $A = 60$. Then: $(60 + \frac{1}{4}\,60) + \frac{1}{4}\,(60 + \frac{1}{4}\,60) = (75 + 18{\cdot}75) = 93{\cdot}75 = C : \frac{C}{\frac{1}{4}(3\frac{1}{8})} = \frac{93{\cdot}75}{{\cdot}78125} = 120$

$= B$: and $\frac{\sqrt{B}}{2} = \frac{\sqrt{120}}{2} = \sqrt{30} = O\,B$.

$\therefore 3\frac{1}{8}(O\,B^2) = 3{\cdot}125 \times 30 = 93{\cdot}75 = C$.

THEOREM 2.

From a given area of the square A C B D inscribed in the circle X, find the area of the squares *c e m a* and ***p d b n***, and prove that the area of these squares $= 3\frac{1}{8}(O\,B^2)$.

Let A denote the area of the inscribed square to the circle X, and B denote the area of the circumscribed square. The following formula solves the theorem :—

$2(A) = B : \frac{\sqrt{B}}{2} = O\,B : \frac{1}{4}(O\,B) = c\,F : 7\,(c\,F) = F\,e$: and, $(c\,F^2 + F e^2) = c\,e^2 = 3\frac{1}{8}(O\,B^2)$.

Proof. Let $A = 60$. Then, $2\,(A) = (2 \times 60) = 120 = B : \frac{\sqrt{B}}{2} = \frac{\sqrt{120}}{2} = \sqrt{120 \div 2^2} = \sqrt{\frac{120}{4}} = \sqrt{30} = O\,B : \frac{1}{4}(O\,B) = \frac{1}{4}(\sqrt{30}) = \sqrt{\frac{1^2}{4^2} \times 30} = \sqrt{\frac{1}{16} \times 30} = \sqrt{{\cdot}0625 \times 30} = \sqrt{1{\cdot}875} = c\,F : 7\,(c\,F) = 7\,(\sqrt{1{\cdot}875}) = \sqrt{7^2 \times 1{\cdot}875} = \sqrt{49 \times 1{\cdot}875} = \sqrt{91{\cdot}875} = F\,e$; therefore, $c\,F^2 + F\,e^2 = \sqrt{1{\cdot}875^2} + \sqrt{91{\cdot}875^2} = 1{\cdot}875 + 91{\cdot}875 = 3\frac{1}{8}\,(O\,B^2)$; that is, $(1{\cdot}875 + 91{\cdot}875) = (3{\cdot}125 \times 30)$, and this equation $= c\,e^2$ and $p\,d^2 = 93{\cdot}75 =$ area of the squares *c e m a* and *p d b n*: and it follows, that these squares are equal to the circle X in superficial area. If this were not so, the solution of these theorems would be utterly impossible.

Hence: The angle *c e* F = half the angle *c* O *d*; and the angle *p d* G = half the angle *p O e*: and similarly, the angle *a c* E = half the angle *a* O *b*, and the angle *d b* F

= half the angle *b* O *a*: and the comparison may be carried to other triangles in the diagram; and it follows, that the angles *c* O *d*, *e* O *p*, *m* O *n*, and *a* O *b* in Fig. 3, and the angles E D F and P C G in the diagram A, page 9, are equal angles of 16° 16′. How could these things be, if the circle X and the squares *c e m a* and *p d b n* were unequal in superficial area?

From the geometrical figure (Fig. 3), we may found other theorems, making the arèa of the circumscribing square to the circle X the given quantity, and prove that the squares *c e m a*, *p d b n*, and the circle X are exactly equal in superficial area. In our proofs, the symbol π "*need not appear;*" but it is not worth my while to waste my time on the existing race of "*recognised Mathematicians,*" and I shall leave it to future Mathematicians to work out the proofs. "*Recognised Mathematicians*" of the present day, may *assume*, and *assert* if they please, that nothing can be true either in Geometry or Mathematics, that they are not pleased to admit: but when common sense shall have overcome the prejudice and absurdities engendered by false teaching, it will become plain enough to Mathematicians and Geometricians, that the ratio between the diameter and circumference in every circle, can be nothing else but 1 to $3\frac{1}{8}$.

Mr. Glaister says, in his Letter of the 27th October: "*The excessive complication of my diagrams totally excludes any one from completely keeping up with my geometrical demonstrations step by step, but it is easy to see I am wrong.*" (In this quotation I have simply substituted *my* for *you*, and *I am* for *you are.*) This is simply ridiculous, and comes with a very bad grace from one who has been at such pains to prove me wrong. If

he is incompetent to follow my "*geometrical demonstrations step by step,*" had he not better have remained silent?

It has often been said to me:—"*Stick to Algebra.*" Well, then, let us see what Algebra will do for us, when tested by "*that indispensable instrument of science, Arithmetic.*"

With reference to the geometrical figure (Fig. 3), take the following algebraical formula.

$\frac{3}{4}$ (O B) = B d, and $\frac{1}{4}$ (O B) = F d.

$\therefore$ (B d + F d) = B F = O B.

Can this algebraical formula prove anything mathematically? Certainly not, until we put an arithmetical value on O B!

Let O B = 1.

Then:

$\frac{3}{4}$ (O B) = $\frac{3}{4}$ (1) = ·75 = B d.

$\frac{1}{4}$ (O B) = $\frac{1}{4}$ (1) = ·25 = F d.

$\therefore$ B d + F d = ·75 + ·25 = 1 = B F = O B.

But:

Let O B = $\sqrt{30}$.

Then:

$\frac{3}{4}$ (O B) = $\frac{3}{4}$ ($\sqrt{30}$) = $\sqrt{16{\cdot}875}$. = B d.

$\frac{1}{4}$ (O B) = $\frac{1}{4}$ ($\sqrt{30}$) = $\sqrt{1{\cdot}875}$ = F d.

But, the sum of $\sqrt{16{\cdot}875}$ and $\sqrt{1{\cdot}875}$ is not equal to B F or O B, that is = $\sqrt{30}$. When will Mathematicians be induced to make a study of the peculiar properties of the square roots of numbers, and apply the higher branches of Mathematics,—such as the differential and integral calculus—to their proper use?

Well, then, by hypothesis, O B = $\sqrt{30}$, and B F = O B, by construction; therefore, $\frac{3}{4}$ ($\sqrt{30}$) = $\sqrt{16{\cdot}875}$ = B d: and, by Euclid: Prop. 47: Book 1: (O B^2 +

$B d^2) = (\sqrt{30^2} + \sqrt{16\cdot875^2}) = (30 + 16\cdot875) = 46\cdot875$ $= O d^2$; therefore, $O d = \sqrt{46\cdot875}$. But, $(O B^2 + B d^2 + O d^2) = (30 + 16\cdot875 + 46\cdot875) = 3\frac{1}{8}\,(O B^2)$, and this equation = area of the squares *cema*, and *pdbn* standing on the circle X.

Finally: Construct an equilateral triangle (Euclid: Prop. 1: Book 1). About the triangle circumscribe a circle (Euclid: Prop. 5: Book 4). In the circle inscribe a square (Euclid: Prop. 6: Book 4). About the circle describe a square (Euclid: Prop. 7: Book 4). Inscribe a regular dodecagon in the circle. Euclid shews us (Prop. 15: Book 4) how to inscribe a regular hexagon in a given circle, and we have merely to double the number of sides to get the inscribed dodecagon.

I must now assume a Mathematician to have constructed the figure, and have it before him.

Let A denote the area of the inscribed square: B denote the area of the circumscribed square: C denote the area of a square on a side of the equilateral triangle: and D denote the area of the inscribed regular dodecagon.

The symbol π is adopted by Mathematicians to denote the length of the circumference of a circle of diameter unity, and it is asserted by them that the value of the symbol π cannot be expressed with arithmetical exactness, which is equivalent to saying, that the value of π cannot be found. This I deny, and maintain on the contrary, that the length of the circumference of a circle of diameter unity, can be arithmetically expressed exactly.

We want to find π's arithmetical value, and if r denote the radius of a circle of diameter unity, it is admitted by Mathematicians, that $2\pi(r)$ or $2\pi(\cdot5)$ is

what we are in search of; but, as the property of one circle is the property of all circles, it follows, that $2\pi(r)$ = circumference in every circle.

It must be self-evident to every Mathematician, that if the arithmetical value of π is to be found, it can only be found by means of something we *know*. Surely! no Mathematician would be so absurd as to say, that we can find it by means of something we don't know. Well, then, what do we *know* with reference to this geometrical figure?

We *know*, that $(A + \frac{1}{2} A) = (B - \frac{1}{4} B)$, and that this equation = C. I may take this as granted by Mathematicians, for, to suppose a Mathematician disputing it, would be equivalent to supposing him to maintain, that B is not the double of A. We *know*, that if r denote the radius of the circle, $6(r \times \frac{1}{2} r) = C$: and from this we get the equation $C = D$. This equation may or may not have occurred to our "*recognised Mathematicians.*" If they dispute it, let them controvert it. If they admit it, let them trace the consequences to which it leads.

THEOREM.

From a given value of A, find r, that is, find the radius of the circle, and prove that $6(r \times \frac{1}{2} r) = C = D$.

Let $A = 60$. Then: $2(A) = (2 \times 60) = 120 = B$: $\sqrt{B} = \sqrt{120}$ = the diameter of the circle. $\frac{\text{Diameter}}{2} = \frac{\sqrt{120}}{2} = \sqrt{30} = r$: and $6(r \times \frac{1}{2} r) = C = D$: and since $(A + \frac{1}{2} A) = (B - \frac{1}{4} B)$, it follows, that C and D = 90, when A = 60.

Proof: $6(r \times \frac{1}{2} r) = 6(\sqrt{30} \times \frac{1}{4} \sqrt{30}) = 6(\sqrt{30} \times \sqrt{7 \cdot 5}) = 6(\sqrt{225}) = \sqrt{6^2 \times 225} = \sqrt{36 \times 225} = \sqrt{8100} = 90 = C = D$. Q. E. D.

Now ,$(D + \frac{1}{24} D) = (90 + \frac{1}{24} 90) = (90 + 3 \cdot 75) = 93 \cdot 75 = 3\frac{1}{8} (r^2)$ = area of the squares *c e m a* and *p d b n* standing on the circle X; and it follows, that the circle X and the squares *c e m a* and *p d b n* are equal in superficial area.

Let P denote the perimeter of a regular hexagon: C the circumference of a circumscribing circle: B the area of the circle: and D the area of a regular dodecagon inscribed in the circle. Then: By analogy or proportion, P : C : : D : B.

I must now conceive some Mathematician to be carefully examining my argument, with a geometrical figure before him, consisting of a circle, with its inscribed and circumscribed squares, an inscribed equilateral triangle, and an inscribed regular dodecagon.

Let A denote the area of the inscribed square. I have proved that the radius of the circle $= \sqrt{30}$, when $A = 60$. Then: $\{(A + \frac{1}{4} A) + \frac{1}{4}(A + \frac{1}{4} A)\} = \{(60 + 15) + \frac{1}{4}(60 + 15)\} = (75 + 18 \cdot 75) = 3\frac{1}{8}(\sqrt{30^2})$; and this equation $= 93 \cdot 75$ = area of the circle. From this deduct $\frac{1}{25}$ part $= \frac{93 \cdot 75}{25} = 3 \cdot 75$. Then: $(93 \cdot 75 - 3 \cdot 75) = 90$ = area of the inscribed regular dodecagon: and if C denote the circumference of the circle: P the perimeter of the inscribed hexagon, B the area of the circle, and D the area of the inscribed dodecagon; then, by analogy or proportion, P : C : : D : B, and since 90 : 93·75 : : 3 : 3·125, it follows, that 3·125 is the circumference of a circle of diameter unity.

Proof:

From 3.125 deduct $\frac{1}{25}$th part $= \frac{3 \cdot 125}{25} = \cdot 125$. Then: $(3 \cdot 125 - \cdot 125) = 3$ = the perimeter of a regular in-

scribed hexagon to a cirele of diameter unity: and $P : C :: D : B$; that is, $3 : 3{\cdot}125 :: 6\,(r \times \frac{1}{2} r) : \frac{3{\cdot}125}{4}$: and it is incontrovertible, that $\frac{\pi}{4}$ = area of a circle of diameter unity, whatever be the value of π : and since every other value of π but 3·125 would make the perimeter of a regular inscribed hexagon to a circle of diameter unity, either greater or less than 3, the conclusion is irrefragable that 3·125 is π's true arithmetical value.

Hence. Let C denote the circumference of a circle of diameter unity. Then: $\frac{C}{6} = \frac{3{\cdot}125}{6} = {\cdot}5208333$ with 3 to infinity, is the value of the arc subtending a chord equal to radius. From this deduct $\frac{1}{25}$th part $= \frac{{\cdot}5208333,\ \&c.}{25}$ $= {\cdot}0208333$, &c.: therefore, (·5208333, &c. — ·0208333, &c.) = ·5 = radius; and it follows, that the ratio of chord to arc is as 24 to 25 : and since the property of one circle is the property of all circles, it follows, that $\frac{24}{25}$ (circumference) = perimeter of a regular inscribed hexagon, in every circle.

It is self-evident that the circumference of a circle is greater than the perimeter of its inscribed regular hexagon. How much greater? Greater, I say, by $\frac{1}{25}$ part of the circumference of the circle exactly. Hence: If C denote the value of an arc of a circle subtending a chord equal to radius, and R denote the difference between the arc and the chord; then, $\frac{1}{25}$ expresses the ratio of R to C, and Mr. Gibbons admits that $\frac{1}{25}$ is a ratio : and yet, in the face of these facts, he and other Mathematicians persist in *asserting*, that I make the perimeter of a certain regular polygon either equal, or greater, than the circumference of

its circumscribing circle. Let Mathematicians go ***honestly*** to work to find the ratio of R to C, and they will soon discover that $\frac{1}{3 \cdot 125}$ expresses the ratio of diameter to circumference in every circle: not only so, but they will also discover the absurdity of their "*exceeding great fancy,*" that they can find π's true arithmetical value, by means of inscribed and circumscribed polygons to a circle.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
28th July, 1869.

MY DEAR SIR,

My object is to convince you, that you are wrong in your application of Mathematics to pure Geometry. This being my object, it is not necessary to wait your solution of the theorems given in my Letters of the 24th inst., and yesterday. Your Letter of the 22nd inst. is amply sufficient for my present purpose.

I *assume* that you are a Geometer, and I have admitted over and over again in the course of our long correspondence, that you are a Mathematician. Well, then, it appears to me that, with reference to the geometrical figure represented by the diagram enclosed in my Letter of yesterday, (see diagram A, page 9), you can have no difficultyin *convincing* yourself, that the angles D E F and P C G are equal angles; and that the angles B E D, B E F, and P A G, are equal angles; and it follows, that the angle P A G = half the angle P C G. It appears to me unnecessary, to work out the proofs.

Now, let the angle D E F denote an angle of 30°. Then: Because the angle D E F is bisected by the line E B, it follows, that the angles B E D and B E F are equal angles of 15°: and, because the line E B is perpendicular to the base D F in the isosceles

triangle E D F; it follows, that B D and B F are equal: and it is self evident, that B D and B F are the *geometrical* sines of the angles B E D and B E F; and the line E B, the *geometrical* co-sine of, and common to, the angles B E D and B E F.

It is self-evident, that an angle of 16° 16′ is intermediate between an angle of 30° and an angle of 15°. "*No Mathematician—I should think—could make a blunder in so simple a thing as this;*" and 'no Mathematician—I should think—could make such a blunder as to deny, that an angle of 16° 16′ may and does exist.

Well, then, let the angle D E F in the isosceles triangle E D F, denote an angle of 16° 16′. Then: $\frac{16° \, 16'}{2} = 8° \, 8'$ = the angles B E D and B E F; and, it is self-evident, that B D and B F are the geometrical sines of the angles B E D and B E F, in the right-angled triangles E B D and E B F.

Now, according to Hutton's Tables, the sine of an angle of 8° 8′ is ·1414772: and according to your reasoning, the double of D B = 2 × ·1414772 = ·2829544, is the sine of the angle D E F. On this point your reasoning and the reasoning of Mr. Walter W. Skeat are identical; or in other words, you both *assume* the correctness of Hutton's Tables. Now, according to Hutton's Tables, ·28295144 is the sine of an angle of 16° 26′ + x.

The figure A D G C is a quadrant of the circle X, and it follows, that the angle A C G is a right angle = 90°. But, the angle A C G is divided into the three angles B C A, B C P, and P C G; and in your Letter of the 22nd instant, you make the equal angles B C A and B C P to be angles of 36° 52′ + x, and, according to Hutton, the angle P C G is an angle of 16° 26′ + x. This makes the sum of the three angles B C A, B C P, and P C G, = 90° 10′ + x, that is, *greater* than a right angle by 10′ + x.

How often have you, my dear Sir, charged me with assuming the thing to be proved? Do you not assume the correctness of mathematical Tables? Is not one of the points at issue between us, the correctness of Tables, whether calculated to 6 or 7 places of decimals? What have I assumed in this communication? I frankly admit that I have assumed the angle P A G to be equal to half the angle P C G, which makes the angle P C G = the angle D E F. But I have assumed more. I have assumed that you are a

Geometrician, and as such, can prove these facts without my assistance. If I am wrong in making these assumptions, you as a Mathematician can surely prove it, and so, vitiate my conclusion, that Hutton's Tables would make the angle A C G greater than a right angle.

Well, then, it is admitted that I have assumed the equality of the angles D E F and P C G, and I can prove that these angles are equal, if you can't. But surely, as a Geometer and fair controversialist, you will make an attempt to prove me wrong, before you expect me to prove that I am right.

I had written so far when your favour of yesterday came to hand. I shall wait the result of to-morrow afternoon's post—which may bring me your reply to my Letter of the 24th instant—before I answer it.

Believe me, my dear Sir,

Yours very truly,

JAMES SMITH.

From the facts given in the second and third paragraphs of the foregoing Letter, it follows, that with reference to the diagram A on page 9, the squares E D M N and A G K L are equal, and exactly equal in superficial area to the circles on which they stand: and it is self-evident, that C A, C D, C P, and C G, are radii of the circle X; and C B the radius of the circle Y. It is also self-evident, that A G K L is an inscribed square to the circle X, by construction.

Now, $\frac{\pi}{\pi} = 1$, whatever be the arithmetical value of π, whether π be commensurable or incommensurable: $\frac{1}{2} = \cdot 5$ = radius of a circle of diameter unity: and $\pi\ (\cdot 5^2) = \frac{\pi}{4}$ = area of a circle of diameter unity, whatever be the value

of π : and because the property of one circle is the property of all circles, it follows, that $\pi(r^2)$=area in every circle ; and consequently, π (C B^2) = area of the circle Y, and π (C D^2) = area of the circle X: whatever be the arithmetical values of C B and C D. It is admitted by Mathematicians, that π (r^2) = area in every circle; but they make no use of this fact, in their search after π ; but dogmatically maintain that C B^2 and C D^2 must be multiplied by their *fanciful* π to find the areas of the circles Y and X, and treat every man as a fool who dares to differ from them.

On page 32 I have given the following theorem, and furnished the algebraical formula that solves it :—

THEOREM.

From a given area of a square, find the area of its circumscribing circle, and prove that it is equal to $3\frac{1}{8}$ times the area of a square on the radius of the circle.

The following is the converse of this theorem :—

From a given area of a circle, find the area of its inscribed square, and prove that the area of the circle is equal to $3\frac{1}{8}$ times the area of a square on the radius of the circle.

Let A denote the area of the circle, B the area of its inscribed square, and r the radius of the circle. The following algebraical formula solves the theorem :—

$$\{(A - \tfrac{1}{5} A) - \tfrac{1}{5} (A - \tfrac{1}{5} A)\} = B.$$

$$\sqrt{\frac{2B}{2}} = r \therefore 3\tfrac{1}{8} (r^2) = A.$$

Proof: Let A = 60. Then : $\{(A - \frac{1}{5} A) - \frac{1}{5} (A - \frac{1}{5} A\}) = \{(60 - 12) - \frac{1}{5} (60 - 12)\} = (48 - 9{\cdot}6) = 38{\cdot}4$

$= B: 2B = (2 \times 38{\cdot}4) = 76{\cdot}8 =$ area of a circumscribed square to the circle ; and it follows, that $\sqrt{76{\cdot}8} =$ diameter of the circle: $\frac{\text{Diameter}}{2} = \frac{\sqrt{76{\cdot}8}}{2} = \sqrt{19{\cdot}2} = r$: and $3\frac{1}{8}\,(r^2) =$ $(3{\cdot}125 \times 19{\cdot}2) = 60 =$ the given area of the circle.

If a "*reasoning geometrical investigator*" apply this theorem to the circle X and its inscribed square A G K L in the diagram A, he will find that $3\frac{1}{8}(CB^2) =$ area of the square A G K L ; and $3\frac{1}{8}(CA^2) =$ area of the circle X ; and it follows, that the square A G K L and the circle Y on which it stands, are exectly equal in superficial area. If Mathematicians attempt to controvert my solution of this theorem, where will they be ? As Professor de Morgan would say :—"*In* π *glory*" !

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

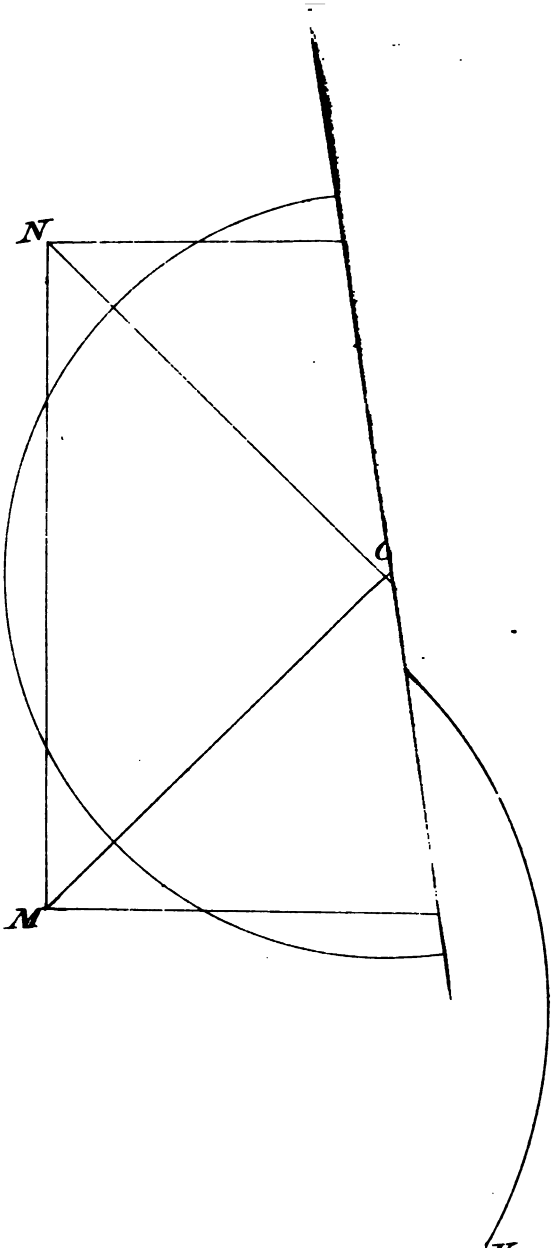
BARKELEY HOUSE, SEAFORTH,
30th July, 1869.

MY DEAR SIR,

I found your favour of the 27th instant awaiting me on my return home rather late last evening.

It appears to me you forget, that in your Letter of the 22nd inst. you admitted my data ; that is to say, I understand you to have admitted, that, with reference to the geometrical figure in my Letter of the 21st inst., we have agreed to adopt as data A B = 3 and B C = 4, which makes A C = 5, B D = 1, and A B to B D in the ratio of 3 to 1. This precludes us from assuming a value of the angle B A C. On our adopted data the angle B A C can but have one value, and the question is, What is that value ?

You say: "*There is a lapsus in your* (my) *Letter. Angle* D A B *is half angle* C. *You mean half the angle* B A C." Indeed, my dear


N
M
K

Sir, I do not mean half the angle B A C. I adhere to what I have said. The angle D A B = half the angle A C D. D B is the geometrical sine, and A B the geometrical cosine of the angle D A B.

You next say: "Given B A C ... 16° 16'."

Then:

$$\frac{BD}{AD} = \text{Sin. } 8^\circ\ 8' = \cdot 1414772.'$$

You surely cannot mean this.

$\frac{BD}{AD}$ is the sin. of the angle D A B, and the trigonometrical sin. of this angle is $\frac{1}{\sqrt{10}} = \cdot 3162277$ to 7 places of decimals; and the angle D A B is an angle of 18° 26', when our data are A B = 3 and B C = 4.

The enclosed diagram *(see diagram B)*, is a fac-simile of that enclosed in my Letter of the 27th inst., with the following addition:—

From the point F draw a straight line parallel to B P, and therefore tangental to the circle Z, to meet and terminate in the circumference of the circle X. This line meets the circumference of the circle X at the point G, and intersects the line C P, a radius of the circle X, at the point T.

THEOREM.

Find the length of the lines F T and T G, and prove that $CT^2 + TG^2 + 2(TF \times TG) = CG^2$, our data being A B = 3 and B C = 4.

Waiting the favour of a solution of this theorem,

Believe me, my dear Sir,

Very truly yours,

JAMES SMITH.

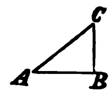
THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

LANEAST, LAUNCESTON,
31st July, 1869.

MY DEAR SIR,

Without any disrespect to you, I am most anxious ***not*** to revive the discussion between us, which has been already so useless and so long. If I write a hundred letters, they would be, in substance, only the same that I have already written, many of them printed in your books.

It is clear beyond question that we cannot agree on the simplest elementary trigonometry—and what is the use of discussing figures and diagrams when we don't agree on the meaning of "a sine?" What use, when you persist in saying that I have confused different things? It is in vain that I remind you that I have only used "sine" as defined by



$$\text{Sin. A} = \frac{\text{C B}}{\text{A C}}$$

—the ratio of one length to another—but which becomes itself a length when A C is given in length. Thus Hutton's natural sines are actual *lengths* to radius 1.

Example.—Sin. $15°$ = (·2588 ......) in feet, if radius = 1 foot.
Sin. $30°$ = ·5 feet.
Sin. $90°$ = 1 foot.—all *lengths*.

Now, when in the face of this I read your page 396 (last book) I feel how utterly futile is any attempt on my part to assert the correctness of my calculations.

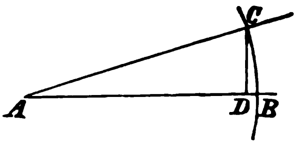
Again: It vexes a mathematician to be met by a proposed refutation based on the most egregious blunder. ***Any*** man may fall into a mistake, but to persist in it when it is obvious, renders all argument hopeless. To Mr. Skeat you oppose reasoning quite fallacious, for you assume that sin. $20°$ = 2 sin. $10°$.

Now, sin. $10°$ = ·1736482..., but sin. $20°$ is *less* than the double of this.

$$\text{Sin. } 2\,\phi = 2 \text{ sin. } \phi \text{ cos. } \phi$$

You have omitted the factor, cos. ϕ, which lessens the value of the term. I have treated it as if sines varied directly as angles. So, in your letter to me, you would make sin. $16° \, 16'$ = 2 sin. $8° \, 8'$, but it is less. Sin. $60°$ is ***not*** 2 sin. $30°$ = 1. One would think ***that***

merely drawing the figure would shew this. As I used the natural sine only in my calculation of chord 15°, it matters nothing to me what other kind of "sine" you may devise; but if natural sine is not the same as trigonometrical, or geometrical, or arithmetical, will you be so kind as to *define each*, and *draw* such sines if possible. Thus $A = 15^\circ$ suppose, the only sine I ever saw mentioned in books on trigonometry $\therefore \frac{CD}{AC} = \sin. A$.



But you have a right to invent a new kind of measurement provided you define or explain it. Now, if your opponents have treated of any other sine than $\frac{CD}{AC}$, which is a *length*, if A C is given in length.

"Log.-sin." is only a short expression for the logarithm of the number which expresses the value of the sine; thus, sin. $30^\circ = \cdot 5$, and by tables the log. of $5 = \cdot 6989700$; therefore, log. $(\cdot 5) = \bar{1}\cdot 6989700$, and 10 being added to avoid negative indices, Log.-sin. $30^\circ = 9\cdot 6989700$. Using *such sines only*, I proved that if radius $= 1$, the chord of 15° is rather greater than $(\cdot 261)$, both expressing *length*. Until we can agree on this, it is useless to go further—a sheer waste of paper and time. Here I rest till I am convinced of my error. To my mind the demonstration is perfect. If you say it is erroneous and cannot make me see where the error lies, surely we are brought to a standstill!

In your last letter you say, there must be some angle that is exactly $16^\circ\ 16'$ or $8^\circ\ 8'$. *No doubt of it;* but then, the *sides* of your triangle will not be *exactly* what you have put them. As to logarithms, they are inexact (so I allow), incorrect, and "fallacious" (so you declare), and yet, with the strangest inconsistency, you employ them to get out *exact* values! You use 7 places of decimals: if you took 8 places the results would be slightly different; but this does not shake your confidence in their power to give exact values. I would fairly put it to you—What possible good can come of further disputation? I can but repeat what I have written too often already.

Yours very truly,

G. B. GIBBONS.

P.S.—The only new thing is—my request for an explanation of the different kinds of sine.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH.
31st July, 1869.

MY DEAR SIR,

In your favour of the 27th inst., you observe: "*I am sure you will agree with me—that it would be wasting time if we were to revive our long controversy, when I have nothing fresh to offer. I don't like to write perpetually the same things over again—nor would you wish to receive such repetitions.*"

The Rev. Professor Whithworth, in his Letter to me of the 5th December, 1868, put the following question: "*Do you not think it is rather impertinent to our investigations to send a question to test my capacity for constructive geometry?*" My answer was: "Certainly not, if I think *that* the best method of convincing you of truth." (See "*Geometry of the Circle*," page 66.)

I thought *our* correspondence had terminated with the Letter written nearly twelve months ago, in which I informed you that I had told your friend Professor Adams, whom I met at the meeting of the British Association, at Norwich, that before the next meeting of the Association I should bring out a work, and demonstrate the true ratio of diameter to circumference in a circle by means of angles. I should not have revived our controversy, but you, my dear Sir, having done so, I shall not close it, until I have either convinced you of geometrical TRUTH, or convicted you of mathematical dishonesty. If my speaking thus plainly offend you, you have only yourself to blame; but my conscience will not suffer me to permit geometrical truth to be *burked* by Mathematicians, so long as I can handle a pen in its defence.

Now, according to Euclid, Prop. 2: Book 2: "If a straight line (A B) be divided into any two parts (A C, C B) the rectangles contained by the whole and each of the parts, are together equal to the square of the whole line: that is, $AB \cdot AC + AB \cdot BC = AB^2$.

Upon this proposition of Euclid, I found the following theorem:—

Divide a straight line (A B) into two unequal parts (A C, C B), so that the ratio of A C to C B shall be such, that from a given value of one of the parts we can find the value of the other, and prove "*that the rectangles contained by the whole and each of the parts, are together equal to the square of the whole line*," that is, $AB \cdot AC + AB \cdot BC = AB^2$.

Now, my dear Sir, can you decline to solve this theorem "*without offence to your own conscience?*" Or, if you can't solve it, can you as a "*Christian and a gentleman*" hesitate to admit it?

I admit that the Proposition of Euclid, Theorem 2, Book 2, is irrefragable. But, I have proved (see, *Geometry of the Circle*, page 187), that Euclid is at fault, inasmuch as his proposition, Theorem 35, Book 3, is inconsistent with it, and it follows, that both propositions cannot be true.

Yours very truly,

JAMES SMITH.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

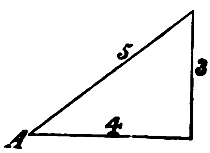
LANEAST, *2nd August*, 1869.

MY DEAR SIR,

I have already said that if the sides of a right-angled triangle are 3, 4, 5,

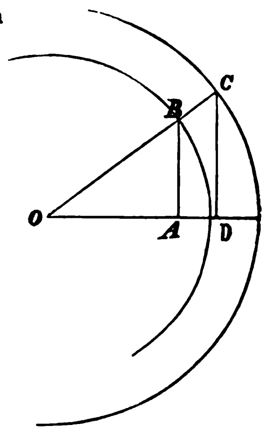
Sin. A = ·6.

A = 36° 52′ + a decimal of a minute.



If this does not satisfy you, I can do no better with other figures, and have no wish to enter on useless investigations.

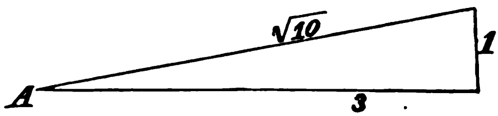
I gather from your Letter received on Saturday, that what you call a geomerical sine is a length, not a ratio—though a ratio must be implied, or else the same angle would have geometrical sines of all degrees of magnitude. B A or C D are equally Geometrical sines of the angle O.



I keep no copy generally of my Letters to you, and therefore cannot refer to them; but my replies to the same questions must always be the same. Treatises on Trigonometry mention only the sine of an *arc* (which is a length dependant on the radius) and sine of an *angle*, which

for the same angle is invariable. It is the ratio $\frac{\text{B A}}{\text{O B}}$. I have used *only this*. If O B = 1, the sine is B A, an actual *length*; and such are Hutton's "natural" sines, lengths of sine to radius unity. But I await your explanation of what *you mean* by the different kinds of sine: Being your own invention, you have a right to define them.

No doubt sin. $A = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$ $= \cdot 3162677$ as you



put it, and the angle lies between 18° 26′ and 18° 27′.

I am engaged in classics and mathematics on heights by barometer, but I would follow you in any *real* investigation of π. I see no use in discussing complicated figures and diagrams, where π is not introduced, and by which its value cannot be found. Indeed if you *could* find it at all, you would do it by figures far simpler than you draw. These only serve to lead you "off the scent," and make you forget that you have *never* proved any equality between a curvilinear and a rectlinear length or area. *This* is really what we aim at. You draw a lot of rectilinear figures, sometimes with circles in or about them, but you never attempt to prove any equality except between rectilinear figures. When you introduce π, it is somewhat thus—" Now *if* $\pi = 3\frac{1}{8}$ so-and-so will follow."

As Mr. Wilson says, your premises and your conclusions are identical, π is assumed $3\frac{1}{8}$ and comes out $3\frac{1}{8}$, because you discuss the triangles correctly (except in assuming *exact* values for the angles, corresponding to given sides), and thus you end where you began. You prove that your results agree with what you *assumed*, but all this shews nothing as to the value of π. You shew correctly what *would be the consequences* if $\pi = 3\frac{1}{8}$.

If you could even be induced to *begin again* your researches, with a resolve *not to assume any value of* π *till such value was proved*, then there might be some hope of your coming to a just conclusion; but adopting your method, *that comes out which was put in*, and you prove nothing but the correctness of your working. At any rate, I must go on, step by step, having proved that in a circle radius 1, chord 15° is rather greater than (·261), I must be con-

vinced of some error in the process. Till then it would be useless for *me* to go further. My proof is short, involves no tables and no logarithms. It is contained in one side of a sheet of paper.

Yours very truly,

G. B. GIBBONS.

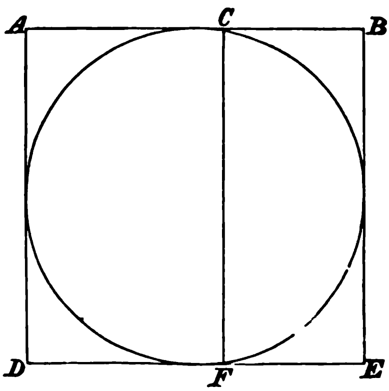
P.S.—I expect to go to the sea-side next week.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
4th August, 1869.

MY DEAR SIR,

Had you replied with your usual punctuality, I should have had an answer to my Letter of the 31st ult. yesterday afternoon: and, since I have no answer, I may fairly infer, that so far as you are concerned, our mathematical correspondence is at an end; and it follows, even if I have succeeded in convincing you of geometrical truth, that you have made up your mind not to admit it.



In the geometrical figure in the margin, let A B denote a straight line divided into two unequal parts at C. On A B describe the square A D E B, and inscribe the circle. From the point C, draw C F parallel to A D or B E.

Now, by Euclid: Prop. 2: Book 2: "If a straight line (A B) be divided into any two parts (A C, C B), the square of the whole line is equal to the sum of the rectangles contained by the whole line and each of its parts:" and I shall proceed to prove that this Theorem of Euclid is irrefragable.

It is self-evident, that the line A B can only be divided into two equal parts, when the line C F divides the area of the square A D E B into two equal parts: and it is also self-evident, that when the line C F divides the square A D E B into two equal parts, it also divides the circumference and area of the circle into two equal parts: but, it is equally self-evident, that A B may be divided into two unequal parts, by altering the position of the line C F. Now, from whatever point in the line A B, the line C F may be drawn parallel to A D or B E; it is axiomatic, if not self-evident, that A C and C B will denote the two terms of a ratio. I may here notice a remark of yours in your favour of the 19th of July You say:—"The sine of an angle is a ratio." Can a ratio have only one term? I trow not! If A C denote the sine, and A B the cosine of an angle, A C and A B denote the two terms of a ratio; and in this case A C and A B *will be* the sides that *include* the right angle in a right-angled triangle.

In the first place, let A C and C B, the two parts into which the line A B in the diagram is divided, be in the ratio of 9 to 7, and let A C = 40.

Then:

As $9 : 7 :: 40 : 31\frac{1}{9}$

$\therefore$ C B = $31\frac{1}{9}$, when A C = 40.

But, if C B = 40,

Then: As $7 : 9 :: 40 : 51\frac{3}{7}$

$\therefore$ A C = $51\frac{3}{7}$, when C B = 40.

Find a common denominator of $31\frac{1}{9}$ and $51\frac{3}{7}$.

Then: By analogy or proportion:

$9 : 7 :: 360 : 280$

$7 : 9 :: 280 : 360$

And, by alternation:

$9 : 360 :: 7 : 280$

$7 : 280 :: 9 : 360$

Now, whatever value we may put upon A C, or C B, the product of the means will be equal to the product of the extremes; and it follows, that the mean proportional of the means is equal to the mean proportional of the extremes; that is to say, $\sqrt{9 \times 280} = \sqrt{7 \times 360}$: but, I know of no other ratio between AC and CB,—nor

do I think your mathematical skill will enable you to find one—in which the derived mean proportionals shall be commensurable quantities.

Well, then, when A C = 40, and A C is to C B in the ratio of 9 to 7, then, C B = the fractional expression $31\frac{1}{9}$: ; but when A C and C B are reduced to a common denominator, A C = 360 and C B = 280; therefore, A C + C B = 360 + 280 = 640 = A B.

Hence:

$A B \cdot A C = 640 \times 360 = 230400$

and, $A B \cdot C B = 640 \times 28^{0} = 179200$

$\therefore (A B . A C + A B \cdot C B) = 230400 + 179200 = 409600 = A B^2$.

This demonstrates beyond the possibility of dispute or cavil by any *honest* mathematician, that the Theorem of Euclid: Prop. 2: Book 2, is irrefragable. Q. E. D.

In the next place, let our DATA be A C to C B in the ratio of 9 to 7, and A C = $\sqrt{40}$.

Then:

As $9 : 7 :: \sqrt{40} : \sqrt{24\cdot1975}$, &c.

$\therefore$ C B = $\sqrt{24\cdot1975}$, &c.

But, $\sqrt{40} = 6\cdot3245$, &c. = A C.

And, $\sqrt{24\cdot1975}$, &c. = $4\cdot9190$, &c. = C B.

$\therefore$ A C + C B = $6\cdot3245$, &c. + $4\cdot9190$, &c. = $11\cdot2435$, &c. = A B approximately.

Hence:

$11\cdot2435^2 = 126\cdot41629225$ = $A B^2$ approximately.

$11\cdot2435 \times 6\cdot3245 = 71\cdot10951575$ = $A B \cdot A C$ approximately.

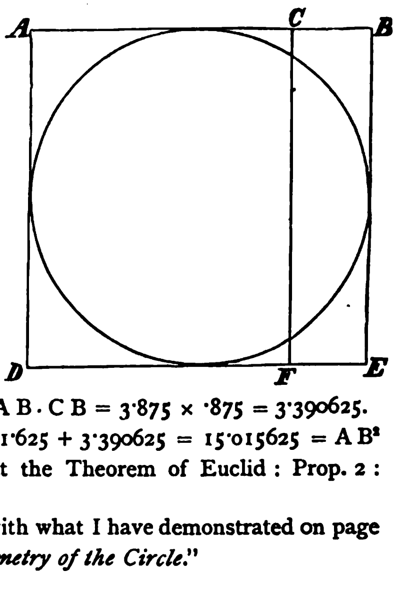
$11\cdot2435 \times 4\cdot9190 = 55\cdot30677650$ = $A B \cdot C B$ approximately.

$\therefore (A B \cdot A C + A B \cdot C B) = 71\cdot10951575 + 55\cdot30677650 = 126\cdot41629225 = A B^2$ approximately: and it is axiomatic, if not self-evident, that no extension of the decimals can "*upset*" the fact that $(A B \cdot A C + A D \cdot C B) = A B^2$.

Now, my dear Sir, although in these computations all the results are merely approximations, they furnish a perfect demonstration, that the square of the whole line A B is equal to the sum of the rectangles contained by the whole line and each of its parts. As a Mathematician, you can readily verify the calculations. Well,

then, if I am right, Euclid is at fault in his Theorems: Prop. 35: Book 3: and Prop. 8: Book 6; and, if I am wrong, surely as a Geometer and Mathematician you can prove it. This demonstration must be taken in connection with the diagram 10, and what I have already called your attention to, on page 187 of my work: "*The Geometry of the Circle*":—

Again:



In the geometrical figure in the margin, let A C and C B be in the ratio of 3 to ·875, and let A C = 3.

Then:

$$CB = \frac{7}{24}(AC) = \frac{7 \times 3}{24} = \cdot 875,$$

$\therefore AC + CB = 3 + \cdot 875 = 3\cdot 875 = AB$; and it follows, that, $3\cdot 875^2 = 15\cdot 015625 = AB^2$.

But, $AB \cdot AC = 3\cdot 875 \times 3 = 11\cdot 625$; and $AB \cdot CB = 3\cdot 875 \times \cdot 875 = 3\cdot 390625$.
$\therefore AB \cdot AC + AB \cdot CB = 11\cdot 625 + 3\cdot 390625 = 15\cdot 015625 = AB^2$ exactly, and again proves, that the Theorem of Euclid: Prop. 2: Book 2: is irrefragable.

Pray compare this proof with what I have demonstrated on page 187 of my work, on "*The Geometry of the Circle*."

Once more:

Let $\frac{28}{96}$ denote the expression of a ratio. The rule is apparently self-evident, that the two terms of a ratio may be multiplied or divided by any number, without altering the ratio itself. This rule is considered axiomatic by Mathematicians, and if we stick to Algebra we may *apparently* prove it. Now, divide the two terms of the ratio $\frac{28}{96}$ by 100 and by 40, and we get the equivalent ratios $\frac{\cdot 28}{\cdot 96}$ and $\frac{\cdot 7}{\cdot 24}$. Now, if we divide the two terms of

the ratio $\frac{\cdot28}{\cdot96}$ by the mystic number 4, we get the ratio $\frac{\cdot07}{\cdot24}$, and not the equivalent ratio $\frac{\cdot7}{\cdot24}$; but, if we multiply the two terms of the ratio $\frac{\cdot28}{\cdot96}$ by 4, we unquestionably get the equivalent ratio $\frac{1\cdot12}{3\cdot84}$, so that, there is an exception to the rule, that the two terms of a ratio may be multiplied or divided by the same number without altering the ratio itself, and the exception arises in dealing with decimal quantities. You may perhaps tell me that $\cdot28$ divided by $4 = \cdot7$ and not $\cdot07$: but if so, 4 times $\cdot7$ must equal $\cdot28$, and I put the following question to you :—Does not $4 \times \cdot7 = 2\cdot8$? Is not $4(\cdot7) = 10(\cdot28)$? Is not $\frac{24}{7}(\cdot28) = \cdot96$? Is not $\cdot28^2 + \cdot96^2 = \cdot0784 + \cdot9216 =$ unity? Is not $\sin.^2 + \cos.^2 =$ unity in every right-angled triangle? Does not $\cdot4 \times \cdot7 = \cdot28$?

Now, with reference to the diagrams, and assuming the line C F to be altered in position, to meet our data, let our data be, A C to C B in the ratio of $3\cdot84$ to $1\cdot12$.

Then :

$AC + CB = 3\cdot84 + 1\cdot12 = 4\cdot96 = AB.$

$\therefore 4\cdot96^2 = 24\cdot6016 = AB^2.$

But, $AB \cdot AC = 4\cdot96 \times 3\cdot84 = 19\cdot0464.$

And, $AB \cdot CB = 4\cdot96 \times 1\cdot12 = 5\cdot5552.$

$\therefore AB \cdot AC + AB \cdot CB = 19\cdot0464 + 5\cdot5552 = 24\cdot6016 = 4\cdot96^2 =$
$AB^2 =$ area of the circumscribing square to the circle.

THEOREM.

Let the area of the circle be represented by any finite arithmetical quantity, say 60. Find the values of the circumference and semi-radius, and prove that, circumference × semi-radius = 60.

In your Letter of the 26th July, in which you acknowledge the receipt of my work on "*The Geometry of the Circle*," you say :—"*This new volume contains in substance, everything I have offered or could suggest, on the subject of* π."

Now, my dear Sir, since you revived our controversy, I have avoided any allusion to π "as needless," and left him "*lurking in his den.*" All my Letters have had reference to the trigonometrical functions and values of angles ; and I have not once

introduced the symbol π. There is one point upon which I think we must be agreed, viz.: circumference × semi-radius = area in every circle. It is this fact that has led me to give you the foregoing theorem for solution, and as a Mathematician, you can surely have no difficulty in solving this theorem. "*π need not appear*," and yet, the arithmetical value of the symbol π, follows as a consequence upon the solution of this theorem. If you decline to give me the solution of this theorem, I, for the present, shall be content to leave your own conscience to answer the following question:—Have you played the part of a fair and candid controversialist, in your controversy with Mr. James Smith?

Believe me, my dear Sir,
Very truly yours,
JAMES SMITH.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
5th August, 1869.

MY DEAR SIR,

I had written my Letter of yesterday, when yours of the 31st July and 2nd instant came to hand. These Letters are no answers to my recent Letters, in which I have given you several simple theorems for solution, none of which have you solved, or even attempted to solve. These two communications are made up of such a mixture of truth and error—the error greatly preponderating—that I shall not waste the time that would be necessary to "*winnow*" them, and separate the wheat from the chaff; or in other words, extract what is true, from such a preponderance of error. It would be a very easy matter to prove to a "*reasoning geometrical investigator*," that in all your latter communications you over and over again make one argument refute another, and so "*upset*" yourself;

but it appears to me, it is an almost hopeless task to attempt to convince you of this ; or at any rate, induce you to admit it.

In the Letter dated 31st July, you say :—"*Any man may fall into a mistake, but to persist in it when it is obvious, renders all argument hopeless.*" You also say :—"*It vexes a Mathematician to be met by a professed refutation based on the most egregious blunder.*" These statements remind me of a most "*egregious blunder*" in your Letter of the 27th July, to which I must call your attention. In doing so, however, I must first remind you of the fact, that we can certainly inscribe a square to any circle, and draw the diagonals of the square. For example : Draw two diameters of a circle at right angles to each other. Join the adjacent extremities of the diameters, and so construct an inscribed square to the circle. You must know, as well as I do, that there are other ways of constructing an inscribed square to a circle. Now, it is self-evident, that by whatever means we may construct an inscribed square to a circle, that the diagonals of the square, subdivide it into four equal isosceles triangles : and it is equally self-evident, that the angles at the centre of the circle are right angles ; that is, angles of 90°, and that these angles are subtended by a side of the inscribed square. In each of these triangles, the angles at the base are angles of 45°, and the two angles are together equal to the angle at the apex ; that is, equal to a right angle = 90° ; and it follows, that in all these triangles, the three angles are together equal to two right angles. "*No Mathematician—I should think—could make a blunder in so simple a matter as this ;*" or have any doubt that the three angles of every plane right-angled triangle are together equal to two right angles. The following copy of a curious note from an old correspondent of mine, touches upon this point :—

DEAR SIR,

. I beg to thank you for your Pamphlet, "*Euclid at Fault.*"

It appears to me, that as concerns the 8th of 6th Book, the real attack, if any, should be on the 32nd of 1st Book, which declares the three interior angles of every rectilinear triangle to be equal to two right angles.

For unless this is disputed, the 8th of 6th Book appears to be as unassailable, as that if interest is at 5 per cent. per annum, the interest for two years will be 10 per cent.

Yours truly,

T. PERRONET THOMPSON.

The gallant General lost sight of the fact, that I have not said in the Pamphlet, "*Euclid at Fault*," that the 8th of 6th Book is not true under any circumstances: but, that it is not of general and universal application, and consequently, is not true, under *all* circumstances.

Let A B C denote a right-angled triangle, of which B is the right angle, A B and B C the sides that *include* the right angle, and A B to B C in the ratio of 3 to 4, by construction; (*See Diagram, Fig.* 2, *in my Letter of the* 21*st July, page* 3).

Now, in your Letter of the 27th July, with reference to this diagram, you say:—"Given: B A C = 16° 16′". If B A C = 16° 16′, it follows, of necessity, that 90° — 16° 16′ = 73° 44′ = the acute angle A C B, and it will not be disputed by any Mathematician that the three angles of the rectilinear triangle A B C are together equal to two right angles. But mark! This would make the acute angle A C B in the right-angle triangle A B C more than four times greater than the obtuse angle B A C. Could absurdity go further? "*Any man may fall into a mistake, but to persist in it when it is obvious, renders all argument hopeless.*" Will you persist in an "*egregious blunder*" so palpable as this? If you refer to the diagram in my Letter of the 21st July, you will find that, C A is the radius of a circle; that C B, the base of the triangle A B C, is produced to meet and terminate in the circumference of the circle at the point D; and that A D is joined; making A C D an isosceles triangle, and C an angle at the centre of the circle. Now, because A B is perpendicular to D C, and A B common to the two right-angled triangles A B D and A B C, it follows, that $AD^2 - DB^2 = AC^2 - BC^2$, and this equation or identity $= AB^2$, but the triangles on each side of A B are not similar triangles. Now, if the angle C, or any other angle at the centre of a circle, is not greater than 60°, and is the apex of an isosceles triangle, a straight line may be drawn from an angle at the base of the triangle, perpendicular to its opposite

side, producing a right-angled triangle, of which the acute angle is equal to half the angle at the apex; thus, the angle D A B in the diagram is equal to half the angle A C B. Produce A B to meet and terminate in the circumference of the circle at a point, say E, and join C E. Then: A E C will be an equiangular and equilateral isosceles triangle, and the three angles will be angles of 60°, and together equal to two right angles. It is axiomatic, if not self evident, that if the apex of an isosceles triangle at the centre of a circle be greater than 60°, a straight line cannot be drawn from an angle at the base perpendicular to its opposite side. Hence, if an angle at the centre of a circle be subtended by a side of an inscribed square, we cannot draw a straight line from an angle at the base, so as to be perpendicular to its opposite side.

By Euclid: Prop. 5: Book 1: the angles at the base of an isosceles triangle are equal. This Theorem of Euclid is irrefragable, and is of general and universal application, and consequently, is true under *all* circumstances.

I shall now shew you, that your reasoning would "*upset*" this theorem of Euclid.

Referring to the Diagram in my Letter of the 21st July, and to what you say in your favour of the 27th July, viz.: "Given: BAC = 16° 16′." Can you dispute that this would make 90° — 16° 16′ = 73° 44′ the value of the angle A C B? It is true a man may dispute anything, although he cannot controvert the thing he disputes, but no Mathematician—I should think—would call this reasoning; certainly, no Logician would. In the same Letter, (July 27th), you observe:—"*There is a lapsus in your* (my) *Letter. You say angle* D A B *is half angle* C. *You mean half the angle* B A C." Thus, on your shewing, the angle D A B is an angle of 8° 8′, if the angle B A C = 16° 16′. And in this case, 90° — 8° 8′ = 81° 52′ is the value of the angle A D B. But, the angles B A C and D A B are together equal to the angle D A C in the isosceles triangle A C D; that is, 16° 16′ + 8° 8′ = 24° 24′ = angle D A C, and is less than the angle A D B. Thus by your reasoning you make the angles at the base of an isosceles triangle unequal, and so "*upset*" the Theorem of Euclid: Prop. 5: Book 1. Q. E. D.

I had written so far when the following curious communication came into my hands:—

16, PERCY STREET, LIVERPOOL,
July 4th, 1869.

SIR,

In my Letter of 28th December, 1868, I told you, that unless you apologized for the false and absurd imputation in your Letter of December 22nd, I should return unopened any subsequent communications. Your next Letter did not convey the apology I demanded, and I consequently had the trouble of returning unopened, a whole bundle of Letters which you subsequently addressed to me.

Still you continued to write. But I have never opened the Letters. They are here at your service if you like to send for them, but I do not care to waste fifteen pence on their postage.

Yours faithfully,
W. A. WHITWORTH.

The following is a copy of my reply:—

BARKELEY HOUSE, SEAFORTH,
5th August, 1869.

SIR,

I am this morning in receipt of your note, bearing the Liverpool post-mark of yesterday, but mis-dated July 4th.

I enclose fifteen pence in postage stamps, and will thank you to return the Letters referred to, which you SAY you have "***never opened***," and are at my service.

Yours faithfully,
JAMES SMITH.

August 7th.

It is axiomatic, if not self-evident, that we may assume the angles B A C and D A B in the diagram in my Letter of 21st July, to be angles of any magnitude less than 90°, so that they be *finite* and *determinate*, and prove that the six angles of the triangles A B C and A B D are together equal to four right angles.

For example: Let the angle B A C $= 60°$. Then: $90° - 60° = 30° =$ the angle A C B: $\frac{30°}{2} = 15° =$ the angle D A B: and

$90^\circ - 15^\circ = 75^\circ =$ the angle A D B: and A B C and A B D are right angles $= 90^\circ$; therefore, the sum of the six angles $= 4$ right angles $= 360.^\circ$ Again: Let the angle D A B $= 20^\circ$. Then: $90^\circ - 20^\circ = 70^\circ =$ the angle A D B: $2(20^\circ) = 40^\circ =$ the angle A C B: $90^\circ - 40^\circ = 50^\circ =$ the angle B A C: and the sum of the six angles $= 4$ right angles $= 360^\circ$. But, this proves nothing as to the true values of the obtuse and acute angles. Now, since the three interior angles of a right-angled triangle are together equal to two right angles, it is obvious that the obtuse angle must be greater in every right-angled triangle, and the acute angle less, than 45°.* How, then, in the name of common sense, can we adopt as a datum, "*Given:* B A C = $16^\circ\ 16'$," to find the values of the angles? Can you fail to see the absurdity of your datum, and the fallacy of your reasoning? The fact is, B A C is an angle of $53^\circ\ 8'$: A C B is an angle of $36^\circ\ 52'$: D A B is an angle of $18^\circ\ 26'$: and A D B is an angle of $71^\circ\ 34'$: and A B D and A B C are right angles; therefore, the sum of the six angles $= 4$ right angles $= 360^\circ$. I have demonstrated that these are the *true* values of the angles, by means of the diagram number 11, page 195, in my work on "*The Geometry of the Circle*," and to my proofs I must refer you. My Letter of 9th November, 1868, to Mr. J. M. Wilson (page 356), ought to convince you on this point.

In your Letter of 31st July you put the question:—"*If natural sine is not the same as trigonometrical, or geometrical, or arithmetical, will you be so kind as define each, and draw such sines if possible?*"

This Letter has run on to such a length, that I must bring it to a close, and reserve my answer to this question, for another communication.

Believe me, my dear Sir,

Very truly yours,

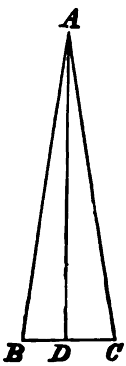
JAMES SMITH.

* Geometrically speaking, an obtuse angle is greater than a right angle: but, a "*reasoning geometrical investigator*" will see, that in this paragraph I have employed the words *obtuse* and *acute*, to distinguish the greater from the lesser of the acute angles, in a right-angled triangle.

J. S.

I might have handled Mr. Gibbons' Letters of July 27, 31, and Aug. 2 with great severity; but at the time of writing the foregoing Letter, I had not entirely lost hope of being able to satisfy that gentleman of the fallacy of his reasoning, and succeed in convincing him of geometrical truth. It is no doubt true, with reference to the diagram, Fig. 2, page 3, that we may assume the angle B A C in the triangle A B C to be an angle of any number of degrees less than than 90; and prove that in the triangles A B C, A B D, and A C D, the three angles are together equal to two right angles; but surely this does not make it consistent with "*geometrical truth,*" to assume the greater of the acute angles in a right-angled triangle, to be less than an angle of 45°, having fixed 90° as the measure of a right angle.

FIG. 1.



Mr. Gibbons, in his Letter of the 31st July, says:—"*It vexes a Mathematician to be met by a proposed refutation based on the most egregious blunder. Any man may fall into a mistake, but to persist in it when it is obvious, renders all argument hopeless. To Mr. Skeat you oppose reasoning quite fallacious, for you assume that sin.* 20° = *2 sin.* 10°. *Now, sin.* 10° = ·1736482..., *but sin.* 20° *is less than the double of this.*"

Let A B C (Fig. 1) denote an isosceles triangle, with the angle A and its subtending chord B C bisected by the line A D. Let B A C be an angle of 20°. Then: D A B and D A C are each angles of 10°, and B C is equal to a side of a regular polygon of 18 sides, inscribed in a circle.

Mr. Skeat reasons thus:—" *Now, the side of the*

polygon (that is, a polygon of 18 sides) *subtends an angle of* 20°, *and if we denote the side by a, we have at once* (*the radius being r*),

$$\tfrac{1}{2}a = r \sin. 10°.$$

Or, since $r = \tfrac{1}{2}$,

$$\tfrac{1}{2}a = \sin. 10°.$$

The value of this sine is given in Hutton's Logarithmic Tables as =·1736482. *Multiply by* 18, *and we get the perimeter of the figure* = 3·1256676;" and Mr. Skeat, assuming the diameter of a circumscribed circle to the polygon to be 1 foot, arrives at the conclusion, that 3·125 "*is less than the perimeter of the regular polygon which it circumscribes.*" Mr. Skeat had the candour to admit that he assumed the value of a sine of 10°, and that he took its value from Hutton's Tables; but Mr. Gibbons has never had the candour to admit anything. Both he and Mr. Skeat assume Hutton's Tables of sines, cosines, &c., to be correct, but neither of them has proved this; and the only reason given by Mr. Gibbons for this assumption is, that "*Tables have been calculated by experts in every country in Europe,*" and all agree, "*clerical errors excepted:*" but if experts have made these calculations on an erroneous principle engendered by false teaching, these Tables cannot be correct.

For the sake of argument, however, let it be admitted that Hutton's Tables of sines, cosines, &c., are correct, and let us examine Mr. Skeat's argument, to which, according to Mr. Gibbons, I have "*opposed reasoning quite fallacious.*"

Now, because the base B C in the isosceles triangle A B C is bisected in D, B C is the double of B D or D C; and it follows, that B D = ½ (B C). So far we are all

agreed. If $1 : 2 :: 2 : 4$, the converse of this proportional holds good; $4 : 2 :: 2 : 1$, and the product of the means is equal to the product of the extremes $\therefore m \times n = n \times m$, whatever values we may put upon m and n, and in either way, works out to the same result; but, $\frac{m}{n}$ is not equal to $\frac{n}{m}$, unless m and n have the same value. Well, then, by hypothesis, B A C is an angle of 20°, and because the angle is bisected by the line A D, D A C and D A B are each angles of 10°. By Hutton's Tables, the sine of an angle of $20° = \cdot3420201 : \frac{\cdot3420201}{2} = \cdot17101005$, and is less than the sine of an angle of 10°. Multiply by 18. Then, $(18 \times \cdot17101005) = 3\cdot0781809$, and is less than $3\cdot125$; so that this argument of Mr. Skeat and Mr. Gibbons proves nothing; or perhaps, I should rather say, that if by this argument they have proved anything, they have proved their incapacity to reason soundly: and yet, Mr. Gibbons is better then an average sample of a "*recognised Mathematician.*"

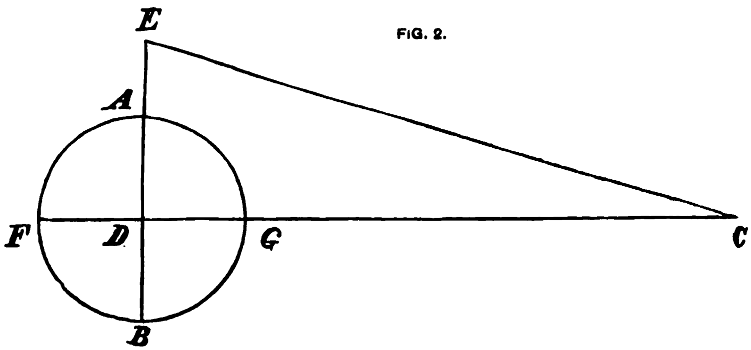
Mr. Gibbons, in his Letter of the 2nd August, says:— "I am engaged in classics and mathematics on heights by barometer, but I would follow you in any *real* investigation of π. I see no use in discussing complicated figures and diagrams, where π is not introduced, and by which its value cannot be found. Indeed, if you *could* find it at all, you would do it by figures far simpler than you draw. These only serve to lead you "*off the scent,*" and make you forget that you have *never* proved any equality between a curvilinear and a rectilinear length or area. *This* is really what we aim at. You draw a lot of rectilinear figures, sometimes with circles in or about them, but you never attempt to prove any equality

except between rectilinear figures. When you introduce π, it is somewhat thus:—"Now if $\pi = 3\frac{1}{8}$, so-and-so will follow."

In the foregoing quotation Mr. Gibbons makes a very bold assertion. At the time of penning the quotation he may have thought that I *had never given him a proof of any* equality between a curvilinear and a rectilinear length or area: but in that case he must have a very treacherous memory. I will once more prove the "*equality between a curvilinear and a rectilinear length,*" by means of a very simple geometrical figure.

Draw two straight lines A B and F G at right angles and of equal length, intersecting and bisecting each other at the point D. With D as centre and D A or D G as radius, describe the circle. Produce D G to C, making D C equal to 6 times D G. Produce D A to E, making A E equal to $\frac{3}{4}$ (D A), and join E C, and so construct the right-angled triangle E D C. (See diagram, Fig. 2.)

FIG. 2.



The construction of the diagram belongs to pure Geometry, but when constructed, what can we prove by it? Simply nothing, unless and until we apply mathematics. Now, whether we make the diameter of the

circle unity, or the radius 1, or put any other arithmetical value on the diameter or radius, we *introduce* mathematics: and whatever value we may put upon the diameter or radius, it can be demonstrated, that the circumference of the circle is exactly equal to the line E C, the hypothenuse of the right-angled triangle E D C.

Let the diameter of the circle be unity, and our unit of length 1. Then: $DA = DG = \frac{1}{2} = \cdot 5$ = radius of the circle: $(DA + AE) = \{\cdot 5 + \frac{3}{4}(\cdot 5)\} = (\cdot 5 + \cdot 375) = \cdot 875 = ED$: $6(DG) = (6 \times \cdot 5) = 3 = DC$: and E D C is a right-angled triangle, by construction; therefore, by Euclid: Prop. 47: Book 1: $(ED^2 + DC^2) = (\cdot 875^2 + 3^2) = (\cdot 765625 + 9) = 9\cdot 765625 = EC^2$; therefore, $\sqrt{EC^2} = \sqrt{9\cdot 765625} = 3\cdot 125 = EC$: and E C = circumference of the circle, when the diameter is unity: and it follows, that the line D C, the base of the triangle E D C, is exactly equal to the perimeter of a regular hexagon inscribed in the circle: not only so, but it also follows, that when the diameter of the circle = 8, D C the base of the triangle E D C = 24, and E C the hypothenuse = 25; and makes $\frac{24}{25}$ (circumference) = the perimeter of an inscribed regular hexagon in every circle.

Now, $\frac{5^2}{2^3} = \frac{25}{8} = \frac{5^5}{1000} = 3\cdot 125$. No Mathematician can controvert this: and whatever the existing race of "*recognised Mathematicians*" may say, $3\cdot 125$ is the π that has been ***lurking in his den*** for ages, and the proof is very simple.

The circumference of a circle is divided into 360° for all practical purposes: and the symbol π is adopted to denote the circumference of a circle of diameter unity. The former statement is known to every educated man,

and I hardly think it will be disputed by a Mathematician: but a Mathematician would probably catch at the latter statement and tell me that Mathematicians adopt the symbol π "*to denote the ratio of the circumference of a circle to its diameter.*" Will Mathematicians be good enough to tell us, what is the difference between the ratio $\frac{\text{circumference}}{\text{diameter}}$ and $\frac{\pi}{1}$?

Well, then, I test the value of π as follows, and in doing so, "*test my work,*" for Mr. Gibbons, in his Letter of the 2nd August, *admits,* that I "*shew correctly what would be the consequences if* $\pi = 3\frac{1}{8}$."

Let c denote the circumference of a circle.

Let d denote the diameter of the circle.

Let r denote the radius of the circle.

Let p denote the perimeter of a regular inscribed hexagon to the circle.

Let $\frac{3{\cdot}125}{1}$ denote the ratio between c and d.

Then:

$$\frac{c}{3{\cdot}125} = d : \frac{d}{2} = r : 6\,(r) = p, \text{ and } (p + \tfrac{1}{24}p) = c.$$

This algebraical formula is infallible, whatever value we may put upon the circumference of a circle.

EXAMPLE I.

By hypothesis, let $c = 360$.

Then:

$$\frac{c}{3{\cdot}125} = \frac{360}{3{\cdot}125} = 115{\cdot}2 = d : \frac{d}{2} = \frac{115{\cdot}2}{2} = 57{\cdot}6 = r : 6\,(r) = (6 \times 57{\cdot}6) = 345{\cdot}6 = p : \text{ and } (p + \tfrac{1}{24}p) = (345{\cdot}6 + 14{\cdot}4) = 360 = c.$$

EXAMPLE 2.

By hypothesis, let $c = 3{\cdot}1416$, a very close approximation to the circumference of a circle of diameter unity, according to orthodoxy.

Then :

$\frac{3{\cdot}1416}{3{\cdot}125} = 1{\cdot}005312 = d : \frac{d}{2} = \frac{1{\cdot}005312}{2} = {\cdot}502656 = r :$ $6\,(r) = (6 \times {\cdot}502656) = 3{\cdot}015936 = p$: and $(p + \frac{1}{24}p)$ $= (3{\cdot}015936 + {\cdot}125664) = 3{\cdot}1416 = c$.

EXAMPLE 3.

By hypothesis, let $c = 3{\cdot}125$.

Then :

$\frac{3{\cdot}125}{3{\cdot}125} = 1 = d : \frac{d}{2} = \frac{1}{2} = {\cdot}5 = r$: $6\,(r) = (6 \times {\cdot}5)$ $= 3 = p$: and $(p + \frac{1}{24}p) = (3 + {\cdot}125) = 3{\cdot}125 = c$.

In examples 2 and 3, $r = \frac{16\,\pi}{100}$; but in example 3, $\frac{1}{4}\left(\frac{16\,\pi}{100}\right) = \frac{1}{4}\left(\frac{50}{100}\right) = \frac{{\cdot}5}{4} = {\cdot}125$, and is equal to the known demi-semi-radius of a circle of diameter unity: and it follows, that $(p + \frac{1}{24}p) = c$: and $(c - \frac{1}{25}c) = p$, in every circle. If $p = 24$, then $\frac{p}{6} = r : \frac{r}{4} = 1$, and 1 is an aliquot part of p. But, $2 \times 3{\cdot}125\,(r) = (6{\cdot}25 \times 4) = 25$ $= c$, and 1 is an aliquot part of c. Hence it is, that $(p + \frac{1}{24}p) = c$, and $(c - \frac{1}{25}c) = p$.

Let x denote an aliquot part of c: and let y denote an aliquot part of p. When Mathematicians can find values of x and y, and prove that $c - x = p$, and $p + y = c$, they will be able to prove that $3{\cdot}125$ is not π's true value, but not till then.

In February, 1867, I received a curious Letter from the Rev. J—— R——, in former days a Fellow of Trinity College, Cambridge, (*see Geometry of the Circle*, page 114). I acknowledged the receipt of this communication, which led to an interview at the house of our mutual friend, Mr. C——. This interview led to my writing a long Letter to the Rev. J—— R—— at his request (*see Geometry of the Circle*, page 115), in which I enclosed a diagram (*see diagram* V., *Geometry of the Circle*, page 91). The triangle E D C in this diagram, and the triangle E D C in Fig. 2, are similar triangles.

In November, 1866, I addressed a Letter to the Rev. Geo. B. Gibbons (enclosing a copy of diagram V.) and embodying all the facts brought out in my Letter to the Rev. J—— R——. I drew Mr. Gibbons' especial attention to the fact, that E C, the hypothenuse of the triangle E D C, is equal to the circumference of the circles X and Z: and B C, the base of the triangle E D C, equal to the perimeter of a regular inscribed hexagon to the circles. Mr. Gibbons may have forgotten this, but it does not alter the fact, that his bold assertion, that "*I have never proved any equality between a curvilinear and a rectilinear length*," is a positive untruth. I think, that, as a controversialist, he should and would have exercised a little more caution.

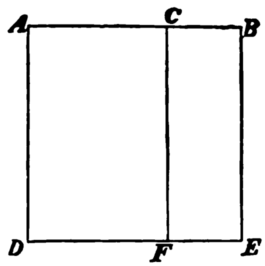
THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

LANEAST, *6th August*, 1869.

MY DEAR SIR,

Nobody doubts Euclid, Prop. 2: Book II. Millions have studied his works for 20 centuries, and never found a flaw. He is indeed "irrefragable."

Euclid, Prop. 2: Book II. is singularly simple, apparent at sight, with scarcely any need of demonstration.



Square on A B is made up on □ms. A F, C E, or of A B· A C, and A B· C B.

If the parts of the line A B are x and y (and therefore the whole line $x + y$).

It is only saying—

$$(x + y)^2 = (x + y)\,x + (x + y)\,y = (x + y)\,(x + y)$$

Numerical illustrations—of course may be given *ad libitum.*

I send back one sheet of your letter, not in rudeness, but simply for correction. Please return it. You have made a "clerical" error in your division.

96 divided by 40 is 2·4 not ·24. All your values of $\frac{28}{96}$ give the same value.

There is *no* exception to the rule that you may multiply or divide *both* terms of a fraction by the same number without altering its value.

$$\frac{a}{b} = \frac{ma}{mb} \text{ multiplying both by } m.$$

$$\frac{am}{bm} = \frac{a}{b} \text{ dividing both by } m.$$

whatever be the value of m.

I am glad π is hid in his den, there is no use in disturbing him.

I can only repeat my assertion that sin. ϕ is a ratio, but when the denominator b, is given in length, the *sine* itself is a length.

$$\text{Sin. } 30° = \frac{\frac{1}{2}r}{r}, \text{ but if } r = 1 \text{ foot,}$$

$$\text{Sin } 30 = \frac{1}{2} \text{ a foot}$$

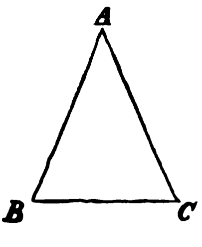
Obviously $\pi r^2 = (2\pi r)\,\frac{r}{2}$: Area of Circle

= circumference × semi-radius.

Given: $AB = AC = 1.$

$A = 15°.$

Find B C.



If you do this, and send me the whole in full, I can go to work and compare it with my own calculations, and correct my error, if I have made one.

It is practical Trigonometry, not involving even any idea of π.

Yours truly,

G. B. GIBBONS.

In the course of the long correspondence I had with Mr. Gibbons, in the years 1866 and 1867, I proved over and over again, that when $A = 15°$, $BC = 14° 24'$ expressed in degrees: and when $A = \frac{\pi}{24}$, $BC = \cdot 125$, and Mr. Gibbons never attempted to refute my proofs. Such an opponent, "*renders all argument hopeless.*"

JAMES SMITH *to the* REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
7th August, 1869.

MY DEAR SIR,

. My Letter dated and commenced 5th instant, was finished and posted this morning, before your favour of yesterday came to hand.

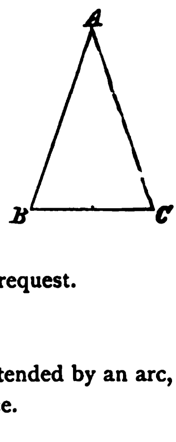
"*Any man may fall into a mistake, but to persist in it when it is obvious, renders all argument hopeless.*" I admit that "96 *divided by* 40 *is* 2·4 *not* ·24," and of course, as an honest man, must admit that I have fallen into a "*mistake*," in making 96 divided by 40 = ·24; and I admit further that you are right in saying: "*There is no exception to the rule, that you may multiply or divide both terms*

of a fraction by the same number without altering its value." But, the question still remains:—Do not the two terms of a fraction, express the two terms of a ratio?

Now, $\frac{\pi}{300}$ = the circular measure of an angle of 36 minutes, whatever be the value of π. For example: By hypothesis, let π = 3·1416. Then: $\frac{36' \times \pi}{180^\circ} = \frac{36' \times 3{\cdot}1416}{10800'} = \frac{113{\cdot}0976}{10800} = {\cdot}010472$ exactly; and this is given as the natural sine of an angle of 36 minutes, in Logarithmic Tables calculated to 6 places of decimals. In one of my pamphlets I have assumed $\frac{\pi}{300}$ to express the natural sine of an angle of 36 minutes. You detected my blunder, and pointed out to me that $\frac{\pi}{300}$ does not express the natural sine, but the circular measure of an angle of 36 minutes. Did I not at once admit that I had fallen into a mistake?

You conclude your Letter of yesterday, by giving me a Theorem for solution.

"Given: $AB = AC = 1$.
$A = 15^\circ$.
Find B C.



If you will do this and send me the work in full, I can go over it and compare it with my own calculations, and correct my error, if I have made one.

It is practical trigonometry, not involving even any idea of π."

I have much pleasure in complying with your request.

$A = 15^\circ$.

$A - \frac{1}{25}(A) = 15^\circ - 36' = 14^\circ\ 24'$.

A denotes an angle at the centre of a circle subtended by an arc, equal to one twenty-fourth part of the circumference.

$BC = 14^\circ\ 24' = 864$ minutes of the circle.

24 (864 minutes) = 20736 minutes.

$\frac{20736 \text{ minutes}}{60} = 345^\circ\ 36'$; and it follows, that $345^\circ\ 36'$ is the perimeter of a regular inscribed hexagon to a circle of circumference $= 360^\circ$.

So far, "*π does not appear, need not appear,*" but I shall now proceed to prove that the value of the symbol π, follows as a consequence. The symbol π denotes the circumference of a circle of diameter 1; or, as Todhunter puts it: "*The symbol π is invariably used to denote the ratio of the circumference of a circle to its diameter; hence, if r denote the radius of a circle, its circumference is $2\pi(r)$.*" For all practical purposes we divide the circumference of a circle into 360 equal parts which we call degrees, and these degrees we again divide into 60 equal parts which we call minutes. Now, the perimeter of a regular inscribed hexagon to a circle of diameter $= 1 = 6$ times radius $= 3$.

Hence:

$345° 36' : 360° :: 3 : 3{\cdot}125$; that is, $345° 36' : 360° :: 3 : \pi$.

Or, $20736' : 21600' :: 3 : \pi$.

$\therefore 24 (BC) = 24 \times {\cdot}864'$, $= 20736$ minutes, is the perimeter of a regular inscribed hexagon to a circle of circumference $= 360°$; and it follows, that $\frac{20736}{21600}$ and $\frac{3}{3{\cdot}125}$ are equivalent ratios, and both express the ratio between the perimeter of every regular hexagon and the circumference of its circumscribing circle.

Divide the two terms of the ratio .... $\frac{20736}{21600}$ by 3.

This gives the equivalent ratio .... $\frac{6912}{7200}$

Divide the two terms of the ratio .... $\frac{6912}{7200}$ by 100.

This gives the equivalent ratio .... $\frac{69{\cdot}12}{72}$

And by analogy or proportion:—

$69{\cdot}12 : 72 :: 3 : 3{\cdot}125,$

And, $69{\cdot}12 : 72 :: 14° 24' : 15°$. $\therefore 14° 24' : 15° : 3 : 3{\cdot}125$.

Now, $\frac{3{\cdot}125}{3} = 1{\cdot}04166$ with 6 to infinity. Hence, $\frac{1{\cdot}04166\ldots}{100} =$ ${\cdot}0104166$ with 6 to infinity, is the circular measure of an angle of 36 minutes. Multiply by 600. Then: ${\cdot}0104166 \times 600 = 6{\cdot}24996 =$

circumference of a circle of radius $1 = 2\pi$ approximately. This arises simply from the fact, that 1 is not divisible by 3 without a remainder, whatever be the value of π. De Morgan has admitted (*see Athenæum*, August 5th, 1865) that 600 times the circular measure of an angle of 36 minutes = the circumference of a circle of radius $1 = 2\pi$, whatever be the value of π. It is obvious that we should get 6·2499 with 9 to infinity by extending the decimals, and that this is not *finite*.

You may tell me that I have assumed the circumference of a circle to be 360°. Granted! Do not you, in fixing the angle A = 15°, and do not all Mathematicians, make the same assumption? Surely we must assume something! Can we get a datum to reason from, without assuming something?

Believe me, my dear Sir,

Very truly yours,

JAMES SMITH.

This Letter ought to have suggested to Mr. Gibbons, that there must be a fallacy in mathematical tables; since tables calculated to 6 places of decimals, make the natural sine, and the circular measure of an angle of 36 minutes equal; which is obviously absurd.

JAMES SMITH *to the* REV. PROFESSOR WHITWORTH.

BARKELEY HOUSE, SEAFORTH,
10*th August*, 1869.

SIR,

Your elaborate Paper entitled "A proof of π, without trigonometry, for the benefit of those who maintain that $\pi = 3{\cdot}125$," (as I write from memory I may not be quite right in this quotation,) came to hand in the afternoon of the 6th instant. This Paper you requested me to return, and I did return it, early in the morning of the 7th, without taking a copy of it, but not without carefully perusing it. Your first Letter of that date, shews that you duly received it.

You commence your Letter, dated 7th August, 2-30 p.m., by observing:—"*I am sorry that you can only meet my proof that π is greater than* 3·126 *with the assertion that it is nonsense.*" Your

elaborate Paper contained what my friend Mr. Gibbons would call a complicated diagram. The construction was perfectly correct, and it contained a regular inscribed dodecagon to a circle. I was struck by your calling the dodecagon a dodecahedron. Is not a dodecahedron a regular solid, having twelve equal bases? (*"Note by Mr. Whitworth:—'Granted.' Dodecahedron was a mistake. In the next line I called the figure correctly a dodecagon."*)

The next paragraph of your Letter runs thus:—"*I was at considerable trouble to devise a proof which should avoid the use of trigonometry, or the adoption of any of the methods which usually present stumbling blocks to the appreciation of arguments.*" "*I am sorry*" I have "*unwittingly*" given you any trouble. I really thought all correspondence between us on the vexed question of the value of the symbol π was at an end. Is it not an axiom in Trigonometry, that $\sin.^2 + \cos.^2 =$ unity, in every right-angled triangle; and is not unity represented by the arithmetical symbol 1? How do we get at this axiom or datum, if not by the 47th Proposition of the first book of Euclid? Did you not introduce this Proposition of Euclid into your fanciful proof that π is greater than 3·126? Now, admitting—for the sake of argument—your fanciful proof to be incontrovertible, how can it be a proof without trigonometry? Introduce Euclid's Theorem: Prop. 47: Book 1: into a proof of anything, and it becomes a proof by trigonometry; or at any rate, trigonometry enters into the proof. (*"Note by Mr. Whitworth. Is this a specimen of Mr. Smith's logic? Because Euclid I. 47 is used in trigonometry; therefore, (he argues,) any proof which is based on Euclid I. 47 is a proof by trigonometry! Similarly we might say Euclid I. 47 is used in astronomy; therefore all these proofs are proofs by astronomy! Mr. Smith may call my proof a proof by astrology if he likes, but nevertheless it involves nothing but arithmetic and Euclid, and may be thoroughly comprehended by a school boy who has never heard of trigonometry. But whatever my proof involves, if Mr. Smith is capable of following it, he ought either to shew where there is a fallacy in it, or else admit its correctness."*)

You next say:—"*I really hoped that the simple proof which I sent you, involving only arithmetic and pure geometry, would have been intelligible to you, and I had some faint hope too that if you went over the work for yourself and found that no flaw existed in it, you would have been induced to reconsider your prejudices that* $\pi =$ 3·125." I can assure you that the arguments in your elaborate Paper were perfectly intelligible to me, and I can "*upset*" your

assertion, that $\pi = 3{\cdot}125$ is a prejudice of mine, without the aid of trigonometry. A proof of this you may see in print some day. ("*Note by Mr. Whitworth.—Then, if intelligible, in what respect was it inconclusive.*")

You then say :—" *I cannot make anything of the remark which you have inscribed on my Paper. Your first two lines are perfectly correct, viz. :—*

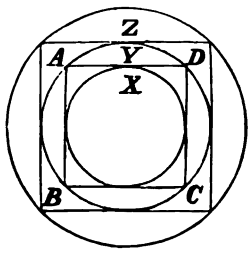
$$25 : 24 = 2 : 1{\cdot}92.$$
$$1{\cdot}92 : 2 = 3 : 3{\cdot}125.$$

but there is surely no connection between this and the conclusion which you append, viz.: therefore, the ratio of chord to arc is the same, whether it be side of a regular VI-gon to its subtending arc, or side of a regular XII-gon to its subtending arc." This is not exactly as I put it, but it will do; for, $25 : 24 :: 2 : 1{\cdot}92$: and, $1{\cdot}92 : 2 :: 3 : 3{\cdot}125$. In your elaborate Paper, you made a side of a regular dodecagon inscribed in a circle, your *datum*, *assuming* its value to be represented by the *finite* and *determinate* arithmetical quantity 2 ? You, Sir, know—or at any rate ought to know—that if you *assume* the side of a regular dodecagon to be *finite* and *determinate*, you would make the circumference, diameter, and area of a circumscribing circle *infinite* and *indeterminate*. (" *Note by Mr. Whitworth.—Nonsense!*")

I shall now adopt your *datum*, that is, a side of a regular polygon inscribed in a circle, and try to make *my* remarks inscribed on *your* Paper intelligible to *you*. Let the side of a regular dodecagon inscribed in a circle = 2. It may be two miles, yards, feet, or inches, in length. Then: 12 times $2 = 24 =$ the perimeter of the dodecagon. Add $\frac{1}{24}$th part. Then: $\{24 + \frac{1}{24}(24)\} = 24 + 1 = 25 =$ circumference of the circle. Again: Let the side of a regular 24-sided polygon inscribed in a circle = 2. Then: 24 times $2 = 48 =$ the perimeter of the polygon. Add $\frac{1}{24}$th part. Then: $\{48 + \frac{1}{24}(48)\} = 48 + 2 = 50 =$ circumference of the circle. Again: Let the side of a regular 6-sided polygon inscribed in a circle = 2. Then: 6 times $2 = 12 =$ the perimeter of the polygon. Add $\frac{1}{24}$th part. Then: $\{12 + \frac{1}{24}(12)\} = 12 + {\cdot}5 = 12{\cdot}5 =$ circumference of the circle. Again: Let the side of a regular dodecagon inscribed in a circle $= 1{\cdot}92$. Then: 12 times $1{\cdot}92 = 23{\cdot}04 =$ the perimeter of the dodecagon. Add $\frac{1}{24}$th part. Then: $\{23{\cdot}04 + \frac{1}{24}(23{\cdot}04)\}$

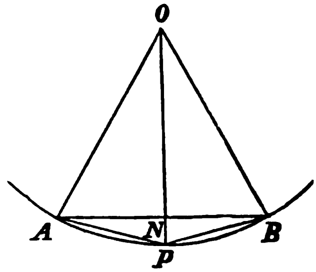
= 23·04 + ·96 = 24 = circumference of the circle. Finally: Let the side of a regular hexagon inscribed in a circle = ·5. Then: 6 times ·5 = 3 = the perimeter of the hexagon. Add $\frac{1}{24}$ th part. Then: $\{3 + \frac{1}{24}(3)\} = 3 + {\cdot}125 = 3{\cdot}125$ = circumference of the circle: and it follows, that $\frac{25}{8} = 3{\cdot}125$ is the true arithmetical value of π; making 8 circumferences = 25 diameters, and 8 to 25 the ratio of diameter to circumference, in every circle. Is not 6 (radius × semi-radius) = area of a regular inscribed dodecagon to every circle? Well, then, $6\left({\cdot}5 \times \frac{{\cdot}5}{2}\right) = 6\,({\cdot}5 \times {\cdot}25) = 6 \times {\cdot}125 = {\cdot}75$ = the area of a regular inscribed dodecagon to a circle when diameter = unity. Add $\frac{1}{24}$ th part. Then: $\{{\cdot}75 + \frac{1}{24}({\cdot}75)\} = {\cdot}75 + {\cdot}03125 = {\cdot}78125$ = the area of the circle = $\frac{\pi}{4}$. Is not $\frac{\pi}{4}$ = the area of a circle of diameter unity, whatever be the value of π?

You are the first "*recognised Mathematician*" I have met with, that ever adopted the side of a regular dodecagon inscribed in a circle, for a *datum*. In doing so—as we say in Lancashire—you have put your foot in it, which—being interpreted—means, you have committed an "*egregious blunder*." You have also "*put your foot in it*," in again intruding yourself upon me. Is it not self-evident, that the hexagon, dodecagon, and 24-sided polygon, may all be inscribed in the same circle?



Your next argument is cleverly put, but I cannot help thinking you will see the fallacy of it, without me pointing it out. We may inscribe and circumscribe circles X and Y to a square A B C D. About the circle Y, we may circumscribe a square, and about this square we may circumscribe a circle Z. The area of the circle Y is double the area of the circle X; the area of the circle Z is double the area of the circle Y; and it follows, that the area of the circle Z = four times the area of the circle X. Now, the radius of the circle Y is not the double of the radius of the circle X; neither is the circumference of the circle Y the double of the circumference of the circle X; and yet, as a

"*recognised Mathematician*," you may readily convince yourself, that the diameter of the circle X = the radius of the circle Z. Now, *you* can inscribe a regular hexagon and a regular dodecagon in the circle Z, and—as a "*recognised Mathematician*"—readily convince yourself, that the perimeter of the hexagon is to the circumference of the circle, as the area of the dodecagon to the area of the circle. These facts are far from exhausting the properties of this very simple geometrical figure. I merely direct your attention to them, as suggestive of other facts.



To your Letter dated August 7th, 1869, 2-30 p.m., you add a postscript, and your argument to disprove my theorem, is based on the geometrical figure in the margin. You give the construction thus:—"*Let* A B *be the side of a regular hexagon inscribed in a circle*." (I think you will hardly raise a quibble because I have not described the whole of the circle). "*Bisect arc* A B *in* P *and join* A P, P B, *so that* A P, P B, *are the sides of a* XII-*gon in the same circle*." (I have simply added the line O P, bisecting the angle O and its subtending arc A P B). You say:—"*Now, by your theorem, sent this morning, arc* A B : *chord* A B : : *arc* A P : chord A P." ("*Note by Mr. Whitwotth.—I quoted this from Mr. Smith, but of course I know it is wrong, and I proved the absurdity of it. Mr. Smith seems now trying to do so too!*")

This fixes what you mean by *my* theorem. You *mean*, my remarks inscribed on your elaborate Paper. On that Paper, the word "*nonsense*" was written in red ink, and *my remarks* in pencil. ("*Note by Mr. Whitworth.—Yes, I mean the theorem recorded in pencil, as Mr. Smith says.*")

Now, you cannot fail to perceive, that O A P and O B P are similar and equal isosceles triangles: that O N A and O N B are similar and equal right-angled triangles: and that A N P and B N P are similar and equal right-angled triangles. But, A P B is an isosceles triangle, and the sides A P and P B are sides of a regular inscribed XII-gon to a circle of which O A, O P and O B are radii:

and it follows, that if the chords A P and P B = 2, A P + P B = 4: and it is self-evident, that A P + P B is greater than the chord A B; therefore, the chord A B must be less than 4, when the chords A P and P B = 2. But, A B is bisected in N, and it follows of necessity, that A N and N B are equal, and must be less than 2, when A P and P B = 2. Find the values of N A, N B, and N P, when the chords A P and P B = 2.

Now, the angles A O P and P O B are angles of 30°, and N A and N B are perpendicular to O P. Hence, the angle P A N = half the angle A O P, and it is self-evident, that A O N is the acute angle in the right-angled triangle O N A; and it follows, that A O N is an angle of 30°: $90° - 30° = 60°$ = the angle O A N: $\frac{30°}{2} = 15°$ = the angle P A N: $90° - 15° = 75°$ = the angle A P N: and A N O and A N P are right angles = 90°: and it follows, that the six angles of the right-angled triangles A N O and A N P are together equal to four right angles. The sum of the angles O A N and P A N is equal to the angle O A P, and the angles O P A and O A P are angles at the base of the isosceles triangle O P A, and are equal angles of 75°. Are not the angles of the equilateral triangle O A B equal angles of 60°, and together equal to two right angles? If you don't mind what you are about, you will "*upset*" the 5th and 32nd propositions of Euclid's first book.

From the point P draw a straight line, P M parallel to A B, and therefore tangental to the arc A P B, (it is self-evident that this line may be drawn on either side of the line O P,) making P M equal to $\frac{3}{4}$(O P), and join O M; and so construct a right-angled triangle O P M. Then: $3\frac{1}{8}(OP^2) = (OP^2 + PM^2 + OM^2)$, and this equation or identity = area of a circle of which O P is the radius. If you can upset this (I will not disturb your equilibrium by calling it a geometrical and mathematical coach) with the value assigned to the symbol π by Mathematicians, you will succeed in "*knocking over*" James Smith. The obtuse and acute angles in this right-angled triangle, are angles of 53° 8′ and 36° 52′. Surely, as a "*recognised Mathematician*," you can controvert this statement if not true. But, I am sure you cannot controvert it; and I have shewn how to demonstrate that these are the true values of the angles, in my work on "*The Geometry of the Circle*," a copy of which I sent you.

The two Letters of mine you returned the other day, you may still read, if you like. You will find one of them in my work on "*The Geometry of the Circle*," commencing on page 215, in which I have demonstrated that parallelograms of equal perimeter may contain unequal areas; and conversely, that parallelograms of unequal perimeter may contain equal areas: and elsewhere, in the same work, I have proved that unequal chords may be subtended by equal arcs.

In conclusion, I can assure you that you have never annoyed me; and I now write you, more in pity than in anger: and "*with this I now wish you farewell, as far as Mathematical correspondence is concerned.*" *("Final note by Mr. Whitworth.—Mr. Smith, by admitting that he understands my proof that* $\pi = 3{\cdot}126$, *and by failing to show any false step in it, practically admits that he is in the wrong.")*

Yours faithfully,

JAMES SMITH.

P.S.—Please return this. I promise, health permitting, that you shall see it in print shortly.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
12th August, 1869.

MY DEAR SIR,

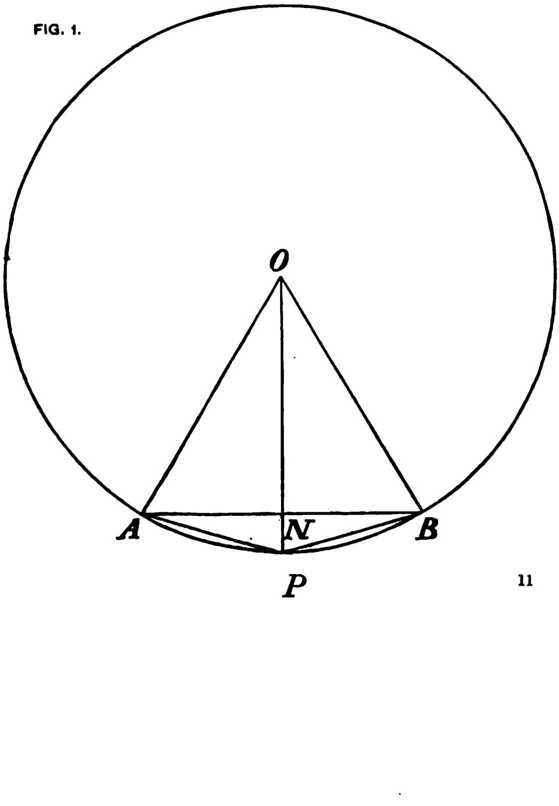
By the afternoon's post of the 6th inst., I received an elaborate Paper from the Rev. Professor Whitworth, evidently written with great care (on one side only), for the press, and professing to prove, without trigonometry, that the arithmetical value of the symbol π, is greater than $3{\cdot}126$. He requested me to return this Paper, and I did so by the morning's post of the 7th, but not before I had given it a very careful perusal.

Since I returned his elaborate Paper, I have received two Letters from that gentleman dated 7th instant, one 2-30 p.m., the other 4 p.m. The first intended for publication, if I chose to publish it; the other marked, "*Private and confidential, not for publication.*" The former I shall certainly publish—health permitting—the latter I returned to him without note or comment: not that I should care to see it published, but I suspect Mr. Whitworth would.

The Rev. and learned Professor, in his elaborate Paper, adopted for a *datum* a side of a regular inscribed dodecagon to a circle, *assuming* the length of it to be represented by the arithmetical symbol 2. He is the first Mathematician I ever met with who adopted such a *datum;* and in doing so, has furnished me with—or perhaps I should rather say has enabled me to furnish—a new proof that the arithmetical value of the symbol π is neither greater nor less than 3·125 ; this proof I have given him.

In a Postscript to the 2-30 p.m. Letter, Mr. Whitworth gives me what he considers a very simple disproof of the truth of my theory, that 8 circumferences of a circle = 25 diameters. With reference to a certain geometrical figure, and apparently with some shew of reasoning, he arrives at the following conclusion: "*Therefore, arc* A B : *chord* A B : : *arcs* A P *and* P B : *chords* A P *and* P B. *But, arc* A B = *arcs* A P *and* P B *together: therefore, chord* A B = *chords* A P *and* P B *together: or one side of a triangle equals the sum of the other two, which is impossible.*" You will thoroughly understand his argument from what follows.

FIG. 1.



In the geometrical figure (Fig. I), let O A B be an equilateral triangle. With O as centre and O A or O B as interval, describe the circle. Draw the line O P bisecting the angle O and its subtending arc A P B. Join A P and P B.

Mr. Whitworth gave no diagram, but he gave me the following construction :—"*Let* A B *be the side of a regular hexagon inscribed in a circle. Bisect arc* A B *in* P, *and join* A P, P B, *so that* A P, P B, *are the sides of a* 12-*gon in the same circle.*" I think you will admit, that I have not done that gentleman any injustice in my construction.

Now, my dear Sir, you must bear in mind that Mr. Whitworth, in his elaborate Paper, adopted A P or P B as a *datum*, say A P, representing its length by the arithmetical symbol 2. Mark the difficulty into which he has brought himself! It is self-evident that A P + P B is greater than A B, and A P + P B = 4, when A P = 2 ; and it follows that A B must be less than 4, when A P = 2. Now, N A and N B are perpendicular to O P, by construction ; and it follows, that A N O and A N P are right-angled triangles. But, A B is bisected in N, and it follows that N A and N B are equal, and that N A must be less than 2 when A P = 2.

THEOREM.

Find the arithmetical values of N A, N B, and N P, when the chords A P and P B = 2.

I have given this Theorem to Mr. Whitworth for solution. I can't solve it. If you can solve it, I am sure I should be greatly obliged if you would shew me how to solve it. I have no object to serve but the cause of scientific truth, and if I am unwittingly perverting it, nothing would please me better than to be convinced of it, and I think the solution of this theorem, if practicable, would convince me.

It is self-evident that O A P is an isosceles triangle, and it is axiomatic, if not self-evident, that the arc A P equals one-twelfth part of the circumference of the circle ; and it follows, that the angle A O P is an angle of 30°. It is self-evident that the angle A O N = the angle A O P ; but, because A N is perpendicular to O P, and the angle A O P less than 60°, the angle P A N = half the angle

O A P, and makes the angles at the base of the isosceles triangle O A P, to be angles of 75°.

I have shewn Mr. Whitworth, that if he does not mind what he is about he will "*upset*" the 5th and 32nd Propositions of the first book of Euclid.

Believe me, my dear Sir,

Very truly yours,

JAMES SMITH

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

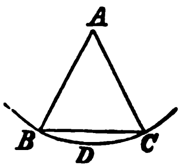
LANEAST, LAUNCESTON.

14th August, 1869.

DEAR SIR,

I found your two Letters on my return home after a few days' absence. I reply as briefly as possible, because you pronounced my recent Letters a jumble of wheat and chaff, truth and error, (though you did not point out the errors).

I gave A = 15°, A B = B C = 1. Find B C.



You forthwith deduct $\frac{1}{25}$ from A, making it thereby 14° 24′, and then *assert* that this equals the straight line B C, expressed in degrees and minutes.

You give no reason for this reduction. You offer no proof whatever, but simply take it as a dogma. I do not allow it to be true. It is, in fact, saying that by deducting $\frac{1}{25}$ from an *arc*, you get the length of its *chord*.

I asked for a calculation, and you begin with assuming the theory to be proved.

It is easy to see why you pitched on $\frac{1}{25}$ rather than $\frac{1}{26}$ or $\frac{1}{24}$. To radius unity, the semi-perimeter of a hexagon is 3. Now, having *fixed beforehand*, that $\pi = 3{\cdot}125$, you soon find, that deducting $\frac{1}{25}$ of this, there remains 3.

In all your writings, *no proof* of the value of π is ever given except *one!* You rightly employ a hexagon in a circle. Then—to radius unity— the semi-perimeter is 3, but the arc must be greater than the chord; therefore, π is greater than 3. So you reason, and reason well. But after this, you forsake *proof*, and apply to *conjecture*, never trying *other* polygons, which would soon have convinced you that π is greater than $3{\cdot}125$.

Yours very truly,
GEO. B. GIBBONS.

The first paragraph of the foregoing Letter is a perversion of what I said in mine of the 5th August. My Letter was not meant offensively, nor did I apply to Mr. Gibbons' recent Letters the offensive words, "*a jumble of wheat and chaff, truth and error.*" Mr. Gibbons must have lost his temper when he wrote his Letter of the 14th August. He complains that I "did not point out the errors." *Cui bono?* In my Letter of the 30th July, I gently suggested to Mr. Gibbons that the angle B A C in Fig. 2, page 3, cannot be less than half a right angle, that is, less than an angle of 45°, the measure of a right angle being fixed at 90°; and so long as 90° is retained as the measure of a right angle, it is a geometrical impossibility, that the angle B A C can be an angle of 16° 16′. Now, by hypothesis, let the measure of a right

angle be 20°, and B A C an angle of 16° 16′. Then: 20° — 16° 16′ = 3° 44′ = the angle A C D, and the three angles of the triangle A B C are together equal to two right angles. The angels A and D at the base of the isosceles triangle C A D are angles of 10°, and the three angles of the triangle C A D are together equal to two right angles. But, these facts do not affect the ratios of side to side in the right-angled triangles A B C and A B D. The data being A B = 3, and B C = 4, the ratios of side to side in these triangles are invariable, whatever we may adopt as the measure of a right angle.

Hence:

$$AB : AC :: 3 : 5.$$
$$AB : BD :: 3 : 1.$$
$$AB : AD :: 3 : \sqrt{10}.$$

Whether the measure of a right angle be 90° or 20°, and upon these facts I founded the theorem which I gave Mr. Gibbons for solution in my Letter of 30th July, page 44, in connection with the diagram B on page 45. I never could make facts suggestive of anything to Mr. Gibbons in the whole course of our very long correspondence. If I gave him a theorem, he never attempted to solve it; and when I pointed out a lapsus, he never had the candour to admit it. Or, if I gave him a theorem and the solution of it, he would either pass it over, or say, "*I can't see it,*" which led to the oft reiterated expression in my Letters to that gentleman:—"*If you can't see it, I can't help it, but the fact remains notwithstanding.*"

After the first paragraph of his Letter of the 14th August, Mr. Gibbons gives his oft-repeated geometrical figure, and observes:—"*I gave* A = 15°, A C = B C

$= 1$. *Find* B C. *You* (I) *forthwith deduct* $\frac{1}{25}$ *form* A, *making it thereby* 14° 24′, *and then assert that this equals the straight line* B C, *expressed in degrees and minutes*." Mr. Gibbons *charges me with taking this as a dogma, and offering no proof whatever of it.*

Let the circumference of a circle be 360°, and it cannot be denied that the circumference of a circle is divided by Mathematicians into 360 equal parts called degrees, and that these degrees are again divided into 60 equal parts called minutes. Now: $\frac{360}{25} = 14{\cdot}4$. Multiply by 24, then, $(24 \times 14{\cdot}4) = 345{\cdot}6$. Again: $\frac{360}{24} = 15$, deduct $\frac{1}{25}$th part $= \frac{1 \times 15}{25} = {\cdot}6$; then, $15 - {\cdot}6 = 14{\cdot}4$. Multiply by 24, then, $24 \times 14{\cdot}4 = 345{\cdot}6$. Again: $\frac{360}{96} = 3{\cdot}75$, deduct $\frac{1}{25}$th part $= \frac{1 \times 3{\cdot}75}{25} = {\cdot}15$; then, $3{\cdot}75 - {\cdot}15 = 3{\cdot}6$. Multiply by 96, then, $96 \times 3{\cdot}6 = 345{\cdot}6$; and it follows, that $345{\cdot}6 = 345°\ 36'$ expressed in degrees, is a constant quantity. Hence $\frac{24}{25}$ (360°) $= 345°\ 36'$, and is the perimeter of a regular hexagon inscribed in a circle, when the circumference $= 360°$.

Now, let c denote the circumference of a circle, and p the perimeter of a regular inscribed hexagon. Then: $(c - \frac{1}{25}\, c) = p$: and $(p + \frac{1}{24}\, p) = c$, whether $c = 360$ or $3{\cdot}125$, or any other arithmetical quantity.

Mr. Gibbons goes on to say:—" It is easy to see why you pitched on $\frac{1}{25}$ rather than $\frac{1}{26}$ or $\frac{1}{24}$. To radius unity, the semi-perimeter of a hexagon is 3. Now, having *fixed beforehand*, that $\pi = 3{\cdot}125$, you soon find that deducting $\frac{1}{25}$ of this there remains 3."

Now, by analogy or proportion.

$3 : 3{\cdot}125 :: 345^\circ\ 36' : 360^\circ$, and because 3 is the semi-perimeter of a hexagon to a circle of radius unity, and the perimeter of a regular hexagon to a circle of diameter unity: and because $\frac{25}{24}(345^\circ\ 36') = 360^\circ$, it follows, that $\frac{25}{24}(3) = \frac{25 \times 3}{24} = \frac{75}{24} = 3{\cdot}125 = \pi$. If "*recognised Mathematicians*," dispute this proof of π's value, let them honestly controvert it. This they may do, by finding the perimeter of a regular hexagon, when the circumference of its circumscribed circle $= 360$: but they will never find the perimeter of the hexagon by *pitching on* $\frac{1}{26}$ *or* $\frac{1}{24}$, *or any other fraction rather than* $\frac{1}{25}$. Has any "*recognised Mathematician*" ever found the perimeter of a regular inscribed hexagon to a circle, when the circumference $= 360^\circ$? Mr. Gibbons would probably say, this reasoning is "*quite fallacious*," and "*renders all argument hopeless.*" This is a matter of very little consequence. I do not now write for his benefit, but for the information of the Mathematicians of a future generation.

The last paragraph of Mr. Gibbons' Letter of the 14th August is utterly unworthy of him. Is not a hexagon a six sided polygon? Is not a dodecagon a twelve sided polygon? Does not circumference $\times$ semi-radius $=$ area, in every circle?

Let c denote the circumference of a circle.

Let a denote the area of the circle.

Let p denote the perimeter of a regular hexagon inscribed in the circle.

Let d denote the area of a regular dodecagon inscribed in the circle.

Let r denote the radius of the circle.

Then:

$6·25\ (r) = c : 3·125\ (r^2) = a : (a - \frac{1}{25} a) = d$: and $6\ (r) = p$: and by analogy or proportion $p : c :: d : a$; and by alternation $a : d :: c : p$; and it follows, that $(c \times \frac{1}{2} r) = a$, whatever be the value of r. For example: Let $r = 4$. Then: $6·25\ (4) = 25 = c : 3·125\ (4^2) = 3·125 \times 16 = 50 = a : (a - \frac{1}{25} a) = \{50 - \frac{1}{25}\ (50)\} = (50 - 2) = 48 = d$: and $6(r) = 6 \times 4 = 24 = p$: and $p : c :: d : a$; that is, $24 : 25 :: 48 : 50$: and it follows, that $(c \times \frac{1}{2} r) = 25 \times 2 = 50 = a$: and that $3\frac{1}{8}$ is π's true value. If Mr. Gibbons can't see these facts, "*I can't help it, but the facts remain notwithstanding.*"

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
16th August, 1869.

MY DEAR SIR,

I leave home this afternoon *en route* for Exeter to attend the meeting of "The British Association," and if I feel well enough to undertake the journey, I shall probably attend the "German Congress for the Advancement of Science," which meets at Innspruck, on the 18th September, so that I may be from home for the next six or seven weeks. Until the 25th, a Letter addressed, Reception Room, British Association, Exeter, will find me.

On reflection, I felt it necessary to reply to the Rev. Professor Whitworth's Letter dated August 7th, 2-30 p.m., which he obviously thought to be unanswerable. This I did in a Letter dated 10th inst., which, at my request, he has returned. He has attached copious notes to it, and on its return, in the same envelope, there was the following communication:—

LIVERPOOL, *13th August*, 1869.

SIR,

If you publish this Letter of yours, I shall be glad if you have the courage to publish the notes I have made on it. I

reserve all rights of publication of all the Letters I have written to you, and caution you against infringing my copyright. If you wish to publish any of the Letters, or Extracts from any of them, I will tell you on what conditions I will consent.

Yours truly,

W. ALLEN WHITWORTH.

I must now refer you to the geometrical figure represented by the diagram in my Letter of the 21st July We are supposed not to know the values of the obtuse and acute angles in the right-angled triangles A B C and A B D, expressed in degrees ; and it stands to common sense that we can only find the values of these angles, if to be found at all, by means of what we do *know*. Well, then, what do we *know* with reference to this geometrical figure? We *know* that A C D is an isosceles triangle, and we *know*, by Euclid, Prop. 5 : Book 1 : that the angles A and D at the base of this triangle are equal. We *know* that A B C and A B D are right-angled triangles, by construction; and we *know* that A B is to B C in the ratio of 3 to 4, and A B to B D in the ratio of 3 to 1, by construction; and we *know* by Euclid, Prop. 32 : Book 1 : that the six angles of the triangles A B C and A B D are together equal to four right angles. We *know* that the angles A and C in the triangle A B C are together equal to the right angle B: and we *know* that the angles A and D in the triangle A B D are together equal to the right angle B. We *know* that a right angle = 90°, and because the angles A and C in the triangle A B C are together equal to a right angle, it follows, that the obtuse angle A must be greater, and the acute angle C less, than 45°: and because the angles A and D in the triangle A B D are together equal to a right angle, it follows, that the obtuse angle D must be greater, and the acute angle A, less, than 45°. We *know* that when A B = 3 : and B C = 4 : then, A C = 5 : B D = 1 : and A D = $\sqrt{10}$. We *know* that $(AB^2 + BC^2 + AC^2) = 3\frac{1}{8}(BC^2)$. Finally: We *know* that A D is shorter than a side of a regular inscribed hexagon to the circle, and it follows, that the angle C is less than 60°.

Now, it stands to common sense that Mathematics can never be made to over-ride the ratio of side to side, by construction, in

any right-angled triangle: and it also stands to common sense, that Mathematics, rightly applied, can never be inconsistent with pure Geometry. In one of your Letters you say: "*I have avoided Logarithms as needless.*" It is true you have avoided Logarithms, for you have certainly never attempted to grapple with any of my proofs by Logarithms, and I have given you many in the course of our long correspondence.

Well, then, my dear Sir, I tell you, not "in rudeness," but distinctly, that it is only by means of Logarithms that we can find the true values of the obtuse and acute angles in the triangles A B C and A B D; and of this fact I shall proceed to give you the proof. I might take either triangle, but as it is not necessary to take both, I shall take the triangle A B D.

Now, $\frac{BD}{AD} = \frac{1}{\sqrt{10}} = \frac{1}{3 \cdot 162277} = \cdot 3162277$ = the trigonometrical sine of the angle A: $\frac{AB}{AD} = \frac{3}{\sqrt{10}} = \frac{3}{3 \cdot 162277} = \cdot 9486834$ = the trigonometrical sine of the angle D. The Logarithm corresponding to the natural number ·3162277 is 9·4999999... and this is the log.-sine of the angle A. The logarithm corresponding to the natural number ·9486834 is 9·9771212... and this is the log-sin. of the angle D.

THEOREM.

Let A D the side subtending the right angle in the triangle A B D be represented by any arithmetical quantity, say 60, and be given to find the values of the sides A B and B D, and prove that they are in the ratio of 3 to 1, that is, in the same ratio as by construction.

Then:

As Sin. of angle B = Sin. 90°	Log.	10·0000000
: the given side A D = 60	Log.	1·7781513
: : Sin. of angle D	Log.	9·9771212
		11·7552725
: the required side A B		10·0000000
= ·9486834 × 60 = 56·9210040	Log.	1·7552725

Again:

As Sin. of angle B = Sin. 90°............................ Log.	10·0000000
: the given side A D = 60 .... Log.	1·7781513
: : Sin. of angle A... Log.	9·4999999
	11·2781512
: the required side B D	10·0000000
= ·3162277 × 60 = 18·97366620... .... Log.	1·2781512

Now, the side A D in the triangle A B D, being incommensurable, and therefore indeterminate with arithmetical exactness, 56·9210040 is slightly greater than the length of the side A B, and 18·9736620 slightly less than the length of the side B D. But, you will observe that the ratio of A B to B D is as 3 to 1, as nearly as possible. For example: By hypothesis, let B D = 18·97367; that is, add 1 at the fifth place of decimals. Then 3 (18·97367) = 56·92101, making A B slightly greater than 56·9210040; and it is obvious, that we cannot alter this conclusion by extending the number of decimals.

Well, then, my dear Sir, we *know* that the angle A in the triangle A B D is less than 45°.

THEOREM.

Given: angle A = 8° 8′, and the side A D in the right-angled triangle A B D, 600 miles in length. Find the lengths of the sides A B and B D, which include the right angle, and prove that they are in the ratio of 3 to 1.

Then: 90° — 8° 8′ = 81° 52′ = angle D, and by Hutton's Tables:

As Sin. of angle B = Sin. 90°............................ Log.	10·0000000
: the given side A D = 600 miles......................... Log.	2·7781513
: : Sin. of angle D = Sin. 81° 52′ .... Log.	9·9956095
	12·7737608
: the required side A B	10·0000000
= 593·97572 miles nearly .... Log.	2·7737608

Again:

As Sin. of angle B = Sin. 90° ....Log.	10·0000000
: the given side A D = 600 miles ....Log.	2·7781513
: : Sin. of angle A = Sin. 8° 8′ ....Log.	9·1506864
	11·9288377
: the required side B D	10·0000000
= 84·89887 miles nearly ....Log.	1·9288377

Thus, the ratio of A B to B D, by construction, is completely "*upset.*"

Now, my dear Sir, you assume Hutton's Tables to be correct, having been "*calculated by experts in every country in Europe,*" and yet you avoid Logarithms. No wonder, so long as you are resolved to bolster up a false theory. I do not intend this offensively, or "*in rudeness.*" I, of course, believe you to be mistaken; the words occur to me as expressing the gravamen of one of Rev. Professor Whitworth's charges against me.

THEOREM.

Given: angle D A B in the triangle A B D = 18° 26′. Find the angles A and C in the triangle A B C.

We *know* that the angle D A B is less than 45°, and we know that the sides that include the right angle in the triangle A B C are in the ratio of 3 to 4, by construction: and I know—if "*recognised Mathematicians*" don't—that Mathematics, rightly applied, can never destroy the ratios of side to side, by construction, in any right-angled triangle.

Now, angle D A B is given = 18° 26′. Hence: 90° — 18° 26′ = 71° 34′ = the angle A D B: 2(18° 26′) = 36° 52′ = the angle A C B in the triangle A B C: and 90° — 36° 52′ = 53° 8′ = the angle B A C. The sum of all the interior angles in the two triangles are equal to four right angles, and the angle D A B = half the angle A C B.

THEOREM.

Given: angle C in the triangle A B C = 36° 52′, and the side

$AC = 600$ miles. Find the values of the sides A B and B C, and prove that they are in the ratio of 3 to 4.

It is self-evident that $90^\circ - 36^\circ\ 52' = 53^\circ\ 8'$ = the angle A. Now, if you go to work by Hutton's Tables, you will find that you destroy the known ratio, by construction, between A B and B C. I know—and you think you know—that if in any right-angled triangle, two angles and a side, or two sides and an angle, be given, we can find the remaining sides and angles. But, I may tell you, that if you make the angle C in the triangle A B C either greater or less than $36^\circ\ 52'$, you can never find the exact lengths of A B and B C from a given length of A C, by existing Logarithmic Tables.

Now, if I can neither induce you nor the Rev. Professor Whitworth to solve the theorem, I have given you in my Letter of the 12th instant, if practicable, shall I not have convicted you both of mathematical dishonesty?

Believe me, my dear Sir,
Very truly yours,
JAMES SMITH.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

EXETER, 18*th August*, 1869.

MY DEAR SIR,

I left home on Monday at 11-30 a.m., en-route for this city, to attend the meeting of the British Association for the advancement of Science. Before leaving home, I had partially written a Letter intended for you, but had not time to finish, make a fair copy, and post it. I brought the rough draft with me, and was just sitting down to write a fair copy, when the post brought me your favour of the 14th instant, forwarded to me from home. I have just read over the rough draft of my intended Letter to you, and if you can take my word—and I give it on the honor of a gentleman—you shall have a copy of it (D. V.) either before, or after my return home.

You now say :—"*I asked for a calculation, and you begin with assuming the thing to be proved.*" I admit that I have made an assumption, but I plead "not guilty" to the charge of assuming the thing to be proved. Can *you* prove anything without assuming something? Will you venture to tell me, that I have not as much right to assume the circumference of a circle = 360°, as a datum, as Mathematicians have to assume the radius of a circle = 1, as a datum, in their endeavours to entice π out of "*his den*"? I know, my dear Sir, that you—"*as a Christian and a gentleman*"—will not venture to tell me anything of the sort.

Well, then, may I request you to take the geometrical figure represented by the diagram in my Letter of the 21st July, (see page 3), and with C as centre, and C B as interval, describe a circle, and in this circle inscribe a square? If I were to say that you were incompetent to make these simple additions to the diagram, you might justly charge me with "*consciously lying.*"

Now, *assuming* you to have made these additions to the diagram, I give you the following theorem for solution :—

THEOREM.

Let X denote the area of the smaller circle, and Y the area of its inscribed square. Given : Y = any *finite* and *determinate* arithmetical quantity you please. Find the value of X, and prove that X is equal to the sum of the squares of the sides of the right-angled triangle A B C, the generating figure of the diagram.

I can solve this theorem, so you must not tell me, the solution of it is impracticable, for that would be equivalent to *asserting* that you possess the intellectual capacity to prove a negative : not only so, but it would be equivalent to telling me that I am "*consciously lying.*"

Well, then, if you can, and will not, solve this theorem ; or, if you can't, and decline to admit your incapacity to solve it, will you not on either alternative be mathematically dishonest

I have had a conversation with your friend Professor Adams this morning.

Believe me, my dear Sir,

Very truly yours,

JAMES SMITH.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

BOSCASTLE, CORNWALL,
19th August, 1869.

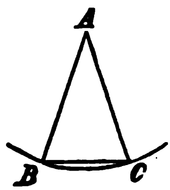
MY DEAR SIR,

I am at the sea side, and have leisure to look into the *results* of your assumption, that—

$$\frac{24}{25} \text{ of the arc} = \text{chord.}$$

Of course, it is soon obvious to every Geometer, that this cannot be true, for the ratio of arc to chord, and the difference between them, will vary with every value of the arc. But assuming your law: If you apply it to an octagon or duodecagon, you will get out a different value of π for every such polygon. I have made these calculations, but need not trouble you with them.

As you deducted $\frac{1}{25}$ for a polygon of 24 sides, I took the reduction the $\left(\frac{1}{n+1}\right)$th part of the arc, giving n the value it had by the number of sides in the polygon, but the result was so far the same, viz.: Each polygon gave a different value to π.



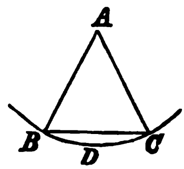
I then thought I would try my hand at this kind of "*proof*." Let A = 60°: A B = A C = 1, the *arc* corresponding to 60° is $\frac{\pi}{3}$.

So I reduced this by its twenty-first part, $\left(\frac{\pi}{3}\right) - \frac{1}{21}\left(\frac{\pi}{3}\right) = \text{chord} = 1$, for chord $60° = 1$; that is, $\frac{20}{21} \cdot \frac{\pi}{3} = 1 : \pi = \frac{63}{20} = 3{\cdot}15$ exactly !!

I am bound to confess that I can offer ***no proof*** that the arc is to be reduced by $\frac{1}{21}$, to get the value of its chord. But then, neither did you offer any proof of your $\frac{1}{25}$ reduction, and my proof is of the same kind as yours, and built of the same sort of foundation. I "***proved***" my π by a reduction $\left(\frac{1}{21}\right)$, purposely contrived to elicit it: similarly your $\frac{1}{25}$ was coined to get out $\pi = \frac{25}{8}$. Both are equally worthless, as proof, because they prove nothing, and fail to hold, for any polygon except that which was their origin. I imagine you will soon see how hopeless it is to deal with me, since we agree not in the simplest fundamentals, and our ideas are so different as to what constitutes a "proof." I have never seen in all your books, a single attempt to ***prove*** the equality in length or area of rectilinear and curvilinear lines or surfaces. Your method is this, "Now on the ***supposition***," that $\pi = \frac{25}{8}$ certain results will follow, ***and they would follow, if π did equal*** 3·125.

The question is: How can we find the length of the circular arc B D C for a given angle A?

The ***straight line*** B C ***can*** be found by trigonometry, which involves no knowledge of π.



Yours truly,
G. B. GIBBONS.

On my THEORY $(\pi - \frac{1}{25}\pi) = (3{\cdot}125 - {\cdot}125) = 3 =$ the perimeter of a regular inscribed hexagon to a circle of diameter unity. To controvert this, Mathematicians must find some other value of π than 3·125, from which they can deduct an *aliquot* part, and get the perimeter of a regular hexagon to a circle of diameter unity.

97

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

BOSCASTLE, 20*th August*, 1869.

My tables of sines and log-sines are from Norie's navigation, and give only to 6 decimals.

$A = 15^\circ$

$AB = AC = 1$

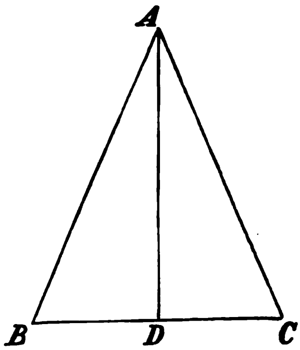
Find BC.

$BC = 2\,BD$

$= 2 \text{ Sin. } 7^\circ\, 30'$

$= 2\,(\cdot 130526)$

$= \cdot 261052.$



Or, by Logarithms—

Log. 2	0·301030
Log.-Sin 7° 30′	9·115698
	9·416728
Thus, Log. ·2610 =	9·416640, difference 166
	88

$$
\begin{array}{l}
166)\,88{\cdot}0\,(53 \\
\quad\;\; 83{\cdot}0 \\
\quad\;\; \overline{\;\;500} \\
\quad\;\;\;\; 498 \\
\quad\;\;\;\; \overline{\quad\;}
\end{array}
$$

This gives $BC = \cdot 261053$.

I see by my note book, that I have already sent Mr. Smith the full calculation of B C, without using any tables of sines or their logarithms, I therefore forbear repeating the detail.

Now, apply Mr. Smith's theorem—

$\text{Arc} - \frac{1}{25}\text{ arc} = \text{chord}$: in this case $A = 15^\circ$:

$\text{Arc} = \frac{\pi}{12} \therefore \frac{24}{25} \cdot \frac{\pi}{12} = \cdot 261052\ldots$:

$\frac{2}{25}(\pi) = \cdot 261052\ldots : \pi = 3{\cdot}2631\ldots$ *

* $\frac{2}{25}(\pi) = \frac{8\pi}{100}$, whatever be the value of π.

In truth there is *no equation* that will *exactly* express the arc in terms of the chord, (*or vice versâ*), nor will any square *exactly* give the diagonal of a square in terms of its side.

So the calculations of "natural sines" (that is *lengths* of the sines to radius 1, which are given in Hutton's and other Tables) will involve *square roots*, which cannot be expressed exactly. Sin. 90° = 1 : Sin. 30° = ½ exactly: but, Sin. 60° = $\frac{\sqrt{3}}{2}$, and so *generally* we fall in with square roots, which are not *exact* quantities.

All efforts to find π, must begin with establishing an equation either (1) between a *straight line* in the circle and its circular boundary, as to length, or (2) between a rectilinear and circular area.

Hitherto, such equation has always been effected by means of a *series*, interminable, but such that the terms decrease rapidly in value, so as to enable the enquirer to get a value of π to any degree of accuracy he may desire.

Another method (and an older) is this—The perimeter, or area of a circle is somewhere *intermediate* to the perimeter or area of its inscribed and circumscribing polygons.

Now, the sides of regular polygons *can be* computed without knowing π, to as great a degree of accuracy as we please: e.g. B C is, in the figure when A = 15°, greater than ·261, but less than ·262.

Thus we can enclose π between two limits, which limits we can bring together as closely as we please.

Yours very truly,
GEO. B. GIBBONS.

LANEAST, SATURDAY MORNING,
21*st August*, 1869.

DEAR SIR,

I have just arrived home and got your Letter. Of course you have a right to assume anything of the kind you name, but you must not assume what involves the thing to be proved. You can assume $\frac{1}{25}$ as a reduction of A, as lawfully as I can assume $\frac{1}{21}$; *both involve* π: and tacitly assume its value as 3·125 and 3·15.

G. B. G.

The difference between the logarithm of ·26105 and ·26106 is unquestionably 166, but I utterly deny Mr. Gibbons' conclusion that $\pi = 3{\cdot}2631\ldots$ According to Hutton, the difference between the sine of an angle of $7^\circ\ 30'$ and $7^\circ\ 31'$ or the difference between the sine of an angle of $7^\circ\ 29'$, and $7^\circ\ 30'$, is 2884. According to my THEORY, $\frac{2}{25}(\pi) =$ ·25, and $25\ ({\cdot}25) = 6{\cdot}25 = 2\,\pi$.* I deny the *assertions* in the two last paragraphs of Mr. Gibbons' Letter dated 20th August, and defy any Mathematician to prove them.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

LANEAST, SATURDAY AFTERNOON,
21st August, 1869.

MY DEAR SIR,

I returned home to-day only in time to add a line to what I wrote at Boscastle, before the post-boy went off.

Of course you can assume the circle divided into 360 equal degrees, indeed, this is not an "assumption" but merely the adoption of a division universally employed.

What I object to is your taking for granted that the arc diminished by its $\frac{1}{25}$ *part equals the chord.*

I know no property of the circle that shews this, and to assert it, is tacitly to assume the value of π. Turning to your Letter of 7th August, I read—

"*It follows*, that $345^\circ\ 36'$ is the perimeter of a regular hexagon in a circle of 360°." I admit that you *may* express a chord which is a straight line, in "degrees," (though this is very unusual,) but passing that by, you assume that :—

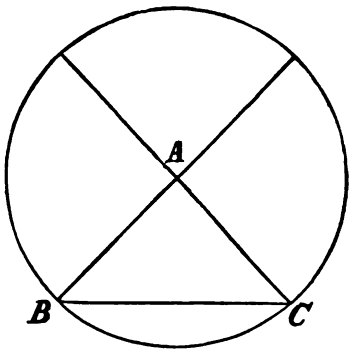
$$\frac{\text{chord}}{\text{arc}} = \frac{345^\circ\ 36'}{360} = \frac{14^\circ\ 24'}{15} = \frac{24}{25}.$$

* 25 times $\frac{2}{25}(\pi) = 2\,\pi$, whatever be the value of π.

Now, chord $60° = 1$, and arc $60° = \frac{\pi}{3}$; that is, $\frac{24}{25} \cdot \frac{\pi}{3} = 1$; $\pi = \frac{25}{8} = 3{\cdot}125$. So that you have, in fact, assumed $\pi = \frac{25}{8}$ in assuming $\frac{24}{25}$ (arc) = chord in a hexagon.

You were discussing A = 15°, but you word your result for A = 60°, so that you assume the law to be general. You say, "*whence it follows*, (from A = 15°), that 345° 36′ is the perimeter of a regular hexagon (A = 60°) in a circle of 360°."

This, then, is the assumption I object to. Where do you learn that, arc — $\frac{1}{25}$ arc = chord? I deny it utterly. If it be so, let A = 90° (radius 1), then, B C = $\sqrt{2}$. Now, by your theorem, since $\frac{\pi}{2}$ is the arc, $\frac{\pi}{2} - \frac{1}{25}\left(\frac{\pi}{2}\right) = \sqrt{2}$. $\frac{24}{25}\left(\frac{\pi}{2}\right) = \sqrt{2}\ \pi = \frac{25}{12}\sqrt{2}$; a value, I think, you will dissent from. But forasmuch as your theorem is false, different polygons will give different values of π.



I am daily expecting that you will give me up as impracticable, but at any rate, it would be useless for me to go into more complicated figures, if I am unable to find B C in the given triangle we are discussing. Our only *chance*, is sticking to one thing, the easiest, till we either agree or quietly cease our dispute. Thus I have not studied the latter part of your letters, wherevei I can't get over the *first* part, for by the nature of the case, the *subsequent* depends on the *previous* demonstration. You can't skip over a difficulty and go on, as you could in items of distinct, *unconnected* information.

And if you suspect my "incapacity" to solve a complicated figure, you ought, the more patiently, to dwell on the "*Elementary*" and simple discussion in which we are now engaged. I have not the smallest objection to your holding the belief, that I am *unable*

to solve your theorems: only, you should deal with me accordingly. Gently led, step by step, I might in time get on. I have calculated B C for A = 15°, and find it rather greater than ·261 to radius 1; that is, if radius be 1000 inches, B C is rather more than 261 inches, consequently it exceeds 3·132.

Yours very truly,

G. B. GIBBONS.

In this Letter Mr. Gibbons says:—"*I admit that you* (I) *may express a chord which is a straight line in "degrees,"* (*though this is very unusual*), *but passing that by you* (I) *assume that:—*

$$\frac{\text{chord}}{\text{arc}} = \frac{345°\ 36'}{360} = \frac{14°\ 24'}{15} = \frac{24}{25}.$$

Now, chord 60° = 1, *and arc* 60° = $\frac{\pi}{3}$; *that is,* $\frac{24}{25} \cdot \frac{\pi}{3} = 1 : \pi = \frac{25}{8} = 3·125$. *So that you have, in fact, assumed* $\pi = \frac{25}{8}$, *in assuming* $\frac{24}{25}$ (arc) = *chord in a hexagon.*"

Now, $\frac{345°\ 36'}{360°} = ·96$: $\frac{14°\ 24'}{15°} = ·96$: $\frac{24}{25} = ·96$: and $\frac{3}{3·125} = ·96$: and it follows, that $\frac{345°\ 36'}{360°}$, $\frac{14°\ 24'}{15°}$, $\frac{24}{25}$, and $\frac{3}{3·125}$ are equivalent ratios: and because the perimeter of a regular inscribed hexagon to a circle of diameter unity = 3, it follows, that $\frac{24}{25}$ (1) = ·96 = the perimeter of a regular inscribed hexagon to a circle of circumference unity. Hence: $\frac{·96}{6} = ·16$ = radius of a circle when the circumference = 1, and $3\frac{1}{8}$ ($·16^2$) = 3·125 × ·0256 = circumference × semi-radius—that is, 3·125 × ·0256 = 1 × ·08 = area of a circle, when the circumference = 1: and it follows, that $\left(\frac{24}{25} \cdot \frac{3·125}{3}\right) = (·96 \times 1·04166666$ with 6 to infinity) = ·9999999936; and if we extended

the decimals to 1000 places, the result would be a decimal quantity, consisting of 1000 nine's and the final decimals 3 and 6. But, $\frac{24}{25} \cdot \frac{3{\cdot}125}{3} = \frac{24 \times 3{\cdot}125}{25 \times 3} = \frac{75}{75} = 1$. Surely! ·9999999936 stands for the *finite* and *determinate* arithmetical quantity 1, as certainly as the series $(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \&c.)$ stands for the *finite* and *determinate* arithmetical quantity 2. Well, then, $\frac{3{\cdot}125}{3} = 1{\cdot}04166666$ with 6 to infinity. Add 1 to the last decimal figure. Then: $({\cdot}96 \times 1{\cdot}04166667) = 1{\cdot}0000000032$, and is greater than unity. So far I have assumed nothing, but have simply dealt with facts. The symbol "*π does not appear, need not appear.*" But, when Mathematicians can prove that $\frac{345^\circ\ 36'}{360^\circ}$, $\frac{14^\circ\ 24'}{15^\circ}$, $\frac{24}{25}$ and $\frac{3}{3{\cdot}125}$ are not equivalent ratios, they will be able to demonstrate that 3·125 is not π's true value, but not till then. What will the Rev. Professor Whitworth say to this? Verily! Verily! With such opponents as Gibbons, Whitworth, Wilson, De Morgan, and Glaister, *all argument is indeed hopeless.*

Mr. Gibbons goes on to say:—"*You* (I) *were discussing $A = 15^\circ$, but you word your result for $A = 60^\circ$, so that you assume the law to be general.*" He then flies off at a tangent, from $A = 15^\circ$ to a circle of radius 1, to $A = 90^\circ$ to a circle of radius 1, and argues, as if the reasoning must be the same, whether we make our datum, in the search after π, A an angle at the centre of a circle of 15°, or A an angle at the centre of a circle of 90°. If $A = 15^\circ$, A is subtended by a chord equal to a side of a regular inscribed polygon of 24 sides. If $A = 60^\circ$, A is subtended by a chord equal to a side of a regular

inscribed hexagon ; or in other words, is subtended by a chord equal to radius. If $A = 90°$, A is subtended by a chord equal to a side of an inscribed square to the circle.

Take Mr. Gibbons' geometrical figure, in his Letter of the 21st August, page 100. B A C is an angle of 90°, to a circle of radius 1 ; therefore, $BC = \sqrt{2}$, as Mr. Gibbons puts it. Bisect B C at a point D, and join A D. Then: D A B and D A C will be angles of 45°, and the trigonometrical sines of these angles $= \frac{1}{2}(\sqrt{2}) = \sqrt{\cdot 5} = \cdot 7071068...$ Now, according to the reasoning of Mr. Skeat and Mr Gibbons, (*see page* 62), (and Mr. Glaister agrees with Mr. Skeat), $2\ (\cdot 7071068) = 1\cdot 4142136 =$ sine of the angle B A C. But, B A C is a right angle, and the sine of a right angle = unity, = the chord subtended by an angle of 60°, and the sine of an arc is half the chord of twice that arc. Hence, $\frac{1}{2}(\sqrt{2})$ is the sine of an angle of 45°. How will Mr. Skeat, Mr. Gibbons, and Mr. Glaister make these facts consistent with their theory?

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

ROYAL CLARENCE HOTEL,
EXETER, *23rd August*, 1869.

MY DEAR SIR,

I am in receipt of your favors of the 19th and 20th inst. Before noticing *them*, I must make some observations on your Letter of the 14th inst. In that communication, you give the old diagram, and repeat the old theorem. "$A = 15° : AB = AC = 1$. Find B C." With reference to what I have said about this theorem you observe:—"You forthwith deduct $\frac{1}{16}$ from A, making it thereby 14° 24′, and then assert that this equals the straight line B C expressed in degrees and minutes. You give no reason for this reduction. You offer no proof whatever, but simply take it as a

dogma. I do not allow it to be true. It is, in fact, saying that by deducting $\frac{1}{25}$ from an arc, you get the length of its *chord*. I asked for a calculation—and you begin with assuming the thing to be proved."

Now, my dear Sir, I have never said,—and I defy you to prove that I have ever said,—that by deducting $\frac{1}{25}$th part from an arc, we get the length of its chord; but, by deducting $\frac{1}{25}$th part from any arc, we can get a ratio. For example: Let an arc of a circle = 1. Then: $\frac{1}{25}(1) = {}'04$, and it follows, that $1 - {}'04 = {}'96$, and we get a ratio. Will you venture to tell me that $\frac{'96}{1}$ is not a ratio? Multiply the two terms of this ratio by 4, and we get the equivalent ratio $\frac{3'84}{4}$, and both these ratios express the true ratio between the perimeter of a regular polygon* and the circumference of its circumscribing circle. Divide the two terms of the ratio $\frac{'96}{1}$ by $3{\cdot}125$, and we get the equivalent ratio $\frac{{\cdot}3072}{{\cdot}32}$. Hence: $\frac{{\cdot}96}{1}$, $\frac{3{\cdot}84}{4}$, and $\frac{{\cdot}3072}{{\cdot}32}$ are equivalent ratios, and by analogy or proportion, $\cdot3072 : \cdot32 :: 3.84 : 4$; and it follows, that all these ratios express the true ratio between the perimeter of every regular hexagon and the circumference of its circumscribing circle.

With you, my dear Sir, there would appear to be some especial charm about a circle of radius = 1. This appears to be the "Alpha and Omega" of your geometrical investigations. For me it has no especial charm, but 'twill do, in fact it will serve the purpose of my argument just as well as *assuming* the circumference of a circle to be divided into 360 equal parts called degrees: and I think you will not venture to dispute or deny that the arithmetical value assigned to π by "*recognised Mathematicians*," (my correspondent, the Rev. Professor Whitworth excepted, who says:—He *knows*, and *always teaches*, that π is a *finite* and *determinate* quantity), is $3{\cdot}14159265$, &c.

*The definition of a polygon is given in Euclid's Elements of Plane Geometry as:—"*A polygon is a rectilinear figure having more than four sides.*" So that, according to Euclid, a rectangle having four equal sides, or in other words, a square, is not a polygon.

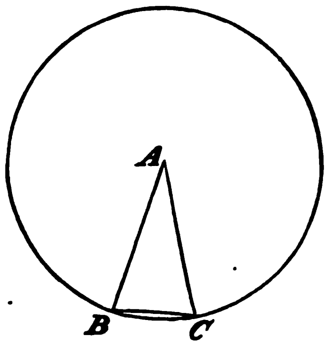
Let π denote the *finite* and *determinate* arithmetical quantity 3·14, which is less than the value assigned to the symbol π by "*recognised Mathematicians*," but greater than my value of the circumference of a circle of diameter unity, and let the radius of a circle = 1. If we want to find the value of a symbol, it may be x, or it may be π—(the symbol denoting an arithmetical quantity we don't know,) it is obvious we must set about finding it by means of something we do know. It is admitted that a circle possesses the three properties of circumference, diameter, and area, and it cannot be disputed that we may assume a value of the circumference, to find the values of the diameter and area. Now, we ***know*** that the circumference of a circle of radius unity = 2π, and on the hypothesis that π = 3·14 = 6·28. From 3·14 deduct $\frac{1}{25}$th part. Then: $\frac{3·14}{25}$ = ·1256, and it follows, that 3·14 — ·1256 = 3·0144, and is greater than the semi-perimeter of a regular inscribed hexagon to a circle of radius unity, or the perimeter of a regular inscribed hexagon to a circle of diameter unity. Now, inscribe a square to a circle of radius unity. We ***know*** that the sides of this square are subtended by arcs = $\frac{\pi}{2}$, or $\frac{2\pi}{4}$, and we also ***know*** that the diagonals of this square are diameters of a circle of radius = 1. "*If you can't see it, I can't help it*," but from these facts it follows, that 3·14 is greater than the true arithmetical value of the symbol π, and I will make an attempt to convince you of this.

I have made the circumference of the circle = 3·14, by hypothesis; and it follows, that $\frac{3·14}{4}$ = ·785 is the value of the arcs subtending the sides of an inscribed square to the circle. From this deduct $\frac{1}{25}$th part. Then: $\frac{·785}{25}$ = ·0314, and ·785 — ·0314 = ·7536. Multiply by 4, and it follows, that 4 × ·7536 = 3·0144, and is greater than the perimeter of a regular inscribed hexagon to a circle of diameter unity. But, 3·0144 is the true value of the perimeter of a regular inscribed hexagon to a circle of circumference = 3·14. Hence: by analogy or proportion, 3·0144 : 3·14 : : 3 : 3·125, and it follows, that $\frac{3·0144}{3·14}$ and $\frac{3}{3·125}$ are equivalent ratios, and both ex-

press the ratio between the perimeter of every regular hexagon and the circumference of its circumscribing circle. "*ij you can't see it, I can't help it,*" but from these facts it follows, that $\frac{25}{8} = 3\cdot125$, is the true arithmetical value of π, making 8 circumferences = 25 diameters, in every circle.

Now, we know that the symbol π not only denotes the ratio of circumference to diameter in a circle, but we also know that it denotes the circumference of a circle of diameter unity; and because the circumferences of circles are to each other as their diameters, it follows, that 2π denotes the circumference of a circle of radius unity, whatever be the value of π. Inscribe a square to a circle of radius unity. Then: $\frac{2\pi}{4} = \frac{6\cdot25}{4} = 1\cdot5625$ is the value of the arcs subtending the sides of an inscribed square. Deduct $\frac{1}{25}$th part. Then: $\frac{1\cdot5625}{25} = \cdot0625$, (*en passant*, $\cdot0625$ is the area of a square on the semi-radius of a circle of diameter unity), and it follows, that $1\cdot5625 - \cdot0625 = 1\cdot5$. Multiply by 4, and it follows, that $4 \times 1\cdot5 = 6 =$ perimeter of a regular inscribed hexagon to a circle of radius unity; and by analogy or proportion, $6 : 6\cdot25 :: 3 : 3\cdot125$. Now, my dear Sir, since we cannot work out these results by any other value of π, whether arithmetically *determinate* or *indeterminate*, or, either greater or less than $\frac{25}{8} = 3\cdot125$, it follows, that $3\cdot125$ *must* be the true arithmetical value of π. "*If you can't see this, I can't help it, but the fact remains notwithstanding.*"

From these facts we learn, that into whatever number of equal arcs we may divide the circumference of a circle, if from one of these arcs we deduct $\frac{1}{25}$ part, and multiply the remainder by the number of arcs, the product is a constant quantity, and exactly equal to the perimeter of an inscribed regular hexagon.



Let the angle A = 15°, and by hypothesis, let the circumference of the circle = 360°. You surely cannot object to my assuming the circumference of the circle to be represented by 360°, since this assumption is made by all Mathematicians, in dealing with a circle for practical purposes. Well, then, the angle A is subtended by an arc of 15°; and since 24 times 15° = 360°, it follows, that the arc B C = $\frac{1}{24}$ part of the circumference of the circle. From the arc B C deduct $\frac{1}{25}$ part. Then: $\frac{15^\circ}{25} = 36'$, and $15^\circ - 36' = 14^\circ\ 24'$. Multiply by 24. Then: $24\,(14^\circ 24') = 345^\circ 36'$, and it follows, that $345^\circ 36'$ is the value of the perimeter of a regular inscribed hexagon to a circle of circumference = 360°; and by analogy or proportion, $345^\circ\ 36' : 360^\circ :: 3 : 3\ 125$. Now, dispense with the symbols denoting degrees and minutes, and let the circumference of the circle = 360. Inscribe a regular hexagon, a regular dodecagon, and a regular 24 sided polygon. Then: $\frac{360}{6} = 60$. From this deduct $\frac{1}{25}$th part; and it follows, that $\frac{60}{25} = 2{\cdot}4$: and $60 - 2{\cdot}4 = 57{\cdot}6$ = radius of the circle, and it follows, that $6\,(57{\cdot}6) = 345{\cdot}6$ = perimeter of the regular inscribed hexagon; and by analogy or proportion $345{\cdot}6 : 360 :: 3 : 3{\cdot}125$. Again: $\frac{360}{24} = 15$. From this deduct $\frac{1}{25}$th part. Then: $\frac{15}{25} = {\cdot}6$, and $15 - {\cdot}6 = 14{\cdot}4$ = radius, and it follows, that $24\,(14{\cdot}4) = 345{\cdot}6$ = the perimeter of the regular inscribed hexagon; and by analogy or proportion, $14{\cdot}4 : 15 :: 3 : 3{\cdot}125$. Again: Divide the circumference of the circle into any number of equal arcs you please. From one of these arcs deduct $\frac{1}{25}$th part, and multiply the remainder by the number of arcs: you will find that the product is a constant quantity = $345{\cdot}6$, and exactly equal to the

perimeter of an inscribed regular hexagon. I cannot compel you to perform this operation, but I may tell you, that you may as well knock your head against a stone wall to improve your intellect, as attempt to controvert my conclusion. I do not say this "*in rudeness*," but you compel me to use strong language in my endeavours to convince you of geometrical and mathematical truth.

Now, we *know*—and I am sure you will not dispute it—that the side of a regular hexagon is equal to the radius of its circumscribing circle: and we also *know* that 6 times radius of the circle = the perimeter of the hexagon. Well, then, let the side of a regular hexagon be represented by any arithmetical quantity, say 60. Then : 6 times 60 = 360 = the perimeter of the hexagon. Add $\frac{1}{24}$ part. Then: $\frac{360}{24} = 15$, and $360 + 15 = 375$ = the circumference of a circumscribing circle, and $\frac{60}{2} = 30$ = semi-radius of the circle. Now, we *know* that circumference × semi-radius $= \pi r^2$, and that this equation = area of the circle, whatever be the value of π. Well, then, $375 \times 30 = 11250$ = area of the circle. $\frac{11250}{\frac{1}{4}\pi} = \frac{11250}{\cdot 78125} =$ 14400 = area of a circumscribing square to the circle, and it follows, that $\sqrt{14400} = 120$ = diameter of the circle, and is equal to twice radius. But, πr^2 = area in every circle, and since the radius of the circle = 60, it follows, that $\pi (60^2)$ = area of the circle. But, $60^2 =$ 3600, and since 3600 cannot be multiplied by any other arithmetical quantity but 3·125 to produce 11250, it follows, that 3·125 must be the true arithmetical vulue of π.

24th Angust.

I had written so far yesterday, and this morning's post brought me your second Letter of the 21st inst. In this communication you say :—"*I am daily expecting that you will give me up as impracticable.*" I am beginning to think that you will weary me out, and that I shall have to give you up "*as impracticable*," but not until I have put you in the wrong, —*your own conscience being judge*—where I may have to leave you, and leave it to a future generation of Mathematicians to draw their own inferences.

Let the angle A denote an angle of $14^\circ\ 24'$. The angle A is subtended by an arc equal $\frac{1}{25}$th part of the circumference of the circle, whatever be the circumference of the circle; and assuming the circle to be represented by $360^\circ = 14^\circ\ 24'$. Now, dispensing with the symbols denoting degrees and minutes, and reducing the value of the angle to its decimal expression, the angle A $= 14{\cdot}4$. To make my meaning perfectly intelligible, $\frac{360}{25} = 14{\cdot}4$, and since there are 60 minutes in a degree, and $\frac{60}{10} = 6$, it follows, that the decimal part of the expression $14{\cdot}4$ is equivalent to $24'$, that is to say $\frac{360^\circ}{25^\circ} = 14^\circ\ 24'$. Now, we cannot directly find $\frac{1}{25}$th part of $14^\circ\ 24'$. We must first reduce this expression to minutes, and it then becomes 864 minutes. $\frac{864}{25} = 34{\cdot}56 = \frac{1}{10}$ part of the perimeter of a regular inscribed hexagon to a circle of circumference $= 360$: for, $10\,(34{\cdot}56) = 345{\cdot}6 = 345^\circ\ 36'$, when expressed in degrees and minutes. *If you can't uuderstand this, I can't help it,* all I can say is, I have endeavoured to make my meaning intelligible. Now, my dear Sir, I think you will find that if you deduct $\frac{1}{25}$th part from $14^\circ\ 24'$, you will get the decimal quantity $13{\cdot}824$. But, by analogy or proportion, $3 : 3{\cdot}125 :: 13{\cdot}824 : 14{\cdot}4$, and $14{\cdot}4 = 14^\circ\ 24'$ when expressed in degrees and minutes. It appears to me that the difficulties and perplexities of Mathematicians may in part be traced to our system of notation, which does not admit of all quantities being expressed in both fractional and decimal notation, with arithmetical exactness.

Have you ever examined the peculiar properties of a geometrical figure called a pelacoid? Pray favour me with an answer to this question.

Believe me, my dear Sir,

Yours truly,

JAMES SMITH.

110

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

LANEAST, LAUNCESTON,
30th August, 1869.

MY DEAR SIR,

I got your Letter on my return home on Saturday.

Of course $\frac{1}{16}$ is a ratio, so are all its equivalents and derivations you named, but it has nothing to do with the value of π: nor does any fraction derived from it, express the ratio of the perimeter of a hexagon to the circumference of its circumscribing circle.

$\frac{\text{Perimeter of hexagon}}{\text{Circumference of circle}} = \frac{6}{2\,\pi}$. Hence, π is greater than 3, but this is no help to finding π's true value.

I am astonished that you don't see that in giving the ratio above, you do absolutely *assume the whole matter of dispute.* If your books established that simple fact, π is found, but without the slightest proof (or even attempt at proof), you simply *state* the ratio.

You began well in this respect. You *proved*, by the above hexagon, that π is greater than 3. Why not pursue this kind of proof further? Manifestly the side of a polygon approximates more and more closely to its subtending circular arc. Now, if you took a polygon of *more* sides than 6, you would get a nearer value of π.

Offer me *any proof* of the value of π, and I will study it carefully; but I am tired of reading mere *assumptions*, and wonder how you can be contented with them. If I weary you, the remedy is in your own power. I should not tease you with letters, or expostulations, if you give me up as hopeless: nor would it distress me at all, if you vote me no mathematician, and incapable of seeing your *proofs*. (I confess my inability on that point).

There is no way of finding π, except by comparing together in length an arc and its chord, (or sine or tangent, or any other trigonometrical line). This you simply *evade:* you never try to tackle that difficulty, but say:—Now, on the assumption that π is $3\frac{1}{8}$, so and so follows.

Hence all your writings (though able and ingenious deductions of what *would* follow if π *did* equal 3·125) don't aid us one bit towards the finding of π's real value. "*If you can't see this, I can't help it, but the fact remains notwithstanding*."

But my chief motive in thus troubling you is to answer the question that concludes your letter. I never heard of the Geometrical figure called a "*pelacoid*," and therefore can know nothing of its properties.

I hope you enjoyed the Exeter meeting.

Believe me, Dear Sir,

Yours truly,

G. B. GIBBONS.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH.
2nd September, 1869.

MY DEAR SIR,

I was seized with an attack of gout in the right hand at Ilfracombe, which obliged me to make the best of my way home. The first part of the following Letter, is that referred to in my Letter of the 18th August, written and posted at Exeter :—

MY DEAR SIR,

I leave home this afternoon *en route* for Exeter to attend the meeting of the *British Association*, and if I feel well enough to undertake the journey, I shall probably attend the "*German Congress for the Advancement of Science*," which meets at Innspruck on the 18th September, so that I may be from home for the next six or seven weeks. Until the 25th, a Letter addressed Reception-Room, British Association, Exeter, will find me.

On reflection, I felt it necessary to reply to the Rev. Professor Whitworth's Letter, dated August 7th, 2.30 p.m., which he obviously thinks to be unanswerable. This I did in a Letter dated 10th inst., which, at my request, he has returned. He has attached copious notes to it, and on its return, in the same envelope, there was the following communication* :—

* The first paragraphs of this Letter are the same word for word as the first paragraphs of my Letter to Mr. Gibbons of August 12. This arises from the fact, that finding I could not finish it to my own satisfaction, and post it before leaving home on the 16th August, I was led to throw off, very hastily, my Letter of that date to Mr. Gibbons. The Two Letters should be taken as one communication.

LIVERPOOL, 13*th August*, 1869.

SIR,

If you publish this Letter of yours, I shall be glad if you have the courage to publish the notes I have made on it. I reserve all rights of publication of all the Letters I have written to you, and caution you against infringing my copyright. If you wish to publish any of the Letters, or extracts from any of them, I will tell you on what conditions I will consent.

Yours truly,

W. ALLEN WHITWORTH.

This is to me really amusing. If I were to publish his malignant and libellous Letter, dated August 7, 4 p.m., and so, *infringe* his *copyright*, I should not be afraid of the verdict of a British jury. It would certainly be equivalent to the verdict of a jury I have somewhere read or heard of—"*serve him right.*"

Mr. Whitworth need not be under any apprehension as to my publishing his notes upon my Letter; they appear to me to be gems too "*rich and rare*" not to be made public. Take an instance:—In my Letter of the 10th instant to that gentleman, I said: "You, Sir, know—or at any rate ought to know—that if you assume the side of a regular dodecagon to be a *finite* and *determinate* arithmetical quantity, on your theory, you would make the diameter, circumference, and area, of a circumscribing circle, to be *infinite* and *indeterminate.*" To this he appends:—"*Note by Mr. Whitworth, Nonsense!*" Mr. Whitworth has made no attempt at a solution of a theorem I gave him; that is to say, he has made no attempt to solve the theorem I have given you for solution in my Letter of the 12th instant. I should like to see a controversy between you and that gentlemen, on the meaning of the words *finite* and *determinate*: and I should like to see a proof from either of you, that the diameter of a circle is *finite* and *determinate*, when the sides of a regular inscribed dodecagon = 2.

I must now refer you to the geometrical figure represented by

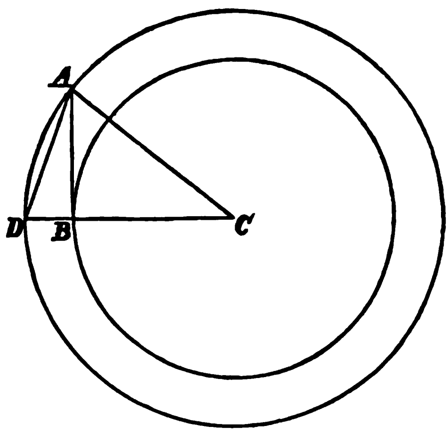
the diagram in my Letter of the 21st July, page 3. We are supposed *not* to *know* the values of the obtuse and acute angles, in the right-angled triangles A B C and A B D, expressed in degrees*: and it stands to common sense, that we can only find the values of these angles—if to be found at all—by means of what we *do know*. Well, then, what do we know with reference to this geometrical figure? *We know* that A C D is an isosceles triangle; and *we know* by Euclid: Prop. 5: Book 1: that the angles A and D at the base of this triangle are equal. *We know* that A B C and A B D are right-angled triangles, by construction: and *we know* that A B is to B C in the ratio of 3 to 4, and A B to B D in the ratio of 3 to 1, by construction: and *we know* by Euclid: Prop. 32: Book 1: that the six interior angles of the triangles A B C and A B D are together equal to four right angles. *We know* that the angles A and C in the triangle A B C are together equal to the right angle B: and *we know* that the angles A and D in the triangle A B D are together equal to the right angle B. *We know* that a right angle = 90°, and it follows, that four right angles = 360°: and half a right angle = 45°. Because the angles A and C in the triangle A B C are together equal to a right angle; *we know*, that the obtuse angle A must be *greater*, and the acute angle C *less*, than half a right angle:* and because the angles A and D in the triangle A B D are together equal to a right angle; *we know* that the obtuse angle D must be *greater*, and the acute angle A less than half a right angle.* Let the line A B, that is, the perpendicular of, and which is also *common* to, the triangles A B C and A B D = 3. Then: *we know*, by computation, that B C = 4 : A C = 5 : B D = 1 : and A D = $\sqrt{10}$. *We know* that an angle at the centre of a circle subtended by a side of a regular inscribed hexagon is an angle of 60°. *We know* that A D, the base of the isosceles triangle A C D, is a shorter line than a side of a regular inscribed hexagon to the circle; and it follows of necessity, that the angle C is less than 60°. *We know* that a side of a regular inscribed hexagon is equal to radius, in every circle.

It is not self-evident, and *we do not know*, until we have made the discovery, by computation, that ($AB^2 + BC^2 + AC^2$) =

* See Footnote, page 61.

$3\frac{1}{8}(B\,C^2)$: but, *we do know* that $(C\,B + B\,D) = C\,D$: and it is self-evident, that $C\,D = C\,A$, for they are the radii of the same circle: and *we know* that $C\,D = 5$, when $A\,B = 3$. Now, from the angle point D draw a straight line tangental to the circle, to meet C A produced at a point, say E, and so construct a right-angled triangle EDC, similar to the right-angled triangle A B C; you will find, by computation, that $(E\,D^2 + DC^2 + EC^2) = 3\frac{1}{8}\,(CD^2)$, and on the THEORY that 8 circumferences = 25 diameters in every circle, $3\frac{1}{8}\,(C\,D^2) = (3{\cdot}125 \times 25) = 78{\cdot}125$ = area of the circle E D C when $A\,B = 3$.

The geometrical figure in the margin is a fac-simile of that in my Letter of the 21st July, with the following addition. With C as centre, and C B as interval, describe the smaller of the 2 circles.



Now, my Dear Sir, you may *assume* $\pi = 3$, or, $\pi = 4$, or you may *assume* π = any arithmetical quantity intermediate between 3 and 4, so that it be *finite* and *determinate;* and if you take the trouble to work out the calculations, you will discover, that the difference between the areas of the two circles is exactly equal to the area of a circle of which A B is the radius. Is not the arithmetical value assigned to π by "*recognised Mathematicians* $3{\cdot}14159265$, &c.? I am sure you will admit this fact. Well, then, "*brush away*" the &c., and make the computations on the assumed value of $\pi = 3{\cdot}14159265$, and you will find that I am right. How then can the true value of π be the indeterminate arithmetical quantity $3{\cdot}14159265$, &c.? Surely you cannot fail to perceive—even granting for the sake of argument that I am *wrong*—that Mathematicians must be *wrong* in assigning to π the *infinite* and *indeterminate* arithmetical value $3{\cdot}14159265$, &c.

Well, then, it stands to common sense that Mathematics can never be made to over-ride the ratio of side to side, by construction,

in any right-angled triangle: and it also stands to common sense, that Mathematics—rightly applied—can never be inconsistent with pure Geometry. In one of your recent Letters you say:—"*I have avoided Logarithms as needless,*" It is true that you have avoided Logarithms, for you have certainly never attempted to grapple with any of my proofs by Logarithms, and I have given you many in the course of our long correspondence.

Now, my dear Sir, I tell you—not "*in rudeness,*" but distinctly—that it is only by means of Logarithms that we can find the values of the obtuse and acute angles in the triangles A B C and A B D; and of this fact I shall proceed to give you the proof. It is not necessary to take both triangles, and I shall take the triangle A B D, of which the side that subtends the right angle is incommensurable with the sides that include the right angle.

When A B = 3, then, B D = 1 and A D = $\sqrt{10}$. Now, $\frac{BD}{AD}$ $= \frac{1}{\sqrt{10}} = \frac{1}{3{\cdot}162277} = {\cdot}3162277$; and ·3162277 is the trigonometrical sine of the angle A. $\frac{AB}{AD} = \frac{3}{\sqrt{10}} = \frac{3}{3{\cdot}162277} = {\cdot}9486834$, and ·9486834 is the trigonometrical sine of the angle D. The logarithm corresponding to the natural number ·3162277 is 9·4999999.. and this is the trigonometrical Log.-sin. of the angle A, (you will observe that the *mantissa* of this Logarithm is a very close approximation to the natural sine of an angle of 30°). The Logarithm corresponding to the natural number ·9486834 is 9.9771212, and this is the trigonometrical Log-sin. of the angle D.

THEOREM.

Let A D the side subtending the right angle in the triangle A B D, be represented by any arithmetical quantity, say 60. and be given to find the lengths of the sides A B and B D which include the right angle, and prove that they are in the ratio of 3 to 1, that is to say, in the same ratio as by construction.

Then:

As Sin. of angle B = Sin. 90° ....Log. 10·0000000

: the given side A D = 60, which may be 60 inches,
feet, yards, or miles, ...Log. 1·7781513

: : Sin. of angle D ....Log. 9·9771212

11·7552725

: the required side A B.................................... 10·0000000

= ·9486834 × 60 = 56·9210040 ....Log. 1·7552725

Again:

As Sin. of angle B = Sin. 90°........................Log. 10·0000000

: the given side A D = 60 ....Log. 1·7781513

: : Sin. of angle A ....Log. 9·4999999

11·2781512

: the required side B D .... 10·0000000

= ·3162277 × 60 = 18·9736620 ....Log. 1·2781512

Now, the side A D in the triangle A B D being incommensurable with the sides A B and B D which include the right angle, the lengths of A B and B D cannot be ascertained so as to give exact arithmetical expression to them. Hence: 56·9210040 is slightly greater than the length of the side A B; and 18·9736620 slightly less than the length of the side B D. But, the ratio of A B to B D is as 3 to 1, as nearly as it is possible to arrive at it by Logarithms. For example: By hypothesis, let B D = 18·97367, that is, strike off the two last decimal figures, and add 1 to the fifth decimal figure. Then: 3 (18·97367) = 56·92101, making A B slightly greater than 56·9210040. It is axiomatic, if not self-evident, that we cannot "*upset*" this conclusion by extending the number of decimals.

Now, my Dear Sir, *we know* that the angle A in the triangle A B D is less than 45°.

THEOREM.

Given: the angle A = 18° 26′, and the side A D 600 miles in length. Find the lengths of the sides A B and B D which include the right angle, and prove that they are in the ratio of 3 to 1. Then: 90° — 18°26′ = 71° 34′ = the angle D.

Then, by Hutton's Tables:

As Sin. of angle B = Sin. 90°	Log.	10·0000000
: the given side A B = 600 miles	Log.	2·7781513
: : Sin. of angle D = Sin. 71° 34′	Log.	9·9771253
		12·7552766
: the required side A B		10·0000000
= 569·2153200 miles nearly	Log.	2·7552766

Again:

As Sin. of angle B = Sin. 30°	Log.	10·0000000
: the given side A D = 600 miles	Log.	2·7781513
: : Sin. of angle A = Sin. 18° 26′	Log.	9·4999633
		12·2781146
: the required side B D		10·0000000
= 189·7206000 miles nearly	Log.	2·2781146

Now, 3 (189·7206000) = 569·1618000, and is less than 3 times A B. In the former example the ratio holds good to the second place of decimals: in this it fails at the first place of decimals. One of two things follows of necessity. Either Hutton's Tables are fallacious, or the angle A in the triangle A B D is not an angle of 18° 26′. It is self-evident that both cannot be true. The question still remains to be answered. Is either true, or are both false? In the course of our very long correspondence, and even in our recent correspondence, you have had to deal with a similar question, and fancy you get rid of all difficulty by assuming Hutton's Logarithmic Tables to be correct, and so, make the angle A to be an angle of 18° 26′ + y. Is not the correctness of existing Logarithmic Tables one of the points in dispute? Could I give you a better proof than I have done, that these tables are fallacious, by shewing you that they "*upset*" the known ratio of side to side, by construction, in a right-angled triangle? I have never disputed the value of Napier's discovery of the Logarithms of numbers, nor do I dispute the Logarithms of numbers as given in Hutton's Tables. What I dispute is, the correctness of natural sines, cosines, and log.sines, &c., as given in existing Logarithmic Tables, whether calculated to 6 or 7 places of decimals.

Now, my dear Sir, you *assume* that existing Logarithmic Tables, "*having been calculated by experts in every country in Europe*," are not fallacious, and yet, oddly enough, you avoid Logarithms. I can only explain this on the principle that you have "*resolved to bolster up a false theory.*" I do not intend this offensively or "*in rudeness*"—I of course believe you to be mistaken—the words occur to me as expressing the gravamen of one of the Rev. Professor Whitworth's charges. I shall now proceed to prove that the angle D A B in the triangle A B D is an angle of 18° 26'.

THEOREM.

Given: the angle D A B in the triangle A B D = 18° 26'. Find the angles A and C in the triangle A B C.

Now, *we know* that the angles D A B and A C B are angles of less than 45°. *We know* that the sides which include the right angle in the triangle A B D are in the ratio of 3 to 1, by construction. *We know* that the sides that include the right angle in the triangle A B C are in the ratio of 3 to 4, by construction. I KNOW, whether "*recognised Mathematician*" know it or know it not, that Mathematics *rightly applied*, can never destroy the ratio of side to side by construction, in any right-angled triangle. *We know* that in every isosceles triangle, the angles at the base are together equal to the angle at the apex; or in other words, the angles at the base of every isosceles triangle are equal to half the angle at the apex. I have never met with a Mathematician, who has turned this fact to any account.

Now, the angle D A B is given = 18° 26'. Hence: 90 — 18° 26' = 71° 34' = the angle A D B: 2 (18° 26') = 36° 52' = the angle A C B in the triangle A B C: and, 90° — 36° 52' = 53° 8' = the angle B A C. The sum of all the interior triangles in the two triangles A B D and A B C are equal to four right angles, and the angle D A B = half the angle A C B.

I have no doubt you would tell me that this proves nothing, since we might put any value we please upon the angle D A B, and work out a similar result. *Granted*, always provided that we do nothing so *absurd* as to *assume* the angle D A B to be greater than 45°, or the angle B A C less than 45°. Let us see what Logarithms will do for us.

THEOREM.

Given: the angle C in the triangle A B C = 36° 52′, and the side A C 600 miles in length. Find the lengths of the sides A B and B C, and prove that they are in the ratio of 3 to 4, that is to say, in the same ratio as by construction. It is axiomatic, if not self-evident, that 90° — 36° 52′ = 53° 8′ = the angle A; and *we know* that the sides A B and B C are in the ratio of 3 to 4, by construction, which makes A C = 5, when A B = 3.

Now, $\frac{AB}{AC} = \frac{3}{5} = \cdot 6$, and ·6 is the trigonometrical sine of the angle C. $\frac{BC}{AC} = \frac{4}{5} = \cdot 8$; and ·8 is the trigonometrical sine of the angle A. The logarithm corresponding to the natural number ·6 is 9·7781513; and the logarithm corresponding to the natural number ·8 is 9·9030900.

Then:

As Sin. of angle B = Sin. 90°	Log.	10·0000000
: the given side A C = 600 miles	Log.	2·7781513
: : Sin. of angle A C B = Sin. 36° 52′	Log.	9·7781513
		12·5563026
: the required sine A B		10·0000000
= ·6 × 600 = 360 miles	Log.	2·5563026

Again:

As Sin. of angle B = Sin. 90°	Log.	10·0000000
: the given side A C = 600 miles	Log.	2·7781513
: : Sin. of angle B A C = Sin. 53° 8′.....................	Log.	9·9030900
		12·6812413
: the required side B C		10·0000000
= ·8 × 600 = 480 miles	Log.	2·6812413

Hence:

A B : B C : : 3 : 4 : that is, 360 : 480 : : 3 : 4.
A B : A C : : 3 : 5 : that is, 360 : 600 : : 3 : 5.
and, B C : A C : : 4 : 5 : that is, 480 : 600 : : 4 : 5.

The angle D A B = half the angle A C D = 18° 26′, and the sum of the angles D A B and B A C = 71° 34′, and are therefore

together equal to the angle A D B in the right-angled triangle A B D.

I shall now give you the computations worked out by Hutton's Tables.

As Sin. of angle B = Sin. 90°	Log.	10·0000000
: the given side A C = 600 miles	Log.	2·7781513
: : Sin. of angle A C B = Sin. 36° 52′	Log.	9·7781186
		12·5562699
: the required side A B		10·0000000
= 359·92663 miles nearly	Log.	2·5562699

Again:

As Sin. of angle B = Sin. 90°	Log.	10·0000000
: the given side A C = 600 miles	Log.	2·7781513
: : Sin. of angle B A C = Sin. 53° 8′	Log.	9·9031084
		12·6812597
: the required side B C		10·0000000
= 480·02593 miles nearly	Log.	2·6812597

At this point I must call your attention to the fact, that in your Letter of the 22nd July last, (see page 4) you say :—" *Tho' it is going over the old ground—trodden by a couple of hundred Letters that have passed between us, I will do as you wish touching the triangles.*" You arrive at the values of the angles A C B and B A C as follows :—

" Sin. A C B = $\frac{3}{5}$ = ·600000 to six decimals.

Sin. 36° 52′ = ·599955

·000045

So that A C B = 36° 52′ + x.

B A C = 53° 8′ − x."

It is obvious or self-evident, that you get the Sine of the angle A C B from some set of Tables calculated to six places of decimals, and assume the Tables to be sufficiently correct for practical purposes, and by your method of applying these Tables, you make the angle A C B greater than 36° 52′, and the angle B A C less than 53° 8′. Can you find a flaw in the computations I have given you worked out from Hutton's Tables? I am sure you cannot. Surely

then, you cannot fail to perceive that Hutton makes the angle A C B $36^\circ\ 52' - x$, and the angle B A C $53^\circ\ 8' + x$. But, according to you, the angle A C B is an angle of $36^\circ\ 52' + x$, and the angle B A C an angle of $53^\circ\ 8' - x$. Which is right? It is self-evident that both cannot be right.

Now, my dear Sir, I know—and you think you know—that if in any right-angled triangle, two angles and a side, or, two sides and an angle, be given, we can find the remaining sides and angles, but I may tell you, that if you make the angle C in the triangle A B C, of which all the sides are known, either greater or less than $36^\circ\ 52'$, you can never find the exact lengths of A B and B C from a given length of A C, by existing Logarithmic Tables.

I must now refer you to my Letter of the 12th August, and I shall repeat the theorem given you for solution, founded upon the geometrical figures represented by the diagrams on pages 78 and 81.

THEOREM.

Find the arithmetical values of the lines N A, N B, and N P, when the chords A P and P B = 2.

Now, my dear Sir, if I can neither induce you nor the Rev. Professor Whitworth to solve this theorem, or admit your incapacity to solve it, shall I not have convicted you both of mathematical dishonesty?

On my return home I found your Letter of the 30th August awaiting me. I will take the earliest opportunity of replying to it, and with this I think our correspondence on the vexed question of the arithmetical value of π may terminate.

Believe me, my dear Sir,
Very truly yours,
JAMES SMITH.

P.S.—Since I commenced writing the above, I have received a communication from the Rev. Professor Whitworth. He, like yourself, never heard of a geometrical figure called a "*pelacoid*." With my gouty hand it has taken me 3 days to write this Letter.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
6th September, 1869.

MY DEAR SIR,

Since my return home from Devonshire, the following correspondence has passed between the Rev. Professor Whitworth and myself. It appears to me so curious and amusing, that I cannot resist the temptation to give you a copy of it.

LIVERPOOL, *1st September*, 1869.

SIR,

I request your permission to publish the notes which you wrote in pencil and red ink on the M.S. (proof that π is greater than 3·126), which I recently sent you.

I am, Sir,
Your obedient Servant,
W. A. W.

BARKELEY HOUSE, SEAFORTH,
2nd September, 1869.

SIR,

Your are quite at liberty to publish the notes which I wrote in pencil and red ink on the M.S. (proof that π is greater than 3·126,) which you recently sent me.

When you publish, you will perhaps have the candour to inform the scientific world that $4\left(\frac{\pi}{2}\right)^2 = \pi^2$, whatever be the value of the π: and that $12\frac{1}{2}\left(\frac{\pi}{50}\right) = \frac{\pi}{4} =$ area of a circle of diameter unity, whatever be the value of π.

Have you ever heard of a geometrical figure called a pelacoid, and have you ever studied the peculiar properties of this remarkable geometrical figure?

I am, Sir,
Your obedient Servant,
J S.

LIVERPOOL, *3rd September*, 1869.

SIR,

I have the honour to acknowledge the receipt of your Letter authorising me to publish your remarks on my proof that π is greater than 3·126.

You ask me to take the opportunity of informing the scientific world that four times the square of half of any quantity is square to the square of that quantity. As nobody doubts this truism, I could not put it forward as a piece of information, but I shall be happy to publish the fact that *you request me* to give this information, as it will form a very pretty illustration of some remarks which I hope to publish some day. What I say about this, applies equally to the further request that you make, viz.: that I shall inform the scientific world also of the undoubted fact, that $\frac{25}{2} \cdot \frac{\pi}{50}$ or $\frac{\pi}{4}$ is the area of a circle, whose diameter is the unit of length.

I never heard of a "*pelacoid*." The word is not formed according to the laws of Greek derivation. Probably the name has been designed by some one ignorant of Greek, and applied to some figure which he developed from his own consciousness. His ideas, perhaps, are of the earth, earthy. The word seems to be the result of a barbarous attempt to express something earth-like, clayey. But we cannot imagine a curve of which this would be a characteristic. Perhaps you can send me its equation; then I can trace it.

I have the honour to be,
Sir,
Your obedient Servant,
W. A. W.

BARKELEY HOUSE, SEAFORTH,
4th September, 1869.

SIR,

I will not be dragged into further correspondence with you. But, in your Letter of yesterday's date, you ask me for a piece of information arising out of a question I put to you in my note of the 2nd inst., and as "*a gentleman and a man of honour*," I feel bound to give you the information you ask for.

Well, then, a "*pelacoid*" is a geometrical figure, derived from an equilateral rectangle, and its external form is composed of *three* circular arcs. It is such, that if P denote the area of the "*pelacoid*," and A denote the area of the square from which the "*pelacoid*" is derived, we get the equation $P = A$. But, we also get another equation. $\left\{(P + \frac{1}{4}P) + \frac{1}{4}(P + \frac{1}{4}P)\right\}$ = area of a circumscribing circle to the generating square. With this information, *you*, Sir, as a "*recognised Mathematician*," will no doubt be able to "*trace*" the figure.

You admit "*that* $\frac{25}{2} \cdot \frac{\pi}{50}$ or $\frac{\pi}{4}$ *is the area of a circle whose diameter is the unit of length.*" But, you lose sight of the "*undoubted fact;*" that π lies within certain limits. The arithmetical value of the symbol π cannot be greater than 4 nor less than 3; since 4 is the perimeter of a circumscribing square, and 3 the perimeter of a regular inscribed hexagon to a circle of which the diameter is 1, when 1 is the unit of length. Hence, the arithmetical value of the symbol π must be intermediate between 3 and 4: and because we can hypothetically *assume* π (or any other symbol) to denote any arithmetical quantity intermediate between 3 and 4 (it may be the integer 3, with a string of decimals that would reach from Queen's College, Liverpool, to St. John's College, Cambridge), and prove that $4\left(\frac{\pi}{2}\right)^2 = \pi^2$; and that $\frac{25}{2}\left(\frac{\pi}{50}\right)$ or $\frac{\pi}{4}$ is the area of a circle of diameter unity, it follows of necessity, that "*recognised Mathematicians*" must be wrong, in assigning to the symbol π, the indeterminate arithmetical value 3·14159265, &c. Can you, or any other "*recognised Mathematician*," divide 3·14159265, &c., by 2, and square the quotient?

Is not 12 times the circular measure of an angle of $30° = 2\pi$, whatever be the value of π? Is not $\frac{\text{angle} \times \pi}{180}$ the formula for finding the circular measure of an angle? Now, $\frac{30° \times 3{\cdot}125}{180} = {\cdot}5208333$ with 3 to infinity. Multiply by 12. Then: $\cdot 5208333 \times 12 = 6{\cdot}2499996$: and by extending the decimals would give 6·249 with 9 to infinity. Pray, work out the computations with the orthodox value of π, and "*where will you be?*

When you publish, what you believe to be your proof, that π is greater than 3·126. "*I should wish this Letter, in its integrity to accompany*" the publication: and I hope "*you will have the courage*" to grant me this favor.

I have the honour to be,

Sir, your obedient Servant,

J. S.

I shall probably wait until I can write with more ease and comfort, before replying to your Letter of the 30th August.

Believe me, my dear Sir,

Very truly yours,

JAMES SMITH.

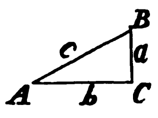
THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

LANEAST, *6th September*, 1869.

MY DEAR SIR,

My conduct was consistent and right in avoiding Logarithms for the simple reason that you do not admit their correctness, while in the same Letter you pretend to offer "*proof*" by means of them! I *know* they are correct, (as far as they go), but if you deny it, and I can do without them, I act well in avoiding them. If they are "fallacious" how can you say that by their means only we can discover certain values?

The fact is, that Logarithms merely facilitate computation, and *everything* they effect, can be done more accurately without them, only with immensely more labour. If $\frac{a}{c} = \sin.a$, and a and c, are expressed in large numbers, it is tedious work to effect the division, and we are much helped by Logarithmic Tables, which gives log-sin. A = log. A · log. C, but the result would be more accurate, if we actually effected the division.*



* In this argument there is the lurking assumption that c is to a in the ratio of 2 to 1; or in other words, that A B : B C : : 2 : 1 : making A an angle of 30°. But, when A B = 5, and B C = 3, then:

Log. 5 = ·6989700
Log. 3 = ·4771213

1·1760913 Logarithm of 15 exactly.

What I say is, Logarithms in our Tables (barring misprints) are accurate *as far as they go:* and every one who has computed them (as I have many) knows that they are found only by *endless series*, where terms are so convergent that each is much smaller than the one preceding it, so that you may extend their accuracy to as many decimal places as you please, 6 or 7 decimals are usually counted sufficient.

For an illustration, find the square of 30, which we know is 900.

Log. 30 = 1·477121		1·47712132	
2	Lynn's Tables.	2·9542426	by Hulse's (German) Tables.
2·954242		2·9542425	
Log. 900 = 2·954243			

So that, using 6 decimals or 7, the logarithmic result is not exact.

Hence, if you want to find any side or angle *exactly*, you will not be able to do it by Logarithms unless there happens a compensation of errors, (an excess neutralising a deficiency). Indeed, nobody ever thinks of seeking exactness, by means of quantities known to be only approximations up to a certain number of decimals.

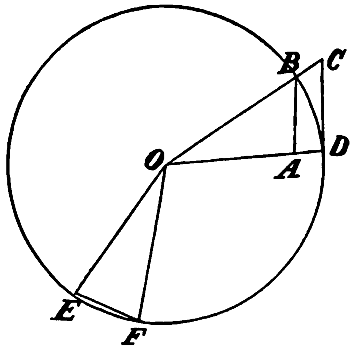
Hutton gives log. 2 = 3010300, but in his next table to 20 places, it is ·3010299956, &c.

"Finite and determinate." I agree with you that this expression of Professor Whitworth is not well chosen: He means that π is not infinite in magnitude, and so may be called "finite," and he calls that "determinate," which we can assign to any degree of accuracy we please.

But π cannot be expressed in finite *terms*, that is, by any finite number of figures. Indeed, his Letter to you, printed in page 9 of your Geometry of the Circle, so explains it, for he gives one of the many endless series, by which π can be determined.

When men debate on religion or politics, they too frequently run into sarcasm, but Mathematics ought to be free from that disadvantage, since they make no appeal to our feelings. On this account, I do not admire the style of Mr. Wilson's Letter to you of 16th November, 1868, (page 381 of your Geometry of the Circle), though it is very cleverly worded. But I quite concur with him, in the *imperfection* of your reasoning (November 18, 1868, page 383). You do "assume the *result* in the *premises*, from which you correctly argue." In every offered proof of yours π is either openly or

implicitly assumed to be $3\frac{1}{8}$, and this invalidates all the subsequent deductions, which (as he says) are correctly drawn. Did you ever think of *another* method, to wit, this: Consider π *unknown* (except that it exceeds 3), and make *no* assumption of its value, but *find* that value?



It will be obviously necessary to find the length of some *arc*, in terms of its sin. A B, with tan. C D, with chord E F, because the ratio of a straight line to a circular one is the very thing we are seeking, and this difficulty cannot be avoided.

If you say, expressly or implicitly: "Now, let π = so much," or, "let the chord E F = $\frac{m}{n}$ of arc E F, this is openly or tacitly assuming π, and though your results will be *consistent* with your hypothesis, you will prove nothing as to the value of π. I should study with the greatest interest and attention any proof you could offer of the value of π, which did not really *assume* the thing we are in search of; but your many writings are, in truth, only correct geometrical deductions of what would follow, if π *did* equal $3\frac{1}{8}$.

You recapitulate a great many things we *know*, but you come at the last to say, and on the theory that 8 circumferences = 25 diameters in every circle, &c. Just so: but that theory is the very thing in dispute: that's the very thing which makes all your writings so inconclusive and unsatisfactory, I quite allow that *on the* theory, &c., all you write down would follow but that is no proof of the correctness of the theory.

You seem to me captivated with the idea, that π *must* be expressed in finite terms. Suppose I took the crotchet into my head, that the diagonal of a square must be expressible in the same way. Let the side = 1, the diagonal (say I) is 1·4 exactly, or $1\frac{2}{5}$.* I am sure that if you followed up such an idea you would *prove* it, by the

* If a side of a square = 1, the diagonal = 1·4, or $1\frac{2}{5}$; so says Mr. Gibbons. I make the diagonal = 1·414, &c. J. S.

same kind of reasoning that you adopt for π: Euclid would not stand in your way, (Euclid is "at fault" some times). Tables of square roots would be easily set aside, (for they are no more correct than Logarithms, (both being only approximations *generally*). The sine of 36° 52′ is given as correctly by our tables ·5999549 as $\sqrt{2} = 1·414213$. In trigonometry we are constantly met by quantities that are not *exact*, and so in every branch of Mathematics, and there is no reason whatever (a priori) for supposing that π can be expressed in finite terms. I have not yet had time to go over your Letter just received, and I heartily hope that your visit to Germany will restore your health and give you satisfaction by your intercourse with the Mathematicians of that very learned *nation*.

Yours very truly,
G. B. GIBBONS.

You intimate the termination of our correspondence, and wisely so. It ends in undisturbed good will on both sides.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
7th September, 1869.

MY DEAR SIR,

I cannot help thinking you would be rather amused with the communication I posted to you yesterday. I commence this on the day it bears date, but I am afraid it may be some days before I get it finished. It is my last effort to convince you of "*geometrical truth*," and I shall write it carefully, and not be in a hurry over it.

In your Letter of the 30th August, you say:—" *Of course* $\frac{1}{16}$ *is a ratio, so are all its equivalents and derivations you name, but it has nothing to do with the value of* π *: nor does any fraction derived from it, express the ratio of the perimeter of a hexagon to the circumference of its circumscribing circle.*

$\frac{\text{Perimeter of hexagon}}{\text{Circumference of circle,}} = \frac{6}{2\pi}$. Hence, π is greater than 3 : *but this is no help to finding* π*'s true value.*"

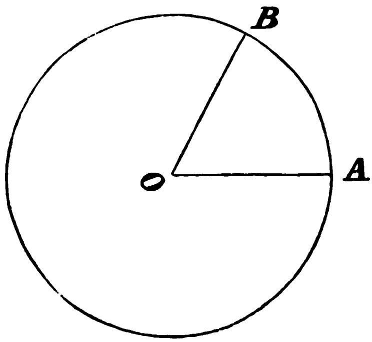
Now, the equation $\frac{\text{Perimeter of hexagon}}{\text{Circumference of circle,}} = \frac{6}{2\pi}$, is not of general and universal application, and is therefore not true under all circumstances; consequently, your conclusion :—"*Hence, π is greater than* 3," is a *non sequitur*. In fact, your equation is *unique*, and true only of a circle of radius 1, when 1 is the unit of length. *We know*, that the arithmetical value of the symbol π is greater than 3, but not from this equation *per se*, as you put it.*

In my Letter of the 4th inst. to the Rev. Professor Whitworth, I have shewn why the symbol π must be greater than 3 and less than 4; and *you know* that in *applying* Mathematics to Geometry, we must take nothing for granted. We *know* that the postulates of Euclid *must* be taken for granted, but are not some of his axioms disputed by modern Mathematicians?

It is obvious that in applying Mathematics to Geometry, we must adopt some unit of length. When we say, let a straight line = 4, do we not mean that the length of the line is 4 times 1, = 4, the unit of length being 1 ?

In the geometrical figure in the margin (Fig. 1), let A O B be an angle subtended by an arc equal to radius, and let r denote radius. Then : "$\frac{\textit{angle } \text{AOB}}{4 \textit{ right angles}} = \frac{\textit{Arc } \text{AB}}{\textit{Circumference of the Circle}} = \frac{r}{2\pi r} = \frac{1}{2\pi}$; *therefore*, *angle* A O B $= \frac{4 \textit{ right angles.}}{2\pi}$"

FIG. 1



* Because 6 times radius is equal to the perimeter of a regular inscribed hexagon, in every circle, (and this is admitted), it follows, that $\frac{6}{2\pi}$ is less than the circumference of a circle of diameter unity. According to "*recognised Mathematicians*," 3·14159 is an approximation, 3·141592 a closer approximation, and 3·1415926 a still closer approximation to π's arithmetical value; and so, according to Orthodoxy π may have as many values as we may please to give it. Can the symbol π have more than one true arithmetical value, according to common sense? It is no doubt true, that π is greater than 3, but as put by Mr. Gibbons, it is an "*unproved premiss.*"

I quote from Todhunter*—a living "*recognised Mathematician*"—but you will find this in any Treatise on Plane Trigonometry; and it follows, that the arithmetical symbol 1 is adopted by Mathematicians as the unit of length, just as certainly as that Mathematicians adopt the algebraical symbol π, to denote the ratio of circumference to diameter in a circle. I have never had these *facts* disputed by any of the numerous Mathematicians with whom I have been in correspondence; and I have taken them as admitted, in all the controversies I have had on the vexed question of the true arithmetical value of π. If you doubt, or dispute, that 1 is adopted as the unit of length, in all Treatises on Plane Trigonometry, pray say so, and prove it?

Now, let the diameter of a circle = 4, and let the unit of length be 2. It is an "*undoubted fact*," that $4\left(\frac{4}{2}\right)^2 = 4^2$: and it is also an "*undoubted fact*," that, "*four times the square of half of any quantity is square to the square of that quantity*," as Mr. Whitworth puts it. But, is not this a base and *inexcusable* perversion of the statement in my Note of the 2nd instant, viz.: $4\left(\frac{\pi}{2}\right)^2 = \pi^2$, whatever be the value of π? It is an "*undoubted fact*," that, by analogy or proportion, $1 : 2\pi :: 2 : 4\pi$: but, according to the *ipse dixit* of Mr. Whitworth, the arithmetical value of π would be 4 when the unit of length is 2. Verily! Verily! Mr. Whitworth is incompetent to perceive the absurdity of his own arguments. I think you cannot fail to perceive, that in Mr. Whitworth's Letter of the 2nd instant, there is a "*lurking*" insinuation, that according to my reasoning, π would be 4 when the unit of length is 2, and π would be 8 when the unit of length is 4.

Let the sides of a regular inscribed dodecagon to a circle = 2, as Mr. Whitworth put it in his elaborate Paper. (I did not keep a copy of the Paper, but I suppose I shall have the opportunity of seeing it in the mathematical Journal which he edits). Then: 12 times 2 = 24 = the perimeter of the dodecagon, and is exactly equal to the

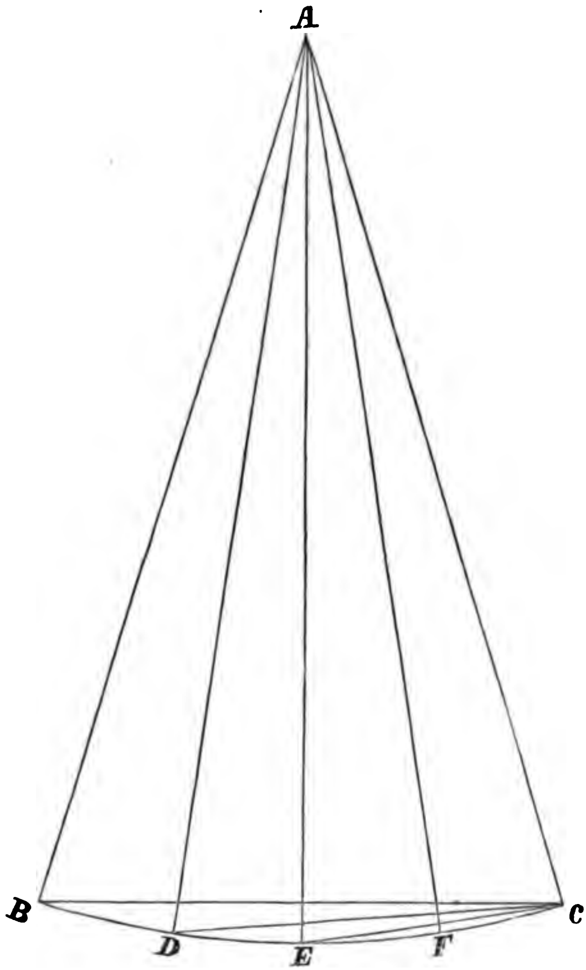
* See PLANE TRIGONOMETRY, BY I. TODHUNTER, M.A., FELLOW AND PRINCIPLE MATHEMATICAL LECTURER OF ST. JOHN'S COLLEGE, CAMBRIDGE. Second Edition, 1860, page 9.

perimeter of a regular inscribed hexagon to a circle of radius = 4. Again: Let the sides of a regular 24 sided polygon inscribed in a circle = 1. Then: 24 times 1 = 24, and is exactly equal to the perimeter of a regular inscribed hexagon to a circle of radius = 4. Now, $\frac{25}{24}(24) = (24 + \frac{24}{24}) = (24 + 1) = 25 = 2\pi$ (radius), when radius = 4, and it is an "*undoubted fact*" that 2π (radius) = circumference in every circle, whatever be the value of π. It would be very absurd, if I were to say that the perimeters of regular 6, 12, and 24 sided polygons inscribed in a circle were of equal length; but the ratio between the sides of all these polygons and their subtending arcs is the same, and the reason is plain enough if Mathematicians could but be brought to reflect. The ratio between a side of any regular polygon inscribed in a circle and its subtending arc is as 3 to π, whatever be the value of π. You, my dear Sir, have overlooked the following facts in your consideration of the questions at issue between us:—Let the radius of a circle be represented by the unit of length = 1. Then: $\frac{25}{24}(1) = \frac{25 \times 1}{24} = 1{\cdot}041666$ with 6 to infinity. From this deduct $\frac{1}{25}$th part = ·041666 with 6 to infinity, and it follows, that $\frac{24}{25}(1{\cdot}041666$ with 6 to infinity$) = 1 =$ radius. Again: Let the side of a regular hexagon inscribed in a circle = 6. Then: $\frac{25}{24}(6) = \frac{25 \times 6}{24} = \frac{150}{24} = 6{\cdot}25$. From this deduct $\frac{1}{25}$th part = ·25, then, 6·25 — ·25 = 6 = radius: and by analogy or proportion 6 : 6·25 : : 3 : 3·125. You will observe that $\frac{25}{24}(1)$ is an interminable repeating decimal, while $\frac{25}{24}(6)$ is a *finite* and *determinate* quantity. This arises from the simple but "*undoubted fact*," that 6 is a multiple of 3, so that 6 can be divided by 3, without a remainder while 1 is not a multiple of 3, and therefore $\frac{1}{3}(1)$ is an interminable decimal; or in other words, cannot be given with arithmetical exactness, in decimal notation.

In the geometrical figure represented by the diagram (Fig. 2), B D E F C is an arc of a circle, of which A B, A D, A E, A F, and A C are radii, by construction. The arc B D E F C is divided into four equal arcs, B D, D E, E F, and F C, by construction. CB, CD, CE, and CF are joined. It is self-evident, that the triangles A B C A D C, and A E C, are isosceles triangles; and it is equally

self-evident that C D, and C E, are divergent lines from the angle C of the isosceles triangle A B C.

FIG. 2.



Now, it would be very absurd, if I were to say that the chords B C, D C, and E C, were lines of equal length: but, it would be

equally absurd, if I were to say, that the chord B C is *not* to its subtending arc B D E F C in the same ratio as the chord D C to its subtending arc D E F C, and the chord E C to its subtending arc E F C.

Let the arc B D E F C be represented by any *finite* arithmetical quantity. For the sake of simplicity, say 4. Then: The arc D E F C = 3: the arc EFC = 2: and the arc F C = our unit of length = 1. From the arc B D E F C deduct $\frac{1}{25}$th part $= \frac{4}{25} = \cdot16$. Then: $4 - \cdot16 = \cdot3\cdot84$ = the chord B C. From the arc D E F C deduct $\frac{1}{25}$th part $= \frac{3}{25} = \cdot12$. Then: $3 - \cdot12 = 2\cdot88$ = the chord D C. From the arc E F C deduct $\frac{1}{25}$th part $= \frac{2}{25} = \cdot08$. Then: $2 - \cdot08 = 1\cdot92$ = the chord E C. From the arc F C deduct $\frac{1}{25}$th part $= \frac{1}{25} = \cdot04$. Then: $1 - \cdot04 = \cdot96$ = the chord to the arc F C. "*π does not appear, need not appear.*" But, $\frac{4}{3\cdot84}$, $\frac{3}{2\cdot88}$, $\frac{2}{1\cdot92}$ and, $\frac{1}{\cdot96}$, are equivalent ratios; and by analogy or proportion, chord B C : arc B D E F C :: chord D C : arc D E F C: that is, $3\cdot84 : 4 :: 2\cdot88 : 3$: and the mean proportional of the means is equal to the mean proportional of the extremes. Again: chord D C : arc D E F C :: chord E C : arc E F C; that is, $2\cdot88 : 3 :: 1\cdot92 : 2$, and the mean proportional of the means is equal to the mean proportional of the extremes. Again: chord E C : arc E F C :: chord F C : arc F C; that is, $1\cdot92 : 2 :: \cdot96 : 1$, and the mean proportional of the means is equal to the mean proportional of the extremes. But, by analogy or proportion, $3\cdot84 : 4 :: 3 : 3\cdot125$; $2\cdot88 : 3 :: 3 : 3\cdot125$; $1\cdot92 : 2 :: 3 : 3\cdot125$; and $3\cdot84 : 1\cdot92 :: 4 : 2$; and in all these analogies, the mean proportional of the *means*, is equal to the mean proportional of the extremes. Hence, the equivalent ratios $\frac{3\cdot84}{4}$, $\frac{2\cdot88}{3}$, $\frac{1\cdot92}{2}$, $\frac{\cdot96}{1}$, and $\frac{3}{3\cdot125}$, all express the ratio between the perimeter of every regular hexagon and the circumference of its circumscribing circle, making $\frac{25}{8} = 3\cdot125$ the true arithmetical value of the symbol π, and 8 circumferences = 25 diameters, in every circle.

Now, my dear Sir, I tell you—not "*in rudeness*," but distinctly, and without the slightest hesitation—that the foregoing facts, and the *conclusion* derived from them, are *incontrovertible.* "*If you can't see this, I can't help it, but the fact remains notwithstanding.*"

I hardly think, however, that you can fail to perceive the fallacy of your assertion, that the ratio $\frac{1}{25}$ has nothing to do with the value of π. You may depend upon it, the ratio $\frac{1}{25}$ is a great "help to finding π's true value."

Well, then, from the foregoing irrefragable truths, we get the following rule for finding the perimeter of a regular inscribed hexagon to a circle, the circumference of the circle being given.

Into whatever number of equal arcs the circumference of a circle be divided, if from one of these arcs $\frac{1}{25}$th part be deducted, and the remainder multiplied by the sum of the arcs, the product is equal to the perimeter of an inscribed regular hexagon.

Apply the rule to a circle of any circumference, and you will find that whether you divide the circumference into 3, 4, 5, or any other number of equal arcs, and from one of these arcs deduct $\frac{1}{25}$th part, and multiply the remainder by the number of arcs, the product is a constant quantity, and exactly equal to the perimeter of an inscribed regular hexagon. I will give you an example:—

Let the circumference of a circle = 360. You surely cannot object to this. Mathematicians make this assumption arbitrarily, for their own purposes, and with far less show of reason than I make the assumption that 8 circumferences = 25 diameters, in every circle. Well, then, $\frac{360}{3} = 120$: $\frac{1}{25}(120) = 4·8$: and $120 - 4·8 = 115·2$. Multiply by 3. Then : $3(115·2) = 345·6 =$ the perimeter of a regular inscribed hexagon. Again: $\frac{360}{5} = 72$: $\frac{1}{25}(72) = 2·88$: and $72 - 2·88 = 69·12$. Multiply by 5. Then : $5(69·12) = 345·6$ = perimeter of the inscribed hexagon. $\frac{345·6}{6} = 57·6 =$ radius, $\frac{57·6}{2} = 28·8 =$ semi-radius, and $c \times sr = 3\frac{1}{8}(r^2)$; that is, $360 \times 28·8 = 3·125 \times 57·6^2 = 10368 =$ area of the circle.

9th September.

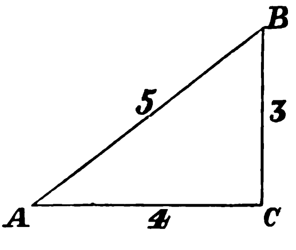
Yesterday afternoon's post brought me your Letter of the 6th instant. I do not suppose you do it intentionally, but nevertheless in the first paragraph of that communication you grossly mis-interpret me. You make the general term *Logarithms* to include not only the Logarithms of numbers, but the Logarithms of natural sines, &c., as given in Logarithmic tables. You cannot charge me with holding this opinion, and yet, *by implication* you do. I have never doubted

or disputed the correctness of the Logarithms of numbers, but I distinctly deny the correctness of natural sines, tangents, &c., as given in Logarithmic tables. I know that the Logarithms of numbers are inexact, but I also know that they are perfect for all the purposes to which they can be legitimately employed. You still fancy you have avoided Logarithms. How can this be? Did you not employ Logarithmic tables, when you made an angle of which the trigonometrical sine is $\cdot 6 = 36^\circ\ 52' + x$? I directed your attention to this fact, in the Letter to which your communication of the 6th professes to be a reply, and you have passed it over without note or comment. Now, my dear Sir, except when I have used Hutton's Logarithmic tables to prove Hutton at fault, I have carefully avoided Logarithmic tables, and in all the proofs I have ever given you by Logarithms, have strictly confined myself to the Logarithms of numbers, finding the Log. sines and Log. cosines of angles by means of them. On this point permit me to refer you to my Letter of the 9th November, 1868, to Mr. J. M. Wilson, in which you will find a copy of your Letter to me of the 15th August, 1868, (See Geometry of the Circle, page 364). You now say:—"*If they (meaning Logarithmic tables, including Logarithms of sines, cosines, &c.,) are fallacious, how can you say that by this means only we can discover certain values?*" I have never said any such thing, and you—unwittingly no doubt—again grossly pervert my language.

It appears to me, that in the next paragraph you "*upset*" yourself.

"Let A B C be a right-angled triangle, of which the sides are 3, 4, and 5, by construction. Then: $\frac{BC}{AB} = \frac{3}{5} =$ ·6, is the trigonometrical sine of the angle A."

FIG. 3.



The Logarithm of 3 is 0·4771213
The Logarithm of 5 is 0·6989700

$\bar{9}$·7781513.

Hence: $\bar{9}$·7781513 is the Logarithm of ·6, and log.-sin of angle A.

The Logarithm of 3 is 0·4771213
The Logarithm of 4 is 0·6020600
The Logarithm of 5 is 0·6989700

1·7781513

And, 1·7781513 is the Logarithm of $3 \times 4 \times 5 = 60$, exactly.

The Logarithm of 5 is 0·6989700.

$0·6989706 \times 2 = 1·3979400$ is Log. of $5^2 = 25$
$0·6989700 \times 3 = 2·0969100$ is Log. of $5^3 = 125$
$0·6989700 \times 4 = 2·7958800$ is Log. of $5^4 = 625$
$0·6989700 \times 5 = 3·4948500$ is Log. of $5^5 = 3125$

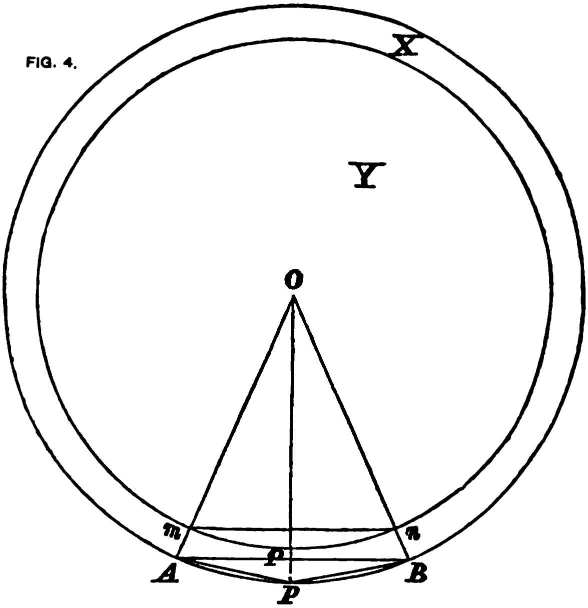
The Logarithm of 30 is 1·4771213.
$1·4771213 \times 2 = 2·9542426$ is Log. of $30^2 = 900$
$1·4771213 \times 3 = 4·4313639$ is Log. of $30^3 = 27000$

Hence: In tables of the Logarithms of numbers to 7 or more places of decimals, we are certain that they are correct to at least the fourth place of decimals.

I mean this communication to terminate our correspondence on the vexed question of the arithmetical value of π, and will make no further comment on your Letter of the 6th instant, but confine my remarks to that of the 30th August, and furnish you with some proofs, in which I will *avoid* the expression.—" On the *theory* that 8 circumferences = 25 diameters in every circle,"—which makes $\frac{25}{8}$ $= 3·125$, the value of π,—which you have so frequently caught at, to fasten upon me the charge of assuming the thing to be proved.

In the geometrical figure represented by the diagram (Fig. 4), let O A B be an isosceles triangle. With O as centre, and O A or O B as radius, describe the circle X. From O A cut off a part *m* A equal to $\frac{1}{25}$th part of O A (it being impossible to divide the line O A into 25 equal parts, in a diagram on so small a scale, the line *m* A as shown in the figure is an exaggeration of its true length: but you will understand that it is meant to denote $\frac{1}{25}$th part of the whole line O A, making O *m* to O A in the ratio of 24 to 25, by construction). With O as centre, and O *m* as radius, describe the circle Y, and join the points *m* and *n* where the arms of the

FIG. 4.



isosceles triangle O A B cut the circumference of the circle Y. Bisect the arc A P B in P, and join O P, A P, and B P.

Now, what can we prove with reference to this geometrical figure, by pure geometry? Because the circumferences of circles are to each other as their radii (and this you have admitted), we can prove, First:—That the circumference of the circle X: the circumference of the circle Y : : O *m* : O A. Second: That, the arc *m p n*: the arc A P B : : the circumference of the circle Y: the circumference of the circle X. Third: That the chord *m n*: the arc *m p n* : : the chord A B: the arc A P B. Fourth: That O *m*: *m n*: : O A : A B; this proportional follows from the fact that, *m n* is parallel to A B. Fifth: That no line can be drawn from the angle O to any other point in the arc A P B but the point P, so as to bisect the chords *m n* and A B, and the arcs *m p n* and A P B, which subtend the angle A O B at the centre of the circles. Sixth: That the angles at the base of the isosceles triangles O *m n* and O A B are equal to half the angle at the apex. Seventh: That the

angles m and n, in the triangle O m n, are equal to the angles A and B in the triangle O A B.

I am not aware that we can get one step further, by pure geometry; and if I am right, it is at this point that Mathematics enters the field for the first time.

In this diagram we have the figure upon which you base a theorem, which I have already told you appears to me, to be the "*Alpha and Omega*" of your geometrical investigations.

Mr. Gibbons' Theorem.

Angle A O B $= 15°$.

O A $=$ O B $= 1$.

Find A B.

You will hardly venture to dispute, that we must find—if to be found at all—anything *we don't know*, by means of something *we do know*. Now, what do we know about the figure upon which you found this theorem? *We know* that the angle A O B $= 15°$, and is equal to $\frac{1}{24}$th part of the circumference of the circle X. But we only know this by *assuming* the circumference of the circle to equal $360°$. *We know* that the circumference of the circle X $= 2\pi$, when O A $=$ O B $= 1$, whatever be the value of π. Well, then, to find the circular measure of the arc A P B from the given angle A O B, involves either a proof of π or an *assumption* of π's value. Or, if we divide 2π by 24 to get the circular measure of the arc A P B, this also involves either a proof of π or an *assumption* of π's value. The sine of an arc is half the chord of double the arc; and it follows, that half the chord A B, is the sine of the arcs A P and B P; and it is self-evident that the chords A P and B P are longer lines than half the chord A B. Now, the angles m, n, A, and B are equal, but it is self-evident that the geometrical sines of these angles are not all equal, and by your arguments and reasonings, you would make the trigonometrical sines of all these angles unequal. "*If you can't see this, I can't help it, but the fact remains notwithstanding.*"

THEOREM.

From a given circumference or radius of the circle X, prove that the perimeter of an inscribed regular hexagon, is exactly equal to the circumference of the circle Y.

Now, if $3{\cdot}14159265$, &c., the arithmetical value assigned by Mathematicians to the symbol π, be correct; it is self-evident that

the solution of this theorem would be impossible: but, if I were to say I cannot solve it, I should be a "*liar*," and a "*liar giveth ear to a naughty tongue.*" If I were to ask you to solve this theorem, I suspect you would do, as you have already done with reference to many theorems I have given you for solution; that is, pass it over without note or comment. I shall therefore solve the theorem, and it will remain for you as an honest Mathematician either to admit my proof, or to controvert it.

Let the circumference of the circle X be represented by any *finite* and *determinate* arithmetical quantity, say 70. Now, 70 is neither a square number, nor divisible by 3 or its multiples without a remainder.

Then: $\frac{1}{25}(70) = \frac{1 \times 70}{25} = \frac{70}{25} = 2 \cdot 8$; and, $70 - 2 \cdot 8 = 67 \cdot 2$ = the perimeter of an inscribed regular hexagon to the circle X: $\frac{67 \cdot 2}{6} = 11 \cdot 2$ = radius: and $\frac{11 \cdot 2}{2} = 5 \cdot 6$ = semi-radius of the circle X. But, circumference × semi-radius = area in every circle; therefore, $c \times s\,r = 70 \times 5 \cdot 6 = 392$ = area of the circle X. But, πr^2 = area in every circle, whatever be the value of π; and since the square of the radius cannot be multiplied by any other arithmetical quantity but $3 \cdot 125$, and produce 392, it follows—not as an *assumption*, but as a *deduction* from logical reasoning upon indisputable geometrical data—*that* $3 \cdot 125$ *must be the true arithmetical value of* π.

Again: Let the circumference of the circle Y = $67 \cdot 2$. Then: $\frac{1}{25}(67 \cdot 2) = 2 \cdot 688$; and $67 \cdot 2 - 2 \cdot 688 = 64 \cdot 512$ = the perimeter of a regular inscribed hexagon to the circle Y. $\frac{64 \cdot 512}{6} = 10 \cdot 752$ = radius; and $\frac{10 \cdot 752}{2} = 5 \cdot 376$ = semi-radius of the circle Y; therefore, $c \times s\,r = 67 \cdot 2 \times 5 \cdot 376 = 361 \cdot 2672$ = area of the circle. But, $3\frac{1}{8}(r^2) = 3 \cdot 125(10 \cdot 752^2) = 3 \cdot 125 \times 115 \cdot 605504 = 361 \cdot 2672$ = area of the circle; and since, by no other value of π but $3 \cdot 125$, can we get the equation $c \times s\,r = \pi r^2$, it follows—not as an *assumption*, but as a deduction arrived at by logical reasoning from indisputable geometrical data—*that* $3 \cdot 125$ *must be the true arithmetical value of* π. From O *m* cut off a part *m a*, making *m a* equal to $\frac{1}{25}$th part of O *m*, and with O as centre and O *a* as radius, describe a circle. The circumference of this circle will be $64 \cdot 512$, and exactly equal to the perimeter of an inscribed regular hexagon to the circle Y, when the

circumference of the circle X = 70: and in this way we might proceed *ad infinitum*, but we could never reach the point O, the centre of the circles. To whatever extent we might carry out the operation, there would still remain a quantity from which we could cut off $\frac{1}{25}$th part.

Again: Let the circumference of the circle X be represented by 400 grades. It was proposed in France to divide the circumference of a circle into 400 grades, in connexion with a uniform system of decimal tables of weights and measures. Then: $\frac{1}{25}(400) = 16$, and $400 - 16 = 384$ = the perimeter of a regular inscribed hexagon. $\frac{384}{6} = 64$ = radius: $\frac{64}{2} = 32$ = semi-radius, and $c \times sr = 3\frac{1}{8}(r^2)$; that is, $400 \times 32 = 3{\cdot}125\,(64^2)$, or, $400 \times 32 = 3{\cdot}125 \times 4096 = 12800$ = area of the circle. *En passant*, I could show that the advantages of the centesimal method would be neutralized by its disadvantages, and that it would be very unwise to attempt to change the sexagesimal method of dividing the circumference of a circle into 360°, which by long use has become thoroughly established.

I shall now solve the theorem, by adopting the radius of the circle X as the given quantity.

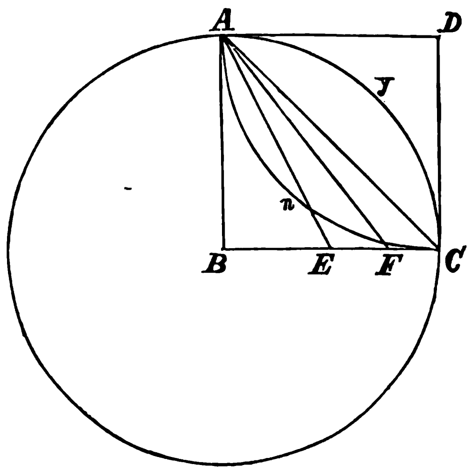
Let the radius of the circle X be represented by any *finite* and *determinate* arithmetical quantity, say 60. Then: 6 times 60 = 360 = the perimeter of an inscribed regular hexagon, and is equal to the circumference of the circle Y. Add $\frac{1}{24}$th part. Then: $\frac{1}{24}(360) = \frac{1 \times 360}{24} = \frac{360}{24} = 15$; $360 + 15 = 375$ = circumference of the circle X; and $\frac{375}{6} = 62{\cdot}5$ = the arcs subtending the sides of the hexagon. But $\frac{60}{2} = 30$ = semi-radius, and, we get the equation, $375 \times 30 = 3\frac{1}{8}(60^2)$, or, $375 \times 30 = 3{\cdot}125 \times 3600 = 11{\cdot}250$ = area of the circle. This establishes the truth of the THEORY that 8 circumferences = 25 diameters in every circle; not as an *assumption*, but as a *logical deduction*, arrived at by sound reasoning from indisputable geometrical data.

Bacon observes:—"*The confirmation of theories relies on the compact adaptation of their parts, by which, like those of an arch or dome, they mutually sustain each other, and form a coherent whole.*" And you speak of my writings as "*able and ingenious deductions*

of what would follow, if π did equal 3·125." Perhaps you are not a believer in Baconian philosophy. If I am wrong in this surmise, and you carefully read this communication, you cannot fail to perceive the fallacy of the following paragraph in your Letter of the 30th August, "*There is no way of finding π except by comparing together in length, an arc and its chord* (*or sine or tangent or any other trigonometrical line*). *This you simply* EVADE. *You never try to tackle the difficulty, but say: Now, on the assumption that π is* $3\frac{1}{8}$, *so and so follows.*"

In comparing the perimeter of a regular hexagon with the circumference of its circumscribing circle, how can I *evade* comparing an arc and its chord? If the radius of a circle = 60, does not each side of a regular inscribed hexagon = 60? I have shewn you that the sides of a regular hexagon to a circle of radius = 60, are subtended by arcs = 62·5. Are not the sides of a regular hexagon, chords to the six arcs into which the angles of the hexagon divide the circumference of its circumscribing circle? I may tell you that the surest way of finding π is by the areas of circles, and having found π, we can then compare the areas of circles with other geometrical figures, standing geometrically in connection with the circles.

FIG. 5.



In the geometrical figure in the margin (Fig. 5), let A B be a straight line. With B as centre and A B as radius, describe the circle. On A B describe the square A B C D, and draw the diagonal A C. Bisect B C at E, and bisect E C at F. Join A E and A F. With D as centre, and D C or D A as radius, describe the arc A *n* C.

What do we know with reference to this geometrical figure? *We know* that B C = A B, for they are sides of the square A B C D. *We know* that B E = E C = $\frac{1}{2}$ B C, by construction. *We know* that E F = F C = $\frac{1}{2}$ E C, by construction, and it follows, that B F = $\frac{3}{4}$ B C = $\frac{3}{4}$ A B, by construction. *We know*, that A *y* C B is a quadrant of the circle, and it follows, that the arc A *y* C = $\frac{1}{4}$th part of the circumference of the circle. Because B is centre, and B A radius of the circle, and because D A = B A and D is centre, and D A radius of the arc A *n* C, it follows, that A *n* C D is a quadrant of the circle. You will hardly think it necessary that I should describe the whole circle, with D as centre, and D A as radius, to establish these facts. *We know*, that the quadrants A y C B and A *n* C D are equal. Because the square A B C D is a fourth part of a circumscribing square to the circle, and because the quadrant A *y* C B is a fourth part of the circle, *we know*, that the quadrants A *y* C B and A *n* C D are exactly equal in superficial area to an inscribed circle to the square A B C D. *We know*, that A B C is a right-angled triangle, and that A E C and A F C are oblique angled triangles. *We know*, that A B F is a right-angled triangle, and A E F an oblique-angled triangle. (With reference to a right-angled triangle in which all the angles are unequal, in one of my Letters I have spoken of the angles subtending the right angle as the obtuse and acute angles. You would understand that I meant this by way of comparing the greater with the smaller of the acute angles. Strictly speaking, *we know*, that *geometrically* an obtuse angle is greater than a right angle). So far, *we know*, by construction and inspection.

We know, by Euclid: Prop. 12: Book 2.

That:

$$\{A E^2 + E F^2 + 2(B E \times E F)\} = A F^2.$$
$$\{A E^2 + E C^2 + 2(B E \times E C)\} = A C^2.$$
$$\{A F^2 + E C^2 + 2(B F \times F C)\} = A C^2.$$

But, Euclid no where proves these facts *mathematically*, nor could he, without travelling out of the domain of pure Geometry. Now, Mathematics—rightly applied—can never be inconsistent with Geometry, and it follows, that these facts can be proved *mathematically*: but, it it is self-evident that to do so, we must put

an arithmetical value upon some line or surface in the figure. For these reasons, I dispute the very common *assertion* of Mathematicians, that Geometry is a mere branch of Mathematics; and maintain that Geometry is a distinct science: a science, to which, no doubt, Mathematics may, and can be, *rightly* applied; and when so applied, become a most valuable adjunct to Geometry. In fact, without the application of Mathematics to Geometry, the science of Geometry would be of little value. De Morgan says, that "*Mathematics and Logic* are the two eyes of exact science." I maintain that *Geometry and Logic* "are the two eyes of exact science."

Well, then, let A B the radius of the circle, and generating line of the diagram = 4. Then: By computation B E = 2: E F = 1: B F = 3: F C = 1: and B C = 4. By Euclid: Prop. 47: Book 1: $AB^2 + BE^2 = AE^2$: $AB^2 + BF^2 = AF^2$: and, $AB^2 + BC^2 = AC^2$: and by computation, $AE^2 = 20$: $AF^2 = 25$: and $AC^2 = 32$: and it follows, that $AC = \sqrt{32}$, and is incommensurable with the sides of the square A B C D. But this is no obstacle to our finding the exact arithmetical value of the symbol π: and yet, it is a fact, that I have never got into a controversy with any Mathematician, yourself included, who has not either asserted, or insinuated, that it is an obstacle. Even some of those who now agree with me (and they are not a few) were at one time of this opinion. Well, then,

$\{AE^2 + EF^2 + 2(BE \,.\, EF)\} = AF^2$: that is, $20 + 1 + 4 = 25$.

$\{AF^2 + FC^2 + 2(BF \cdot FC)\} = AC^2$: that is, $25 + 1 + 6 = 32$.

$\{AE^2 + EC^2 + 2(BE \cdot EC)\} = AC^2$: that is, $20 + 4 + 8 = 32$.

Now, referring to the diagram, Fig. 5, let A denote the area of the right-angled isosceles triangle A B C. Let B denote the area of the quadrant A y C B. Let x denote the difference between A and B. And let y denote the difference between B and the area of the square A B C D.

Then: The following things are self-evident:—

$A + x = B$: $x + y = A$. $B + y =$ area of the square A B C D: and A = half the area of the square A B C D. It is axiomatic, if not self-evident, that B = the area of an inscribed circle to the square A B C D, and it follows, that A = the area of an inscribed square to this circle. All these facts we arrive at by geometrical analysis, without the aid of Mathematics. You, my dear Sir, know

as well as I do, that we can demonstrate by pure geometry, that the area of a square inscribed in any circle is equal to half the area of a circumscribing square to the circle: and that the area of a circumscribing square to any circle is equal to four times the area of a square on the radius of the circle.

Now, if we can find the exact ratios between A and B, A and x, B and y, and x and y, it follows of necessity, that we can find the true arithmetical value of the symbol π; and that we can also construct, isolate, and exhibit, a circle and a square of exactly the same superficial area. But, how are we to prove whether it is possible, or impossible, to find these ratios, without the aid of Mathematics? Must we not put some value on a side of the square A B C D, and then compute the superficial area of the square, before we can take another step in search of these ratios? I may tell you that at the late meeting of the British Association, I met with more than one Mathematician, whose faith in Orthodoxy is shaken: and more than one Mathematician, *who now agrees with me*, that did not agree with me twelve months ago.

I will here relate an incident that occurred at the late meeting of the British Association, which bears upon the last paragraph. A gentleman, well known in the Mathematical Section, came up to me in the Reception-room one morning, and politely asked me if I would answer a question he wished to put to me. I told him, that if his question were a proper one, and it was in my power to answer it, I should be happy to do so. He sat down at one of the tables in the Reception-room, and constructed a square, with its diagonal and the two quadrants, as shewn in Fig. 5. He then said, "if x denote the area contained by the diagonal of the square and either of its subtending arcs, can you give me the value of x?" If so, Mr. Smith, you have certainly squared the circle. I called his attention to the fact, that x may have as many values as we please, since its value must necessarily depend upon the value we may put on a side of the square, or on the area of the square, and told him, that if he would put a value on the side of the square, I would give him the value of x in one minute. He fixed upon a value, and I was about to give him the proof, when his son, quite a youth, interrupted us in a very unseemly manner, chattering—

after the Whitworth fashion—as if *he* had attained the *ne plus ultra* of geometrical and mathematical knowledge. The old gentleman did not attempt to put a stop to his son's interruption, and at the moment, a doubt crossed my mind, whether his question was put in good faith, and I told him, he had better get his son to give him the information he required, and that if he failed to get it in that quarter, I would give it him the next time we met. I did not happen to see him again during the meeting of the Association. I regretted this, for, although the old gentleman (who is many years my senior), and I, had hitherto agreed to differ on Geometry and Mathematics, we had for years been on the most friendly terms, at our British Association re-unions, and I now regret that I acted on the sudden impression, that his question was not put in good faith.

I will now state the ratios, and then give you some proofs of their correctness:—

$A : x :: 16 : 9$: therefore, $A : x :: AB^2 : BF^2$.

$A : B :: 16 : 25$.

$B : y :: 25 : 7$.

$x : y :: 9 : 7$.

Now, A denotes the area of the right-angled isosceles triangle A B C = half the area of the square A B C D. It is obvious, that we may put any arithmetical value we please on the line A B, a side of the square, and the generating line of the geometrical figure represented by the diagram (Fig. 5); and B F the base of the right-angled triangle A B F = $\frac{3}{4}$ (A B), by construction.

Let A B be represented by the *mystic* number 4. Then: $AB^2 = 4^2 = 16$ = area of the square A B C D: and $\frac{AB^2}{2} = \frac{16}{2} = 8 = A$.

$A : B :: 16 : 25$: and by inversion, $16 : 25 :: A : B$. But, $16 : 25 :: 8 : 12{\cdot}5$; therefore, $B = 12{\cdot}5$. $B - A = 12{\cdot}5 - 8 = 4{\cdot}5 = x$; or, $A : x :: 16 : 9$: and by inversion, $16 : 9 :: A : x$. But, $16 : 9 :: 8 : 4{\cdot}5$; therefore, $x = 4{\cdot}5$. $B : y :: 25 : 7$; and by inversion, $25 : 7 :: B : y$. But, $25 : 7 :: 12{\cdot}5 : 3{\cdot}5$; therefore, $y = 3{\cdot}5$; or, $x : y :: 9 : 7$: and by inversion, $9 : 7 :: x : y$. But, $9 : 7 :: 4{\cdot}5 : 3{\cdot}5$; therefore, $y = 3{\cdot}5$. Hence: $x + y = 4{\cdot}5 + 3{\cdot}5 = 8 = A$ = half the area of the square A B C D.

Now, $BE = \frac{1}{2}$ (A B), by construction, and when $AB = 4$, then, $BE = 2$. Hence: $3\frac{1}{8}(BE^2) = 3\frac{1}{8}(2^2) = 3{\cdot}125 \times 4 = 12{\cdot}5 = A + x$

$= B =$ area of the quadrant $A y C B$. It is self-evident, that $4(A + x) =$ area of the circle; therefore, $4(B) = 4(12\cdot5) = 4 \times 12\cdot5 = 50$, area of the circle. But, $3\frac{1}{8}(A B^2) = (A B^2 + B F^2 + A F^2)$, and this equation $= 50$, when $A B = 4$; and it follows—not as an *assumption*, but as a *deduction* arrived at by logical reasoning upon indisputable geometrical data—that $3\cdot125$ is, and that nothing else can be, the true arithmetical value of the symbol π; and establishes the truth of the THEORY that 8 circumferences = 25 diameters, in every circle.

Hence: The area of every circle is equal to the sum of the areas of squares on the sides of a right-angled triangle, of which the sides that include the right angle are in the ratio of 4 to 3, and the longer of these sides the radius of the circle. In how many ways have I proved this fact, in the course of our long correspondence?

With you, my dear Sir, as "*a reasoning geometrical investigator*," and an honest Mathematician, it is hardly necessary I should give you further proof of the truth of my THEORY, that 8 circumferences of a circle are exactly equal to 25 diameters; but I have other proofs in store, and will give you one or two more, founded on the diagram, Fig. 5.

When $A B = 4$, which makes $A = 8$; then: $\{(A + \frac{1}{4} A) + \frac{1}{4}(A + \frac{1}{4} A)\} = \{(8 + 2) + \frac{1}{4}(8 + 2)\} = (10 + 2\cdot5) = 12\cdot5 = A + x$. But, $A + x = B$: and $B : y :: 25 : 7$: and by inversion, $25 : 7 :: B : y$. But, $25 : 7 :: 12\cdot5 : 3\cdot5$; therefore, $y = 3\cdot5$; therefore, $B + y = (12\cdot5 + 3\cdot5) = 16 = A B^2 = 2 A$; and all the other ratios follow.

Again: $B F$ is to $F C$ in the ratio of 3 to 1, by construction, and $B F + F C = B C$. Let $B F$ be represented by any arithmetical quantity not divisible by 3 without a remainder, say 7. Then: $(B F + F C) = (7 + \frac{1}{3} 7) = (7 + 2\frac{1}{3}) = 9\frac{1}{3} = B C$; but we cannot give exact arithmetical expression to $B C$ in decimal notation. You will observe, that $B F + F C$ is a binomial; therefore, $B F^2 + 2(B F . F C) + F C^2 = B C^2$.

Now, it is self-evident, that the area of the figure composed of, or described by, the arcs of the quadrants $A y C B$ and $A n C D = 2x$: and I shall now proceed to prove that $2x = B F^2$.

Let $FC = 7$: Then: 3 times $7 = 21 = BF$: and $FC + BF = 7 + 21 = 28 = BC$. But, $(BF + FC)$ is a binomial; therefore, $\{BF^2 + 2(BF \cdot FC) + FC^2\} = BC^2$: that is, $\{21^2 + 2(21 \times 7) + 7^2\} = \{441 + 294 + 49\} = 784 = BC^2$. $\frac{BC^2}{2} = \frac{784}{2} = 392 = A$. $\frac{BF^2}{2} = \frac{441}{2} = 220{\cdot}5 = x$: and $A + x = 392 + 220{\cdot}5 = 612{\cdot}5 = B$. But, $B : y :: 25 : 7$: and by inversion, $25 : 7 :: B : y$. But, $25 : 7 :: 612{\cdot}5 : 171{\cdot}5$; therefore, $y = 171{\cdot}5$. But, $B + y = BC^2$; that is, $612{\cdot}5 + 171{\cdot}5 = 784 = BC^2$. $A : B :: 16 : 25$: and by inversion, $16 : 25 :: A : B$. But, $16 : 25 :: 392 : 612{\cdot}5$; therefore, $B = 3\frac{1}{8}\left(\frac{BC}{2}\right)^2 = 3\frac{1}{8}\left(\frac{28}{2}\right)^2 = 3\frac{1}{8}(14^2) = 3{\cdot}125 \times 196 = 612{\cdot}5$, and the other ratios follow.

Again: Let x be represented by any arithmetical quantity, say 8. Then: $x : y :: 9 : 7$: and by inversion, $9 : 7 :: x : y$. But, $9 : 7 :: 8 : 6\frac{2}{9}$; therefore, $y = 6\frac{2}{9}$. Here we are met with an apparent difficulty, since the value of y is not arithmetically expressible in decimal notation. Reduce x and y to a common denominator. Then: $x = 72$ and $y = 56$; therefore $x + y = 72 + 56 = 128 = A$. But, $A : x :: 16 : 9$: and by inversion, $16 : 9 :: A : x$. But, $16 : 9 :: 128 : 72$; therefore $x = 72$. Divide by 9. Then, $\frac{72}{9} = 8 =$ the given value of x, and the other ratios follow.

Again: Let A be represented by the number of the apocalyptic beast $= 666$. Then: $A : x :: 16 : 9$: and by inversion, $16 : 9 :: A : x$. But, $16 : 9 :: 666 : 374{\cdot}625$; therefore, $x = 374{\cdot}625$, and it follows, that $A + x = 666 + 374{\cdot}625 = 1040{\cdot}625 = B$. $B : y :: 25 : 7$; and by inversion, $25 : 7 :: B : y$. But, $25 : 7 :: 1040{\cdot}625 : 291{\cdot}375$; therefore, $y = 291{\cdot}375$. But, $B + y = 1040{\cdot}625 + 291{\cdot}375 = 1332 = BC^2$; therefore, $BC = \sqrt{1332}$, and it is self-evident, that $BC^2 = 2A$. Now, $\frac{BC}{4} = \frac{\sqrt{1332}}{4} = \sqrt{\frac{1332}{4^2}} = \sqrt{\frac{1332}{16}} = \sqrt{83{\cdot}25} = FC$; and, 3 times $FC = 3(\sqrt{83{\cdot}25}) = \sqrt{3^2 \times 83{\cdot}25} = \sqrt{9 \times 83{\cdot}25} = \sqrt{749{\cdot}25} = BF$; therefore, $\overline{\sqrt{749{\cdot}25}}^{\,2} = 749{\cdot}25 = BF^2$; therefore, $\frac{BF^2}{2} = \frac{749{\cdot}25}{2} = 374{\cdot}625 = x$. In these computations, "π

does not appear, need not appear." But mark! $B + y$ = area of the square A B C D; and the area of the quadrant AyC B = area of an inscribed circle to the square A B C D; and it follows, that $\sqrt{1332}$ is the diameter of the inscribed circle, when A = 666; therefore, $\frac{1}{2}(\sqrt{1332}) = \sqrt{\frac{1}{2^2} \times 1332} = \sqrt{\cdot 25 \times 1332} = \sqrt{333}$ = radius of the circle. But, $\pi(r^2)$ = area in every circle, whatever be the value of π; and since no other arithmetical quantity but 3·125 can be multiplied into r^2, so as to produce 1040·625, the arithmetical value of $(A + x)$, which is equal to B = 1040·625, it follows—*not as an assumption*, but as an *incontrovertible* deduction—that the true arithmetical value of π *must* be 3·125, and can be nothing else.*

Now, 2π = circumference of a circle of radius = 1, whatever be the value of π; and since the property of one circle is the property of all circles; it follows, that $2\pi(r)$ = circumference in every circle. But, radius = 2 (semi-radius) in every circle, and, because the area of a square on the radius of a circle is equal to four times the area of a square on the semi-radius, and because the area of a circumscribing square to every circle is equal to four times the area of a square on the radius; it follows, that $4\pi(sr) = 2\pi(r)$, and that this equation = circumference in every circle, whatever be the value of π. But circumference × semi-radius = $\pi(r^2)$, and this equation = area in every circle, whatever be the value of π. We can prove this equation by any hypothetical value of π within the limits of 3 and 4. We may assume $\pi = 3$, or $\pi = 4$, or π = any *finite* and *determinate* arithmetical quantity, intermediate between 3 and 4. I will give you two examples. Let $\pi = 3$. Then: $2\pi(1) = 6$ = circumference of a circle of radius = 1, and $\frac{1}{2}(1) = \frac{1}{2} = \cdot 5$ = semi-radius; therefore, $c \times sr = 6 \times \cdot 5 = 3 = 2\pi(r)$. Let $\pi = 4$. Then: $2\pi(1) = 8$ = circumference of a circle of radius unity, and $\frac{1}{2}(1) = \cdot 5$ = semi-radius; therefore, $c \times sr = 8 \times .5 = 4 = 2\pi(r)$. The former value of π would make the circumference of a circle and the perimeter of its inscribed regular

* With reference to Fig. 5, it is self-evident, that 4 (A) = area of an inscribed square to the circle. But, B denotes the area of the quadrant AyC B, and y denotes the difference between B and the area of the square A B C D; and it follows, that $4(B + y)$ = area of a circumscribing square to the circle. J. S.

hexagon equal; and the latter value of π would make the circumference of a circle and the perimeter of its circumscribing square equal; and proves that π must be greater than 3, and less than 4: and you would say, "*this is no help to finding π's true value.*"

Referring to the paragraph in your Letter of the 30th August, which I have already quoted, and in which you say:

"$\left.\dfrac{\text{Perimeter of hexagon}}{\text{Circumference of Circle}} = \dfrac{6}{2\pi}\right\}$. Hence π is greater than 3."

I deny that your equation proves your conclusion, and I have already said that your conclusion is a *non sequitur*. According to Mathematicians, $\dfrac{6}{3{\cdot}14159} = 954$, &c., but this proves nothing; and no extension of the decimals can make it prove anything. But, when π's true value is known, the equation, $\dfrac{6}{2\pi} = \dfrac{6}{6{\cdot}25} = {\cdot}96 =$ the perimeter of a regular inscribed hexagon to a circle of circumference $= 1$; and because the property of one circle is the property of all circles, it follows, that $\dfrac{\text{Perimeter of every regular hexagon}}{\text{Circumference of its circumscribing circle.}}$ is a constant quantity $= {\cdot}96 =$ the perimeter of a regular inscribed hexagon to a circle of circumference unity $= 1$.*

Now, the equation, circumference $\times$ semi-radius $= \pi\,(\text{radius}^2)$, and it is admitted by every Mathematician, that this equation $=$ area in every circle. This is an inference—and rightly an inference—from the "*undoubted fact*," that $2\pi =$ circumference of a circle of radius unity; but Mathematicians cannot prove this with the value they assign to π. I will now give you a proof, that $\pi\,(r^2) =$ semi-radius $\times$ circumference.

Referring to the diagram (Fig. 5), let A denote the area of the right-angled isosceles triangle A B C = half the area of the square A B C D. Let B denote the area of the quadrant A y C B. Let x denote the difference between A and B. And let y denote the difference between B and the area of the square A B C D, as before.

I have already proved, that when A = 666, then, A + x = 1040·625 = B; and it cannot be disputed that B = area of an

* Let the reader take this fact, in connection with the equivalent ratios to which I have directed attention, on page 101. J. S.

inscribed circle to the square A B C D. I have also proved that $y = 291 \cdot 375$, when $A = 666$; and $B + y = 1040 \cdot 625 + 291 \cdot 375 = 1332 = 2A$ = area of the square A B C D; and it follows, that $\sqrt{1332}$ = diameter of an inscribed circle to the square A B C D; therefore, $\frac{\sqrt{1332}}{2} = \sqrt{333}$ = radius of an inscribed circle to the square A B C D. But, $FC = \frac{1}{4}(BC)$, by construction; and it follows, that $FC = \frac{\sqrt{333}}{2} = \sqrt{83 \cdot 25}$ = semi-radius of an inscribed circle to the square A B C D. Now, 2π (radius) = circumference in every circle; and it follows, that 4π (semi-radius) = circumference in every circle. (On pp. 145–6, I have given you an ***incontrovertible deduction***, arrived at by logical reasoning from indisputable premisses, that π can be nothing else but $3 \cdot 125$; and I hope I shall hear no more of your ***absurd*** charge that I assume the value of π; that is, assume the thing to be proved. You cannot by your mere ***ipse dixit*** prove that my deduction is either false or absurd, and if you dispute it, you should prove its absurdity, either by pointing out a fallacy in my reasoning, or a flaw in my premisses). Hence: $4\pi\,(sr) = 4\pi\,(\sqrt{83 \cdot 25}) = \sqrt{12 \cdot 5^2 \times 83 \cdot 25} = \sqrt{156 \cdot 25 \times 83 \cdot 25} = \sqrt{13007 \cdot 8125}$ = circumference of an inscribed circle to the square A B C D; therefore, $\sqrt{83 \cdot 25} \times \sqrt{13007 \cdot 8125} = \sqrt{(\sqrt{83 \cdot 25 \times 13007 \cdot 8125})} = \sqrt{1082900 \cdot 390625} = 1040 \cdot 625 = A + x = B$ = area of an inscribed circle to the square A B C D, when the area of half the square = 666. If we inscribe a circle in the square A B C D, is not the area of an inscribed square to this circle equal to half the area of the square A B C D?

Once more: It cannot be disputed that squares can be circumscribed and inscribed to a circle, and I am sure you know as well as I do, that there are more ways than one of geometrically constructing such a figure, and that the area of the circumscribing square is double the area of the inscribed square. Well, then, let A denote the area of a circumscribed, and let B denote the area of an inscribed square to a circle; and let O denote the area of the circle. You will hardly consider it necessary that I should prove that $A = 2B$, and I shall take this fact as admitted. Upon this very simple figure I found the following theorems:—

THEOREM I.

From a given value of A find O, and from O find B, and prove that $B = \frac{1}{2} A$.

Now, *we know* that $A = 2 B$, but it is self-evident that "*this is no help*" to solving the theorem: for, by the terms of the theorem B is supposed to be unknown. *We know* that π is involved in the solution of this theorem, and *we know* that the area of a circle is found by multiplying the area of its circumscribing square by $\frac{1}{4}(\pi)$. If r denote the radius of the circle, *we know*, that $A \times \frac{1}{4}(\pi) = \pi (r^2)$, and this equation $= O$. This may be proved by means of any hypothetical value of π intermediate between 3 and 4. For example: Let $A = 128$, and let $\pi = 3{\cdot}14$. Then: $\sqrt{128} =$ diameter of the circle, and $\frac{1}{2}(\sqrt{128}) = \sqrt{32} =$ radius; therefore, $128 \times \frac{3{\cdot}14}{4} = 3{\cdot}14 (\sqrt{32})^2$; that is, $128 \times {\cdot}785 = 3{\cdot}14 \times 32$, and this equation $= 100{\cdot}48$. This is no proof of π's value, and you would say, "*is no help to finding π's true value*," and it is self-evident that until we have found π's true value, the solution of this theorem is impossible; but having found the true value of π, we can solve the theorem in more ways than one. I will give you two solutions of the theorem.

Now, because $A \times \frac{1}{4}(\pi) = O$, it follows, that $\frac{O}{\frac{1}{2}\pi} = B$. Hence: the area of an inscribed square to any circle is found by dividing the area of the circle by $\frac{1}{2}(\pi)$; and it follows, that the ratio between the area of any circle and the area of its inscribed square is as the $\frac{1}{2}(\pi)$ to 1. From this, having found the true arithmetical value of π, we get the following solution of the theorem:—

Let A be represented by any arithmetical quantity, say 128. Then: $A \times \frac{1}{4}(\pi) = 128 \times \frac{1}{4}(3{\cdot}125) = 128 \times {\cdot}78125 = 100 = O$; therefore, $\frac{O}{\frac{1}{2}\pi} = \frac{100}{\frac{1}{2}(3{\cdot}125)} = \frac{100}{1{\cdot}5625} = 64 = \frac{1}{2}(A) = B$. Q.E.D.

Another solution: $A \times (\frac{1}{4}\pi) = 100 = O$. $\{(O - \frac{1}{5} O) - \frac{1}{5}(O - \frac{1}{5} O)\} = \{(100 - 20) - \frac{1}{5}(100 - \frac{1}{5}100)\} = \{80 - \frac{1}{5}(80)\} = (80 - 16) = 64 = \frac{1}{2}(A) = B$. Q.E.D.

THEOREM 2.

From a given value of B find O, and from O find A, and prove that A = 2 B.

It is obvious that this theorem is the converse of theorem 1, but there is this difference between them. We must know the true arithmetical value of π, before we can solve theorem 1, but the solution of theorem 2 can be solved, and π not appear in the solution; but, from the solution of the theorem, it follows—as a deduction, ***not as an assumption***—that the true arithmetical value of π can be nothing else but $\frac{25}{8} = 3{\cdot}125$, making 8 circumferences = 25 diameters, in every circle.

Let B = 64. Then: $\{(B + \frac{1}{4}B) + \frac{1}{4}(B + \frac{1}{4}B)\} = \{(64 + \frac{1}{4}64) + \frac{1}{4}(64 + \frac{1}{4}64)\} = (80 + \frac{1}{4}80) = (80 + 20) = 100 = O$. The difference between O and A is in the ratio of 25 to 7, as I have already proved in a previous part of this communication, and 25 : 7 : : 100 : 28 : therefore, 100 + 28 = 128 = A = 2 B. Q.E.D.

It is self-evident that π "*does not appear*," but I must shew you how π's true value follows, from the solution of this theorem, ***not as an assumption***, but as a logical deduction. Well, then, it is self-evident, that the diameter of any circle is equal to a side of its circumscribing square; and it follows, that when A = 128 the diameter of an inscribed circle = $\sqrt{128}$, and $\frac{\sqrt{128}}{2} = \sqrt{32} =$ radius of the circle, and $\pi(r^2)$ = area in every circle, whatever be the value of π; therefore, $\pi(\sqrt{32}^{\,2}) = O$: and since no other value of π but 3·125 will multiply into $\sqrt{32}^{\,2}$ and produce 100, it follows as a logical deduction, that 3·125 must be π's value, and can be nothing else.

In your Letter of the 30th August, you say:—"*Offer me any proof* of the value of π and I will study it carefully." Well, then, I offer you this proof, and if you "*study it carefully*," I think you cannot fail to be convinced, that we have found π's true value.

I will now shew you how to solve theorem 1, by ratios:—

By hypothesis, let A = 128. Then: As 4 : 3·125 : : 128 : 100; therefore, O = 100. But, 128 — 100 = 28 = the difference between

A and O: aud if X denote this difference, X : O : : 28 : 100; and by inversion, 28 : 100 : : X : O; therefore, X : O : : $\frac{28}{4}$: $\frac{100}{4}$; that is, X : O : : 7 : 25. But, (7 + 25) = 32 = area of a square on the radius of a circle of which the diameter is $\sqrt{128}$, and the area of a square on the radius of any circle is equal to half the area of an inscribed square to the circle; therefore, twice 32 = 2 × 32 = 64 = area of an inscribed square to a circle of diameter = $\sqrt{128}$. Hence: The ratio between the area of any circle and the area of its inscribed square is as 100 : 64, or, as 25 : 16. Therefore, O : B : : 25 : 16: and by inversion, 25 : 16 : : O : B: that is, 25 : 16 : : 100 : 64; therefore, B = 64 = $\frac{1}{2}$ (A). Q.E.D.

It is self-evident, that a ratio exists between the circumference of a circle and the perimeter of its circumscribing square; and because the perimeter of a circumscribing square to a circle of diameter unity = 4, it follows of necessity, that the ratio between the circumference of any circle, and the perimeter of its circumscribing square, is as π to 4, whatever be the value of π.

Now, let the circumference of a circle = 100. Then: 3·125 : 4 : 100 : 128, and 128 is the perimeter of a circumscribing square to a circle of which the circumference is 100; therefore, $\frac{128}{4}$ = 32 = side of the square and diameter of the circle. $\frac{32}{2}$ = 16 = radius, and $\frac{16}{2}$ = 8 = semi-radius of the circle. Now, circumference × semi-radius = $\pi (r^2)$, and 100 × 8 = $3\frac{1}{8}$ (16^2); that is, 100 × 8 = 3·125 × 196 = 800, and this equation = area of a circle of circumference = 100; and demonstrates, beyond the possibility of dispute or cavil, that $\frac{25}{8}$ = 3·125 is the true arithmetical value of π; for, by no other value of π can we get this equation.

In your Letter of the 30th August, you say:—"*I should not tease you with Letters, or expostulations, if you give me up as hopeless; nor would it distress me at all, if you vote me no Mathematician, and incapable of seeing your* PROOFS. *I confess my inability on that point.*" Now, if you tell me you can't see the proofs I have given you in this communication, I shall take you at your word, and not attempt to disprove it; but I might VOTE you "no Mathematician," and feel perfectly justified in doing so. I suspect, that this would hurt your feelings, but that I cannot help: truth admits

of no compromise. I understand the term Mathematician to include the term Geometrician; does not every Mathematician look upon Geometry as a branch of Mathematics? It follows, that to be a Mathematician, a man must be a Geometer.

18th September.

The evening's post of the 16th inst., brought me the following communication. It was addressed to the care of Messrs. Simpkin and Marshall, and came to me through my Liverpool publisher.

September 14th, 1869,
11, LYNDHURST ROAD,
PECKHAM, SURREY, S.E.

DEAR SIR,

I have just finished the perusal of your splendid work entitled "The Geometry of the Circle." I am quite sure, that you are at present in the class of UN-recognised Mathematicians: in fact you are beyond the age. However, it is not often we get a good jest on so dry a subject as that of the 'Quadrature of the Circle,' and with many thanks for the pleasure which your very comical production has afforded me,

I remain,
Yours truly,
R. F. GLAISTER,
(Late Mathematical Scholar of King's College, London).

P.S.—Page viii. of the introduction entitled "To the Reader" is charming. I also much admire your certainly gentlemanlike tone towards a somewhat humble Mathematician, named "De Morgan."

There is a P.P.S. to this Letter not worth troubling you with, but I may tell you, that the numerous erasures and interlineations, stamps the document as a very suspicious one. The following was my reply:—

BARKELEY HOUSE, SEAFORTH,
Near Liverpool,
17th September, 1869.

DEAR SIR,

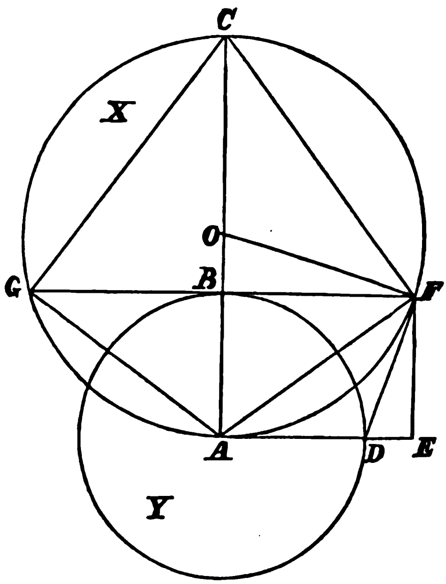
Your favor of the 14th inst. came into my hands last evening.

It may be construed in two ways, but whether it is, or is not, intended to be ironical, one thing is certain, that like myself, you have a spice of humour in your composition; and I send you by this day's post, a little *brochure* * of mine, that I think will amuse you.

I remain,
Yours truly,
JAMES SMITH.

R. F. GLAISTER, Esq.

FIG. 6.



I construct the geometrical figure represented by the diagram in the margin (Fig. 6), in the following way. From the point A, I draw a straight line, and mark off three equal parts, together equal to A B. I produce A B to C, making B C = $\frac{5}{8}$ (A B). I bisect A C at O, and with O as centre and O C or O A as radius, describe the circle X, and with A as centre and A B as radius, describe the circle Y. From the point A, I draw a straight line perpendicular to A C, and therefore tangental to the circle X, to meet and terminate in the circumference of the circle Y at the point D. I produce A D to E, making D E equal to $\frac{7}{16}$th parts of A D, and from the point E draw a straight line parallel to B A, to meet another straight line drawn from the point B perpendicular to A C in the circumference of the

* *The British Association in Jeopardy, and Professor de Morgan in the Pillory,* without hope of *escape.*

circle X, at the point F. I produce F B to meet and terminate in the circumference of the circle X at the point G, and join G C, G A, C F, F O, F A, and F D.

I shall now prove that Euclid, in attempting to make his fifth book on proportion alike applicable to commensurables and incommensurables, attempted an impossibility; and consequently, that many of his theorems, particularly in the sixth book, are not of general and universal application, and therefore, ***are not true***, under all circumstances.

Now, C B is to B A in the ratio of 5 to 3, by construction; and when $CB = 5$, then, $BA = 3$: $FO = 4$: and $OB = 1$.

Then:

By Euclid: Prop. 47: Book 1.

$(FO^2 - OB^2) = (4^2 - 1^2) = 16 - 1 = 15 = FB^2.$

$\therefore FB = \sqrt{15}.$

$(FB^2 + BA^2) = (\sqrt{15}^2 + 3^2) = (15 + 9) = 24 = FA^2.$

$\therefore FA = \sqrt{24}.$

By Euclid: Prop. 12: Book 2.

$CF^2 = FO^2 + OC^2 + 2(OC \cdot OB).$

$\therefore \{4^2 + 4^2 + 2(4 \times 1)\} = (16 + 16 + 8) = 40 = CF^2.$

$\therefore CF = \sqrt{40}.$

By Euclid: Prop. 8: Book 6.

$CB \times CA = 5 \times 8 = 40 = CF^2$ or CG^2.

$\therefore CF$ and $CG = \sqrt{40}.$

By Euclid: Prop. 47: Book 1.

$(CB^2 + BF^2) = (5^2 + \sqrt{15}^2) = (25 + 15) = 40 = CF^2.$

$\therefore CF = \sqrt{40}.$

Now, B A E F is a rectangular parallelogram; therefore, the side A E equals the side B F, and the side F E equals the side B A. Well, then, Mathematics—rightly applied—can never be inconsistent with pure Geometry: and because all Mathematics are founded upon that "*indispensable instrument of science, Arithmetic,*" it follows of necessity, that Arithmetic—rightly applied—can never be inconsistent with pure Geometry.

Now, if two lines be constructed in ratio—the ratio may be as 2 to 1, or as 4 to 3, or as 4 to $\sqrt{32}$, and it is an "*undoubted fact*"

that 4 to $\sqrt{32}$ is the ratio between a side and the diagonal, in every square. It is self-evident, that neither Mathematics nor any of its branches—such as Algebra and Trigonometry—can ever "*upset*," or affect the geometrical ratio between a side and the diagonal in any square.

Well, then, when C B = 5, then, A B = 3, and A D = A B, for they are radii of the circle Y. But, A D : D E : : 24 : 7, by construction: and by inversion, 24 : 7 : : A D : D E. But, 24 : 7 : : 3 : ·875 : therefore, (A D + D E) = (3 + ·875) = (3·875) = A E. But, A E = B F, by construction, and according to Euclid, B F = $\sqrt{15}$ when A B = 3. But, the extracted root of the definite arithmetical expression $\sqrt{15}$ is 3·872, &c., and is less than the geometrical length, by construction, of the line A E. Now, A D = A B, and F E = A B, for they are opposite sides of a parallelogram; and when A B = 3 : F E = 3, and D E = ·875, and F E D is a right-angled triangle; therefore, by Euclid: Prop. 47: Book 1 : (FE^2 + DE^2) = 3^2 + $·875^2$) = (9 + ·765625) = 9·765625 = FD^2; therefore, $\sqrt{9·765625}$ = 3·125 = F D. Now, by Euclid: Prop. 47: Book 1: FA^2 = 24, when A B = 3. But, by Euclid: Prop. 12: Book 2: $\{FD^2 + DA^2 + 2(DA \cdot DE)\} = FA^2$; therefore, $\{3·125^2 + 3^2 + 2(3 \times ·875)\} = FA^2$. But, when we work out the calculations by that "*indispensable instrument of science, Arithmetic*," we find that $\{FD^2 + DA^2 + 2(DA \cdot DE)\}$ is greater than 24; that is, $\{3·125^2 + 3^2 + 2(3 \times ·875)\} = \{9·765625 + 9 + 5·25\} = 24·015625$, and is greater than 24. Hence: If a straight line be represented by the square root of any number (unless the number happen to be a square number) the extracted root will not give the true length of the line: and it follows, that Euclid attempted an impossibility in attempting to make his fifth book on proportion, alike applicable to commensurables and incommensurables; and consequently, that many of his theorems are not of general and universal application.

Referring to the diagram (Fig. 6), C G A F is a quadrilateral, and O B is the line that joins the middle points of the diagonals, by construction. In every modern Treatise on Elementary Geometry, an exercise is given on the first and second books of Euclid, founded on these geometrical data.

J. Radford Young, a living "*recognised Mathematician*" gives the exercise in the following terms :—

Prove that "*in any quadrilateral, the sum of the squares of the four sides, is equal to the sum of the squares of the diagonals, together with four times the square of the line joining the middle points of the diagonals.*"

J. M. Wilson, Mathematical Master of Rugby School, and the most recent author of a Treatise on "*Elementary Geometry,*" puts it :—

Prove that "*the squares on the diagonals of a quadrilateral are together less than the squares on the four sides, by four times the square on the line joining the points of bisection of the diagonals.*"

Whether put in Mr. Young's or Mr. Wilson's terms, it comes to the same thing; and no geometrical pupil who had mastered the first and second books of Euclid, would have any difficulty in seeing what he had to prove, if the exercise were given to him. I hardly think you will venture to take exception to the diagram, (Fig. 6), or object to my use of it in the solution of this theorem.

Now, according to Mr. J. M. Wilson: when $CA = 8$, then, $CF = \sqrt{40} : BF = \sqrt{15}$: and the line $OB = 1$. I have Mr. Wilson's admission (in his own handwriting) of the facts, so far. Well, then, I admit that taking $BF = \sqrt{15}$; and $BA = 3$ (and it is an "undoubted fact" that $BA=3$ when $CA=8$, by construction), we make $AF^2 = 24$: and it is self-evident that $GF = 2(BF)$; therefore, $2(\sqrt{15}) = \sqrt{2^2 \times 15} = \sqrt{4 \times 15} = \sqrt{60} = GF$; therefore, $\{2(CF^2) + 2(FA^2)\} = \{CA^2 + CF^2 + 4(OB^2)\}$; that is, $(80 + 48) = (64 + 60 + 4) = 128$; and this equation = the sum of the squares of the four sides of the quadrilateral; and is equal to the area of an inscribed square to a circle of which CA is the radius.

I will now prove to you that this is not the only way of solving this theorem.

Referring to the diagram (Fig. 6), FE and ED, the sides that include the right angle in the triangle F E D, are in the ratio of 24 to 7, by construction; the ratio may be proved by Logarithms, but you avoid Logarithms as "*needless,*" and consequently, it is not worth while to trouble you with the computations. And when $AC = 8$: $BA = 3$. Now, $FE = BA$; therefore, $FE = 3$. But, $FE : ED$

: : 24 : 7 ; and by inversion, 24 : 7 : : F E : E D, and 24 : 7 : : 3 : ·875 ; therefore, E D = ·875 : and by Euclid, Prop. 47 : Book 1 : $(FE^2 + ED^2) = (3^2 + \cdot875^2) = (9 + \cdot765625) = 9\cdot765625 = FD^2$; therefore, $\sqrt{9\cdot765625} = 3\cdot125 = FD$. By Euclid : Prop. 12 : Book 2 : $\{FD^2 + DA^2 + 2(AD\,.\,DE)\}\ FA^2$: and I have worked out the computations, in a previous part of this communication, and proved that $FA^2 = 24\cdot015625$, when $BA = 3$. Now, by Euclid : Prop. 31 : Book 3 : C F A is a right-angled triangle ; therefore, $CA^2 - FA^2 = (8^2 - \sqrt{24\cdot015625^2}) = (64 - 24\cdot015625) = 39\cdot984375 = CF^2$; therefore, $CF = \sqrt{39\cdot984375}$, and $FA = \sqrt{24\cdot015625}$. $(FO^2 - OB^2)\ (4^2 - 1^2) = (16 - 1) = 15 = BF^2$; therefore, $BF = \sqrt{15}$. But, it is self-evident, that $GF = 2(BF)$; therefore, $2(BF) = 2(\sqrt{15}) = \sqrt{60} = GF$: and we get the equation: $\{2(CF^2) + 2(FA^2)\} = \{CA^2 + GF^2 + 4(OB^2)\}$; that is, $\{2(39\cdot984375) + 2(24\cdot015625)\} = \{79\cdot96875 + 48\cdot03125\} = \{64 + 60 + 4\} = 128$: and this equation = the sum of the squares of the four sides of the quadrilateral, and is exactly equal to the area of an inscribed square to a circle of which C A is the radius.

If one of Mr. Wilson's pupils were to offer him this solution of the theorem, what would *he* say ? What could *he* say ?

Geometry is a natural science, and within the grasp of human reason. To prove this, and bring out the truths we discover by means of a quadrilateral, involves the construction of another problem.

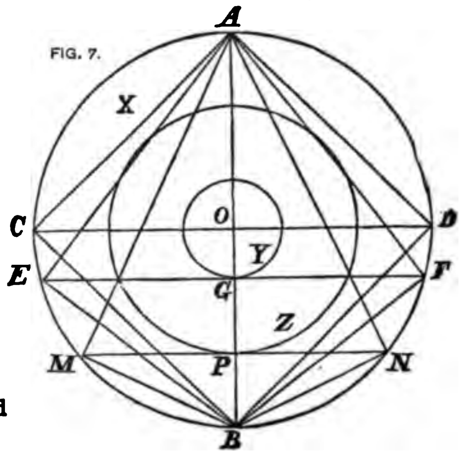


FIG. 7.

I construct the geometrical figure in the margin (Fig. 7) in the following way : I draw two lines at right angles intersecting at O. With O as centre and any radius, I describe the circle X. It is self-evident that A B and C D are diameters of the circle X. I join the extremities of these diameters, and so construct the inscribed square A C B D. With

O as centre and one-fourth part of O A or O B as radius, I describe the circle Y, and draw the line E F parallel to C D, and passing through the point G, and join A E, A F, E B, and F B, and so construct the quadrilateral A E B F. With O as centre and any radius, greater than O G, I describe the circle Z, and draw the line M N parallel to C D and E F, passing through the point P, and intersecting the line G B, and join A M, A N, M B, and N B, and so construct the quadrilateral A M B N.

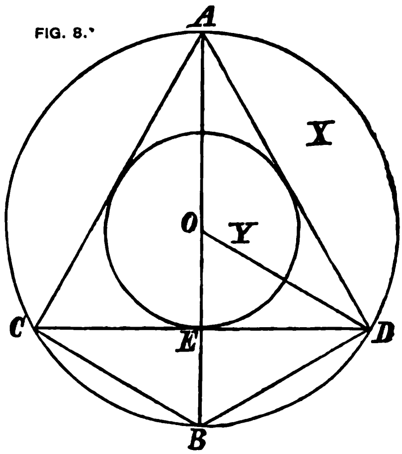
Now, A O is to O G in the ratio of 4 to 1, and A G is to G B in the ratio of 5 to 3, by construction: but the ratio between A P and P B is unknown by the terms of construction. The line O G joins the middle points of the diagonals in the quadrilateral A E B F; and the line O P joins the middle points of the diagonals in the quidrilateral A M B N. There is no line joining the middle points of the diagonals in the quadrilateral A C B D, nor can there be, since the diagonals of this quadrilateral intersect and bisect each other at the point O. This is unique, and is true only of a square. Will you or any other Mathematician venture to tell me that a square is not a quadrilateral?

Now, it is self-evident, that we may inscribe as many regular quadrilaterals within a circle, as we please: and it is equally self-evident, that every quadrilateral inscribed within a circle (the square excepted) will have two sides longer and two sides shorter than the sides of an inscribed square, (even the Rev. Professor Whitworth would hardly have the *audacity* to pretend to mistake my meaning, and raise a "*quibble*" by asserting that an inscribed quadrilateral to a circle may have four unequal sides); but the difference between a longer and a shorter side of the quadrilateral is compensating, so that the sum of a longer and shorter side of any regular quadrilateral inscribed in a circle, is equal to the sum of two sides of a square inscribed in the same circle. Hence: The sum of the squares of the four sides of any regular quadrilateral inscribed in a circle, is equal to the area of an inscribed square to a circle of which the longer diagonal of the quadrilateral is the radius.

Euclid proves by pure Geometry, Prop. 35: Book 3: that $(A\,O \times O\,B) = O\,D^2$: and $(A\,G \times G\,B) = G\,F^2$: and it follows of necessity, that $(A\,P \times P\,B) = P\,N^2$. Now, although we may have $(A\,P + P\,B) = 8$, and therefore know the value of A B, how are we

to find the arithmetical values of P N, and P M, the ratio between A P and P B being unknown? Again: Because $\{AB^2 + EF^2 + 4\,(OG^2)\}$ = the sum of the squares of the four sides of the quadrilateral A E B F, and is equal to the sum of the squares of the four sides of the inscribed rectangle A C B D, it follows of necessity, that $\{AB^2 + MN^2 + 4\,(OP^2)\}$ = the sum of the squares of the four sides of the quadrilateral A M B N; but, how are we to prove this *mathematically*, the ratio between A P and P B being unknown? Well, then, *we know*, that $AB^2 + EF^2 + 4(OG^2) = \{AC^2 + CB^2 + AD^2 + DB^2\}$. It follows, that $\{AB^2 + MN^2 + 4\,(OP^2)\} = \{AB^2 + EF^2 + 4\,(OG^2)\}$, and that this equation is equal to the sum of the squares of the four sides of the inscribed square A C B D. More we cannot know with reference to these quadrilaterals without the aid of Mathematics. Euclid demonstrates all his Theorems by angles and ratios. Can you find an instance in which Euclid puts an arithmetical value upon a line?

FIG. 8.



In the geometrical figure in the margin, (Fig. 8), let A B be a straight line, bisected at O. With O as centre and O A or O B as radius, describe the circle X. With O as centre and $\frac{1}{2}$(O B) as radius, describe the circle Y. It is self-evident that the circumference of the circle Y cuts the line A B and bisects O B a part of it at E. Draw the straight line C D perpendicular to A B and passing through E, and join A C, A D, C B, and D B, and so construct the quadrilateral A C B D. Join O D. It is self-evident, that the line A B is divided into two parts A E and E B, and that A E is to E B in the ratio of 3 to 1, by construction.

According to Euclid: Prop. 32: Book 1: the three internal angles of every triangle, are together equal to two right angles.

In some Treatises on "Elementary Geometry," the following is given as a corollary: "*In an equilateral triangle, each angle is a third of two right angles, and is therefore equal to two-thirds of a right angle.*"

To this I take exception, and maintain that the three angles in the equilateral triangle A C D in Fig. 8, are subtended by arcs of 120°, and are, therefore, each equal to one-third of four right angles. Proof: From the angles C and D draw straight lines through the point O, to meet and terminate in the circumference of the circle X at two points, say *m* and *n*. The arcs A C and A D will be bisected at *m* and *n*. Join A *m*, C *m*, A *n*, and D *n*, and so construct a regular inscribed hexagon to the circle X. The six angles at the centre of the circle will be subtended by sides of the hexagon, and in this case, the six angles at the centre of the circle, will each be equal to two-thirds of a right angle, and together equal to four right angles. But, if an equilateral triangle be inscribed in a circle, with all the angles touching the circumference, each of these angles will be equal to one-third of four right angles, and together equal to four right angles. Can I make this plainer? I will try, if possible. With A as centre and A C or A D as radius, describe a circle, say Z. Then: C D will be a side of a regular inscribed hexagon to the circle Z. Complete the construction of a hexagon. Then: the angle C A D in Fig. 8, will become an angle at the centre of the circle Z, and equal to two-thirds of a right angle: but, as an angle at the circumference of the circle X, it is equal to one-third of four right angles. J. Radford Young and J. M. Wilson would appear to have seen this, for neither of them has given the corollary I have quoted; but they both maintain, that the three interior angles of every triangle are together equal to two right angles. Well, then, is not an equilateral triangle, when inscribed to a circle, with all its angles touching the circumference, an exception to Euclid's Theorem: Prop. 32: Book 1?

By Euclid: Prop. 31: Book 3: the triangles A C B and A D B are right-angled triangles; and it is self-evident, that A B, a diameter of the circle, is the hypothenuse of, and common to, the two triangles.

By Euclid: Prop. 35: Book 3: $(AE \times EB) = ED^2$ and EC^2.

By Euclid: Prop. 47: Book 1: $(AE^2 + ED^2) = AD^2$: $(DE^2 + EB^2) = DB^2$: $(AE^2 + EC^2) = AC^2$: and $(CE^2 + EB^2) = CB^2$.

By Euclid: Prop. 12: Book 2:

$$DO^2 + OA^2 + 2(OA.OE) = AD^2.$$

By Euclid: Prop. 8: Book 6: the triangles on each side of D E are similar triangles, and similar to the whole triangle A D B: and the triangles on each side of C E are similar triangles, and similar to the whole triangle A C B.

By Euclid: Prop. 13: Book 6:

$AE : ED :: ED : EB$: and, $AE : EC :: EC : EB$.

It is self-evident, that the quadrilateral A C B D is divided by the diagonal A B into two similar triangles A C B and A D B. It is also self-evident, that these triangles are each divided into two triangles, by the diagonal C D: and it is demonstrable, but not self-evident, that the triangles A C B, A D B, A E C, C E B, A E D, and D E B, are all similar triangles. I admit, and I do not suppose that either Whitworth or Wilson would dispute, that $\{AB^2 + CD^2 + 4(OE^2)\}$ = the sum of the squares of the four sides of the quadrilateral A C B D.

We know that A E is to E B in the ratio of 3 to 1, by construction: and because the equilateral triangle A C D is bisected by the line A E, it follows, that A C is to C E, and A D to D E in the ratio of 2 to 1: and I admit that in every triangle (the equilateral triangle excepted when inscribed to a circle) the three interior angles are together equal to two right angles. It is self-evident, that the circles X and Y are circumscribed and inscribed circles to the equilateral triangle A C D. Euclid's fourth book is made up of problems exclusively, and he shews us how to circumscribe and inscribe circles to any triangle. I do not think we can get a step further by pure geometry.

I do not dispute, that $(AE \times EB) = EC^2$ and ED^2: nor do I dispute, that $AE : ED :: ED : EB$: and, that $AE : EC :: EC : EB$. What I say is this:—It depends upon what line in Fig. 8,

we put an arithmetical value, whether we can prove these facts *mathematically* with arithmetical exactness; and it is at this point that Mathematics enters into our enquiry as to the properties of this geometrical figure.

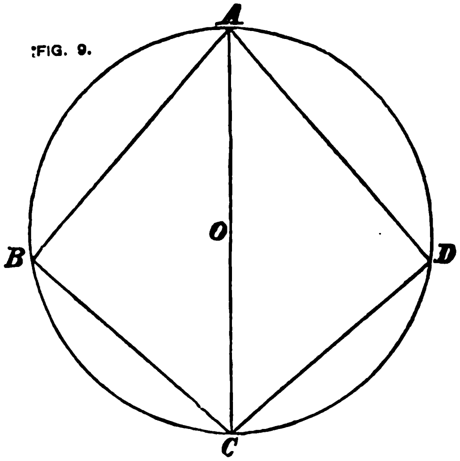
Let A D a side of the equilateral triangle A C D = 2. Then: $\frac{AD}{2} = \frac{2}{2} = 1 = ED$: and by Euclid: Prop. 47: Book 1: $(AD^2 - ED^2) = (2^2 - 1^2) = (4 - 1) = 3 = AE^2$; therefore, $AE = \sqrt{3}$. But, $EB = \frac{1}{3}(AE)$, by construction; therefore, $\frac{AE}{3} = \frac{\sqrt{3}}{3} = EB$. How can we find the exact arithmetical value of E B? Now, *we know*, that ED = 1, when AD = 2: and $\sqrt{3} = 1{\cdot}7320$, &c. $\frac{1{\cdot}7320}{3} = {\cdot}5773$, &c. = E B, approximately: and, $(AB \times EB) = (1{\cdot}7320 \times {\cdot}5773) = {\cdot}99988360$: and is a very close approximation to unity: and $1{\cdot}7320 + {\cdot}5773 = 2{\cdot}3093$, and is a close approximation to the arithmetical value of A B. Now, A E : E D : : E D : E B. But, $1{\cdot}7320 : 1 :: 1 : {\cdot}5773$, &c. This may be taken as a proof, that A E : E D : : E D : E B; but is certainly not a demonstration with arithmetical exactness.

Again: Let $AB = \sqrt{20}$. Then: $\frac{3}{4}(AB) = \frac{3}{4}(\sqrt{20}) = \sqrt{\frac{3^2}{4^2} \times 20} = \sqrt{\frac{9}{16} \times 20} = \sqrt{{\cdot}5625 = 20} = \sqrt{11{\cdot}25} = AE$. $\frac{1}{4}(AB) = \frac{1}{4}(\sqrt{20}) = \sqrt{\frac{1^2}{4^2} \times 20} = \sqrt{\frac{1}{16} \times 20} = \sqrt{{\cdot}0625 \times 20} = \sqrt{1{\cdot}25} = EB$; and by Euclid: Prop. 35: Book 3: $(AE \times EB) = ED^2$; therefore, $\{\sqrt{11{\cdot}25} \times \sqrt{1{\cdot}25}\} = \sqrt{(\sqrt{11{\cdot}25 \times 1{\cdot}25})} = \sqrt{\sqrt{14{\cdot}0625)}} = \sqrt{3{\cdot}75} = ED$: and AE : ED : : E D : E B; that is, $\sqrt{11{\cdot}25} : \sqrt{3{\cdot}75} :: \sqrt{3{\cdot}75} : \sqrt{1{\cdot}25}$.

In my Letter of the 30th November, 1868, to the Rev. Professor Whitworth, I have given a demonstration of "*The Quadrature of the Circle*," by means of quadrilaterals. (*See* "*Geometry of the Circle*," page 41.)

In the geometrical figure in the margin (Fig. 9), let A C be a straight line, bisected at O. With O as centre and OA or OC as radius, describe the circle. It is self-evident that A C is a diameter of the circle. From the extremities of A C draw straight lines to meet the circumference of the circle at the points B and D, and so construct the quadrilateral A B C D, of which the sides are unequal.

FIG. 9.



Now, by Euclid : Prop. 31 : Book 3 :

A B C and A D C are right-angled triangles: and it is self-evident that A C is the hypothenuse of, and common to, the two triangles. By Euclid : Prop. 47 : Book 1 : $(A\,B^2 + B\,C^2) = (A\,D^2 + D\,C^2)$: and this equation $= A\,C^2$. It is self-evident that if we join the angles B and D in the quadrilateral A B C D, that B D will not be perpendicular to A C. So much for pure Geometry.

THEOREM.

Prove that the sum of the squares of the four sides of the quadrilateral A B C D is equal to the area of an inscribed square to a circle, of which A C is the radius.

How are we to set about the solution of this theorem? We neither know the radius of the circle, nor the sides of the quadrilateral; it is obvious, therefore, that we can prove nothing Arithmetically with reference to this figure, without the aid of Mathematics.

What do we know with reference to this geometrical figure? *We know* that the sum of the squares of A B and B C = the sum of the squares of A D and D C : and both $= A\,C^2$. It is self-evident,

that A B is longer than B C, and A D longer than D C. Now, let A C the diameter of the circle = 8. Then: $AC^2 = 8^2 = 64 = AC^2$ = area of a circumscribing square to the circle: and *we know* that if we construct an inscribed square to the circle, the area of this square will = $\frac{64}{2} = 32$; and it follows, that the sides of an inscribed square = $\sqrt{32}$; therefore, A B and A D must be greater than $\sqrt{32}$, and B C and D C must be less than $\sqrt{32}$.

Then: By hypothesis, let A B = $\sqrt{36}$: B C = $\sqrt{28}$: A D = $\sqrt{34}$: and D C = $\sqrt{30}$. Then : The sum of the squares of the four sides of the quadrilateral A B C D = $(36 + 28 + 34 + 30) = 128$: and is equal to the area of an inscribed square to a circle of which A C is the radius. Q.E.D.

If you dispute my solution of the theorem, pray let me have yours? Hence: If from the extremities of the diameter of a circle, straight lines be drawn to points in the circumference on each side of the diameter, the sum of the squares of the four sides of the quadrilateral so constructed, is equal to the area of an inscribed square to a circle, of which the diameter of the given circle is the radius.

September 21st.

According to the "*usual courtesies of good society*," Mr. R. F. Glaister should have acknowledged the receipt of the pamphlet I sent him on the 17th instant. He has not done so, and I conclude that his "*very comical production*," was intended to be ironical. The following is a copy of a Letter I have just addressed to him :—

BARKELEY HOUSE, SEAFORTH,
21st September, 1869.

SIR,

Doubtless you consider yourself a "*man of honour*" and a "*gentleman;*" and as such, ought to know the usual "*conventions among gentlemen.*" According to the "*conventions among gentlemen*," I should have had your acknowledgement of the receipt of the pamphlet I sent you on the 17th instant. If I were compelled to offer an opinion, as to whether Whitworth, Wilson, or Glaister,

is the greater "*man of honour*" and the more perfect "*gentleman*," I should be in a fix; but I think *you* would do well, if, for the future, you seek for "*pleasure*" in "*burlesque theatres*," such as "The Strand," "Gaiety," "Globe," and "Charing Cross," which it appears to me, would be more congenial to your taste and scientific knowledge, than troubling yourself about "*The Quadrature of the Circle.*"

I am, Sir,
Yours truly,
JAMES SMITH.

R. F. GLAISTER, ESQ.

P.S.—When I next publish, I shall give a fac-simile of your "*very comical production.*" (See, Appendix B).

This morning's post brought me a Letter, of which the following is a copy:—

LANGLEY HOUSE,
GROVE LANE, CAMBERWELL,
September 20th, 1869.

DEAR SIR,

Just a year ago, as you may perhaps remember, I wrote asking you a question relative to your pamphlet, "*Euclid at Fault.*" You kindly replied, and, at the same time, were so good as to send me a copy of your pamphlet, "*British Association in Jeopardy.*"

Now, towards the end of your Letter, you referred to your belief in π being 3·125, and quoted from a communication which had been sent you, which particularly praises your argument on page 14 of the "*British Association in Jeopardy.*"

I have read that page, and, as I have discovered what I conceive to be a fatal error in the argument there, I now write to point it out to you.

You take two angles, namely 90° and $\frac{360°}{25}$, and (at the bottom of page 14) imply, that the circular measure of 90° divided by the

circular measure of $\frac{360^\circ}{25}$ ought to equal 2π. You make the division and find the result to be 6·25, and hence you infer that $\pi = \frac{6{\cdot}25}{2}$ $= 3{\cdot}125$.

Now, your error lies in supposing that that quotient should equal 2π. For, on examination, you will find that it is altogether independent of π. In short, the reason why it comes out 6·25 is simply because you have selected two angles, one of which is 6·25 times the other. 90° is 6·25 times $\frac{360^\circ}{25}$. That is why

$$\frac{\text{The circular measure of } 90^\circ}{\text{The circular measure of } \frac{360^\circ}{25}} = 6{\cdot}25.$$

To make this still clearer if possible, $\frac{\text{The circular measure of } 90^\circ}{\text{The circular measure of } \frac{360^\circ}{25}}$

$$= \frac{90 \times \frac{\pi}{180}}{\frac{360}{25} \times \frac{\pi}{180}} = \frac{90}{\frac{360}{25}} = 6{\cdot}25.$$

I remain, my dear Sir,

Faithfully yours,

THOMAS S. BARRETT.

JAMES SMITH, ESQ.

The following is a copy of my reply:—

BARKELEY HOUSE, SEAFORTH,
21st September, 1869.

DEAR SIR,

You are wrong in supposing there is any flaw in the argument to which you refer; and I may tell you that, $4\left(\frac{\pi}{2}\right)^2 = \pi^2$, whatever be the value of π.

The perimeter of every regular hexagon, is equal to six times the radius of its circumscribing circle. Hence:

$$\frac{\text{The perimeter of every regular hexagon}}{\text{The circumference of its circumscribing circle,}}$$

is a constant quantity = ·96 ; therefore, $\frac{\cdot 96}{6} = \cdot 16$ is the radius of a circle of which the circumference is 1, our unit of length being 1 ; therefore, $\pi(\cdot 16^2) = 3\cdot 125 \times \cdot 0256 = \cdot 08$ = area of a circle of which the circumference = 1.

Can you, my dear Sir, find the area of a circle, of which the circumference = 1 ?

I am, dear Sir,
Faithfully yours,
JAMES SMITH.

THOMAS S. BARRETT, ESQ.

24th September.

Can you, my dear Sir, find the area of a circle, of which the circumference = 1 ? You will remember, that in our former correspondence, I extracted from you the admission, that a circle has a diameter, circumference, and area ; just as certainly as a square has four sides, a perimeter, and area : and it follows, that *if Mathematics be not at fault*, the following theorem can be solved :—

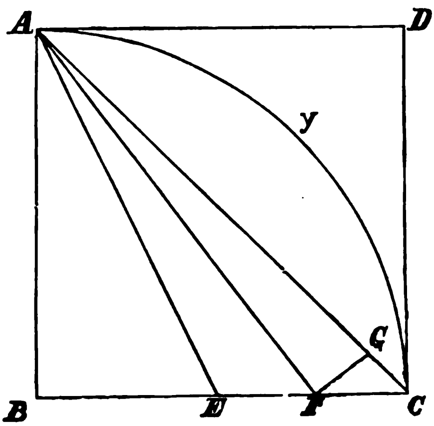
THEOREM.

Circumference of a circle = 1. Find the area of the circle.

Will you kindly favor me with the solution of this theorem ?

In the geometrical figure in the margin (Fig. 10), let A B be a straight line, and A B C D a square described on that line. Bisect B C at E, and bisect E C at F. Join A E, A F, and A C. From the angle F in the triangle A F C, draw the line F G perpendicular to A C. With B as centre and B A or B C as interval, describe the arc A y C.

FIG. 10.



Now, it is self-evident, that A B F and A G F are right-angled triangles, by construction ; and it is equally self-evident that A F is the hypothenuse of, and common to, both triangles. Is it not self-evident that the line A C divides the square A B C D into two equal parts ? Does it not follow, that A C is a diagonal of the square A B C D ? Is it not self-evident, that the perimeter of the square A B C D = four times A B ? Is it not self-evident, that the sides that include the right angle in the triangle A B E, are in the ratio of 4 to 2 ? Is it not self-evident, that the sides that include the right angle in the triangle A B F are in the ratio of 4 to 3 ? Is it not self-evident, that the sides that contain the right angle in the triangle A B C are equal ? Does it not follow, that A B is to B C in the ratio of 4 to 4 ? Might we not express this ratio as 1 to 1, just as well as 4 to 4 ?

Upon the geometrical figure represented by the diagram (Fig. 10), I found the following theorem :—

THEOREM.

Find the ratio between A G and G F, the sides that contain the right angle in the triangle A G F, and prove that they are in the ratio of 7 to 1.

I have shewn you how to solve this theorem by Logarithms, but you avoid Logarithms as "*needless*," and it follows, that so far as you are concerned, it would be a piece of folly on my part to give you the solution of this theorem by Logarithms, and I will solve it for you without Logarithms.

Now, the definite arithmetical expressions 1 and $\sqrt{1}$ have the same arithmetical value. Hence, twice $1 = 2(\sqrt{1}) = \sqrt{2^2 \times 1} = \sqrt{4 \times 1} = \sqrt{4} = 2$. You may wonder what this has to do with the solution of the theorem. I will shew you that it has much to do with it.

It is self-evident, that we must put some arithmetical value on the line A F, the common hypothenuse of the triangles A G F and A B F, as our first step in the solution of this theorem. Well, then, let A F be represented by any finite arithmetical quantity, say 70. Then : $70^2 = 4900 = \text{A F}^2$. $\sqrt{(\frac{4900}{50})} = \sqrt{98} = \text{G F}$: and 7 (G F)

$= 7(\sqrt{98}) = \sqrt{(7^2 \times 98)} = \sqrt{(49 \times 98)} = \sqrt{4802} =$ A G ; therefore, $(AG^2 + GF^2) = (\sqrt{4802^2} + \sqrt{98^2}) = (4802 + 98) = 4900$ $= AF^2$; therefore, $\sqrt{4900} = 70 = AF$. Now, *we know* that A B and B F, the sides that include the right angle in the triangle A B F, are in the ratio of 4 to 3, by construction ; and it follows, that $\frac{4}{5}(AF)$ $= \frac{4}{5}(70) = \frac{4 \times 70}{5} = \frac{280}{5} = 56 = AB$: and $\frac{3}{5}(AF) = \frac{3}{5}(70) =$ $\frac{3 \times 70}{5} = \frac{210}{5} = 42 = BF$; and it cannot be disputed that A B F is a right-angled triangle ; therefore, $(AB^2 + BF^2) = (56^2 + 42^2)$ $= (3136 + 1764) = 4900 = AF^2$; therefore, $\sqrt{4900} = 70$, and is exactly equal to the given value of A F, and demonstrates that A G and G F are in the ratio of 7 to 1. Q.E.D.

For the benefit of Mathematicians of a future generation, I will now give a solution of the theorem by Logarithms, without any reference to Logarithmic tables, except as regards the Logarithms of numbers, the value of which I have admitted. I shall employ Hutton's Tables of the Logarithms of numbers, the correctness of which I have never disputed.

Now, when $AF = 70$, then, $AF^2 = 4900$: $GF^2 = 98$: and AG^2 $= 4802$.

THEOREM.

Let AF, the side subtending the right angle in the triangle AGF, be 4900 miles in length, and be given to find the lengths of the sides A G and G F, the sides that contain the right angle, and prove that they are in the ratio of 7 to 1.

I have proved, that when $AF^2 = 4900$, then, $AG^2 = 4802$, and $GF^2 = 98$; therefore, $AG = \sqrt{4802}$, and $GF = \sqrt{98}$. Now, 4900 is a square number; therefore, $AF^2 = \sqrt{4900} = 70 = AF$. But, 4802 and 98 are not square numbers, and it follows, that A G and G F are incommensurable. But, $\sqrt{4802} = 69{\cdot}29646\ldots = AG$: and $\sqrt{98} = 9{\cdot}899494\ldots = GF$; therefore, 7 times $GF = 7 \times 9{\cdot}899494$ $= 69{\cdot}296458 = AG$ approximately, and, you will observe, agrees to the fourth place of decimals.

Now, $\frac{AG}{AF} = \frac{69{\cdot}29646}{70} = {\cdot}9899494\ldots$, and ·9899494 is the tri-

gonometrical sine of the angle A F G, and is equal to the square root of $\frac{98}{100} = \sqrt{\cdot 98}$. $\frac{GF}{AF} = \frac{9\cdot 899494}{70} = \cdot 1414213\ldots$, and ·1414213 is the trigonometrical sine of the angle F A G, and is equal to the square root of $\frac{2}{100} = \sqrt{\cdot 02}$; and, $\sqrt{\cdot 98}^2 + \sqrt{\cdot 02}^2 = \cdot 98 + \cdot 02 = 1 =$ unity, and meets the requirement of the trigonometrical axiom sin.^{2} + cos.^{2} = unity, in every right-angled triangle.

Well, then, the Logarithm corresponding to the natural number ·9899494 is 9·9956129; and 9·9956129 is the log.-sine of the angle A F G. The Logarithm corresponding to the natural number ·1414213 is 9·1505148; and 9·1505148 is the log.-sine of the angle F A G.

Then :

As Sin. of angle G = Sin. of 90°	Log.	10·0000000
: the given side A F = 4900 miles	Log.	3·6901961
: : Sin. of angle A F G	Log.	9·9956129
		13·6858090
: the required side A G		10·0000000
= ·9899494 × 4900 = 4850·75206 miles	Log.	3·6858090

Again :

As Sin. of angle G = Sin. 90°	Log.	10·0000000
: the given side A F = 4900 miles	Log.	3·6901961
: : Sin. of angle F A G	Log.	9·1505148
		12·8407109
: the required side G F		10·0000000
= ·1414213 × 4900 = 692·96437 miles	Log.	2·8407109

The Logarithm of 692·96437 is 2·8407109 exactly; but 7 (G F) = 7 × 692·96437 = 4850·75059, and is slightly less than the side A G, as ascertained by Logarithms. Now, we know that Logarithms are inexact, but by means of them we find the length of A G, from a given length of the side A F, correct to the second place of decimals.

Now, let G F = $\sqrt{\cdot 02}$. Then: 7 (G F) = 7 ($\sqrt{\cdot 02}$) = $\sqrt{(7^2 \times \cdot 02)}$ = $\sqrt{(49 \times \cdot 02)}$ = $\sqrt{\cdot 98}$ = A G; therefore, (G F^2 + A G^2) = ($\sqrt{\cdot 02^2}$ + $\sqrt{\cdot 98^2}$) = (·02 + ·98) = 1 = A F^2; therefore, A F = $\sqrt{1}$. $\frac{4}{5}$ (A F) = $\frac{4 \times 1}{5} = \frac{4}{5}$ = ·8 = A B: $\frac{3}{5}$ (A F) = $\frac{3 \times 1}{5} = \frac{3}{5}$ = ·6 = B F: and B F = $\frac{3}{4}$ (A B), by construction; therefore, $\frac{3}{4}$ (A B) = $\frac{3 \times \cdot 8}{4} = \frac{2 \cdot 4}{4}$ = ·6 = B F. But, A B F is a right-angled triangle, by construction; and the sides A B and B F contain the right angle; therefore, (A B^2 + B F^2) =) ·8^2 + ·6^2) = (·64 + ·36) = 1 = A B^2; therefore, $\sqrt{1}$ = 1 = A B.

From a given length of G F, we can find the length of A F, whatever value we may put on G F; but, by no other value of G F but $\sqrt{\cdot 02}$ can we make A F = 1.

This morning's post brought me a Letter, of which the following is a copy :—

LANGLEY HOUSE,
GROVE LANE, CAMBERWELL,
LONDON, S.E.

MY DEAR SIR,

I am at a bewilderment to comprehend the connection between my Letter and your reply. It seems like "*cross questions, and crooked answers.*" I must have failed in expressing myself clearly. I will therefore try again.

On page 14 of your "*British Association in Jeopardy,*" there occurs the following argument that π = 3·125 :—

$$\frac{\text{Circular measure of } 90^\circ}{\text{Circular measure of } \frac{360^\circ}{25}} = \text{(you say) } 2\,\pi.$$ It also = 6·25;

therefore, (you argue) 6·25 = 2 π : and π = 3·125.

Now, my answer is this : The reason is logical ; but unfortunately, it is based on an unproved premiss, and consequently, the conclusion that π = 3·125 is unproved. The unproved premiss is,

$$\frac{\text{Circular measure of } 90^\circ}{\text{Circular measure of } \frac{360^\circ}{25}} = 2\,\pi.$$

You assume this without proof; consequently the argument, as it stands, is merely a *petitio principii.*

I remain, dear Sir,
Faithfully yours,
THOMAS S. BARRETT.

The following is a copy of a Letter I have posted to Mr. Barrett in reply :—

BARKELEY HOUSE, SEAFORTH,
24th September, 1869.

MY DEAR SIR,

Your Letter, without date, but bearing the London post-mark of yesterday, is to hand.

To prove that the argument to which you refer is a "*petitio principii,*" you must prove that 2π (radius) is not the circumference of a circle, when radius $= 1$. Pray furnish me with the proof. Is not $\frac{2\pi}{4}$ the circular measure of a right angle? Surely, my dear Sir, if you are right, you can tell me the area of a circle, when the circumference $= 1$.

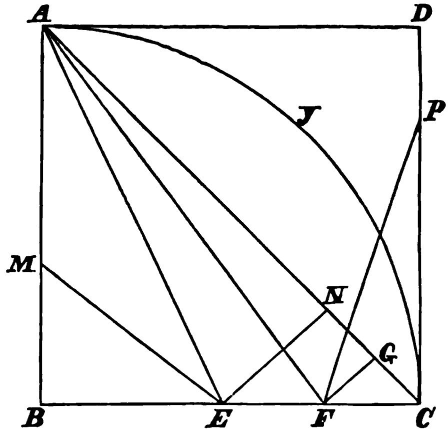
Did you attend the late meeting of the "British Association" at Exeter? If so, were you one of the party in the excursion to Gittisham Hill?

I remain, dear Sir,
Faithfully yours,
JAMES SMITH.

THOMAS S. BARRETT, Esq.

The geometrical figure (Fig 11), is a fac-simile of Fig. 10, with the following additions: From A B cut off a part B M, making B M $= \frac{3}{8}$ (A B), and join M E. From the angle E, in the triangle A E C, draw a straight line perpendicular to, and meeting its opposide side A C, at the point N. From D C cut off a part P C, equal to $\frac{3}{4}$ (D C), and join P F.

FIG. 11.



It is self-evident, that E N and F G are parallel lines ; but it is equally self-evident, that if, from the angle E in the triangle A E F, we draw a straight line perpendicular to its opposite side A F, this line will not be parallel either to E N or F G.

Now, my dear Sir, you make the angle B A F in the triangle A B F, to be an angle of $36^\circ\ 52' + x$; and I have shewn you, in my Letter of the 2nd inst., that Hutton makes B A F to be an angle of $36^\circ\ 52' - x$. You and Hutton cannot both be right, and you may be both wrong. By your method of applying Tables, Hutton is right. By my method of applying Tables, you are both wrong. It cannot be denied, that both you and Hutton make B A F an angle of $36^\circ\ 52'$ nearly, though that is not exactly its value according to either of you. I shall proceed to prove that B A F is an angle of 36° $52'$, and can have no other value when expressed in degrees and minutes.

Euclid proves (Prop. 13, Book 1), that the angle A E B and the angle A E F are together equal to two right angles; and that the angle A F B and the angle A F C are together equal to two right angles. But A E B and A F B are acute angles, and A E F and A F C are obtuse angles ; and it follows, that A E B is less, and A E F greater than a right angle, and A F B less, and A F C greater than a right angle.

But, because A B C is an isosceles triangle, it follows, that the angles B A C and B C A are equal; and, because the triangle A B C is right angled at B, and the angle B A C and B C A equal; it follows, that the angles B A C and B C A are each equal to half a right angle. It also follows, that the angles B A E and B A F are less than half a right angle, and the angles A E B and A F B greater than half a right angle, and the angles B A E, E A F, and F A C are together equal to half a right angle. But, Euclid nowhere gives the values of these angles *mathematically*—that is, expressed in degrees—nor could he, without travelling out of the domain of pure Geometry.

We know, that for practical purposes, we must *assume* a right angle to have some value. To say that a right angle is a right angle, means nothing, and can be "*no help to finding*" anything. Well, then, for practical purposes, Mathematicians, whether "*recognised*" or UN-recognised, *assume* a right angle to equal 90°; and it follows, that four right angles = 360°, and represents the circumference of a circle, when expressed in degrees. Can you require me to prove this conclusion? I think not. Well, then, I do not think that Mathematicians could have assumed anything more convenient than 90° to represent the value of a right angle; since 90 will divide by all single numbers but 7, without a remainder, and even when divided by 7, the quotient can be expressed fractionally, if not decimally.

Having adopted 90° as the measure of a right angle, we can now demonstrate that in every right-angled triangle, the three interior angles are together equal to two right angles.

Now, A G F is a right-angled triangle; and *we know* that the angle F A G is less than half a right angle. By hypothesis, let the angle F A G be an angle of 8° 8′. Then: *we know*, that 90° — 8° 8′ = 81° 52′ = the angle A F G, and G is a right angle; therefore, the three angles are together equal to two right angles. But, A B F is a right-angled triangle, and because B A C is half a right angle, B A F is less than half a right angle; and if F A G be an angle of 8° 8′, then, 45° — 8° 8′ = 36° 52′ = the angle B A F, and B is a right angle; therefore, the three interior angles of the triangle A B F, are together equal to two right angles. It is obvious, that

I might have assumed the angle F A G to be either greater or less than 8° 8′ for the proof that the three interior angles of a right-angled triangle are together equal to two right angles: and I *admit*, that this is no proof that B A F is an angle of 36° 52′, and FAG an angle of 8° 8′.

Euclid proves, (Prop. 13: Book 1), that A F B and A F C are together equal to two right angles. But, F G is perpendicular to A C, by construction; and it follows, that A G F and F G C are right angles; therefore, the angles A G F, F G C, A F G, and G F C, are together equal to four right angles. E N C and F G C are similar isosceles triangles to the isosceles triangles A B C and A D C, by construction. Hence: Straight lines drawn from any point in the line B C, perpendicular to, and meeting in, the line A C—a diagonal of the square A B C D—will produce a right angled isosceles triangle, similar to the right-angled isosceles triangles A B C and A D C.

Well, then, you make the angle B A F an angle of 36° 52′ + x. Hutton makes it 36° 52′ — x. I make it 36° 52′ exactly. I make the angle F A G an angle of 8° 8′, and its trigonometrical sine $\sqrt{\cdot 02} = \cdot 1414213$: Hutton gives the sine of an angle of 8° 8′ as ·1414772. I have proved, that the sides that include the right angle, in the triangle A G F, are in the ratio of 7 to 1. Hence: If $AG = 7$, and $GF = 1$: by Euclid: Prop. 47: Book 1: $(AG^2 + GF^2) = (7^2 + 1^2) = (49 + 1) = 50 = AF^2$; therefore, $AF = \sqrt{50}$; therefore, $\sqrt{\frac{50}{100}} = \sqrt{\frac{1}{2}} = \sqrt{\cdot 5} = \cdot 7071068$: and ·7071068 is the trigonometrical sine of an angle of 45°. Now, $\frac{GF}{AF} = \frac{1}{\sqrt{50}} = \frac{1}{7\cdot 071068} = \cdot 1414213$ = the trigonometrical sine of the angle FAG, true to 7 places of decimals.

Hence:

The angle F A G is an angle of 8° 8′;

∴ 90° — 8° 8′ = 81° 52′ = the angle A F G.

But, G F C is an angle of 45°;

∴ Angle A F G + angle G F C = 81° 52′ + 45° = 126° 52′ = angle A F C.

But, angle A F B + angle A F C = 2 right angles;

∴ 180° — 126° 52′ = 53° 8′ = angle A F B.

But, A B F is a right-angled triangle, and B is the right angle;
$\therefore 90^\circ - 53^\circ 8' = 36^\circ 52' =$ angle B A F.
$\therefore$ Angle B A F + angle F A G $= 36^\circ 52' + 8^\circ 8' = 45^\circ =$ angle B A C.

Again:
The angle B A F is an angle of $36^\circ 52'$;
$\therefore 90^\circ - 36^\circ 52' = 53^\circ 8' =$ the angle A F B.
But, angle A F B + angle A F C = 2 right angles;
$\therefore 180^\circ - 53^\circ 8' = 126^\circ 52' =$ the angle A F C.
But, G F C is an angle of 45°;
$\therefore 126^\circ 52' - 45^\circ = 81^\circ 52' =$ the angle A F G.
But, A G F is a right-angled triangle, and G is the right angle;
$\therefore 90^\circ - 81^\circ 52' = 8^\circ 8' =$ the angle F A G;
$\therefore$ the angle F A G + the angle B A F $= 8^\circ 8' + 36^\circ 52' = 45^\circ =$ the angles B A C and B C A.

Now, if you are right, F A G will be an angle of $8^\circ 7' + x$: and if Hutton is right, the angle F A G $= 8^\circ 8' + x$. Take either as the value of the angle F A G, and find the value of the angle A F G, and prove that the angle A F G + half a right angle = the angle A F C.

I may tell you that the angle F P C in the right-angled triangle P C F = half the angle B A F, and its trigonometrical sine is $\frac{1}{\sqrt{10}}$ = ·3162277, and not ·3162010 as Hutton gives it. I proved this in several ways, in our former correspondence. You may remember, that you admitted ·3162277 to be the sine of the angle F P C, but denied that it was an angle of $18^\circ 26'$.

I must now refer you to the geometrical figure in my Letter of the 21st July, (page 3), and ask you to compare it with the geometrical figure represented by the diagram, Fig. 11 (page 175). You will find that the triangle A B C in the former, is similar to the triangle A B F in the latter: and the triangle A B D in the former, similar to the triangle P C F in the latter; but, the two figures are very widely different.

You might put the question:—What has this to do with the ratio of diameter to circumference in a circle? As I intend this to be my last controversial communication, I shall, on the hypothesis of your putting such a question, proceed to answer it.

It cannot be denied, that the quadrant AyC B, and an inscribed circle to the square A B C D, are equal in superficial area: neither can it be denied, that the radius of an inscribed circle to the square A B C D = E B and E C: and I am sure you will admit, that $\frac{2\pi}{4}$ = the circular measure of a right angle, whatever be the value of π.

Now, B is a right angle; therefore, $\frac{2\pi}{4}$ = the arc Ay C, when A B and B C = 1, our unit of length being 1; and it follows, that $\frac{4\pi}{2}$ is the circular measure of the arc AyC, when A B and B C = 4. E B = E C = $\frac{1}{2}$(B C) = 2, when A B and B C = 4: and E B M is a right-angled triangle, of which the sides E B and B M, that include the right angle, are in the ratio of 4 to 3, by construction; therefore, $\frac{3}{4}$ (E B) = $\frac{3 \times 2}{4} = \frac{6}{4}$ = 1·5 = B M; and by Euclid: Prop. 47: Book 1: (E B^2 + B M^2) = (2^2 + 1·5^2) = (4 + 2·25) = 6·25 = E M^2; and it follows, that the sum of the squares of the three sides of the triangle E B M = $\frac{6\cdot25}{2}$ (E B^2); that is, (4 + 2·25 + 6·25) = 3·125 × 4; and this equation = 12·5 = 2 (E M^2).

The arc Ay C is equal to one-fourth part of the circumference of a circle, of which A B and B C are radii; and it is self-evident, that E C which is equal to E B, by construction, must be equal to a semi-radius of such circle: and I am sure you will admit, that circumference × semi-radius = π (r^2), and that this equation = area in every circle.

Now, when A B and B C = the MYSTIC number 4, then, E B and E C = 2: and 4 (E M^2) = 25. Hence: 4 (E M^2) × E B = 25 × 2 = 50; and it follows, that 4 (E M^2)] × E B = 3$\frac{1}{8}$ (E B + E C)^2; that is, 25 × 2 = 3·125 × 16 = 50 = 3$\frac{1}{8}$ (B C^2). Now, with B as centre and B E as radius, describe a circle. It is axiomatic, if not self-evident, that this circle, and the quadrant Ay C B, are exactly equal in superficial area. But, E M^2 × E B = 3$\frac{1}{8}$ (E B^2), that is 6·25 × 2 = 3·125 × 4; and this equation = $\frac{50}{4}$ = 12·5; and it follows, not as an *assumption*, but as a *logical deduction*, that the arithmetical value of

the symbol π, is $\frac{25}{8} = 3{\cdot}125$, making 8 circumferences = 25 diameters in every circle; and 8 to 25, or, 1 to 3·125, the ratio between diameter and circumference in every circle. Hence: when AB = 4, then, 6·25 (AB) = 25 = circumference of a circle of which AB and BC are radii: and E C = 2, is the semi-radius of such circle: and circumference × semi-radius = area; that is, $25 \times 2 = 50$ = area = $3\frac{1}{8}$ times BC^2; and it follows, that, $\frac{50}{4} = 12{\cdot}5$ = area of the quadrant A*y* C B: and $\frac{12{\cdot}5}{2} = 6{\cdot}25$ = the arc A*y* C. Is not the area of a circle of radius 4 equal to twice the circumference, whatever be the value of π?

27th September.

I have received another Letter from Mr. Barrett, of which the following is a copy :—

LANGLEY HOUSE, GROVE LANE,
CAMBERWELL, LONDON, S.E.,
September 25th, 1869.

MY DEAR SIR,

You say (in your "British Association in Jeopardy") that

$$\frac{\text{Circular measure of } 90^\circ}{\text{Circular measure of } \frac{360^\circ}{25}} = 2\pi: \text{ and thence—}$$

$$\left(\text{As } \frac{\text{circular measure of } 90^\circ}{\text{Circular measure of } \frac{360^\circ}{25}} = 6{\cdot}25\right), \text{deduce that } \pi = 3{\cdot}125.$$

Now, what I say is, that, as you do not furnish a proof of your starting point, viz., that $\frac{\text{Circular measure of } 90^\circ}{\text{Circular measure of } \frac{360^\circ}{25}} = 2\pi$, the argument is thereby rendered defective and inconclusive.

I remain, dear Sir,
Yours faithfully,
T. S. BARRETT.

I have replied to this communication, and the following is a copy of my reply —

BARKELEY HOUSE, SEAFORTH,
27th September, 1869.

SIR,

Mathematicians "*arbitrarily*" divide the circumference of a circle into 360 equal parts, called degrees. Is not this an "*unproved premiss?*" Might not Mathematicians as well divide the circumference of a circle into any other number of equal parts, say 400, and call these parts by some other name, say *grades?* But, would not this be an "*unproved premiss?*" Mathematicians adopt as a "*starting point*," in their search after π, a circle of radius 1, adopting 1 as the unit of length. Why not make radius 2, or 4, or 6, or any other single number? What is 1? Is it not a mere symbol? Can you prove it to be anything else? Would not *one* called by any other name, "*smell as sweet?*" One is one, and that appears to be all you know about it. Would not any little boy tell you, that he would rather have two apples, or two oranges, than one? Why? Because he knows that twice one make two, although he may not have learned his multiplication table. Is it more certain that twice one make two, than that $\frac{2\pi}{4}$ = the circular measure of a right angle? Is not 90° the measure of a right angle, according to Mathematicians? Is not this an "*unproved premiss?*" Mark!! I find no fault with Mathematicians for "arbitrarily" *assuming* 90° as the measure of a right angle. They must *assume* a right angle to be represented by something, and I do not think they could have hit upon a better *assumption* than 90°.

The perimeter of a regular inscribed hexagon to any circle, is equal to three times the diameter, or six times the radius: and the symbol π denotes the circumference of a circle of diameter unity, the unit of length being 1; and it follows, that $\frac{\pi}{3}$ expresses the ratio between the circumference of every circle, and the perimeter of its inscribed regular hexagon. Can you prove that the number 3 does

not represent the perimeter of a regular inscribed hexagon to a circle of diameter unity? Now, the property of one circle, is the property of all circles; and it follows, that three times diameter is equal to the perimeter of an inscribed regular hexagon, in every circle.

Hence:

$$\frac{\text{The perimeter of every regular hexagon}}{\text{The circumference of its circumscribing circle}} \text{ is a constant}$$

quantity = ·96; therefore, $\frac{\cdot 96}{6}$ = ·16 = radius of a circle of circumference = 1: and the measure of the difference between the circumference of any circle, and the perimeter of its inscribed regular hexagon, is exactly equal to $\frac{1}{25}$th part of the circumference of the circle.

Can *you* tell the measure of the difference between the perimeter of a regular hexagon, and the circumference of its circumscribing circle?

I cannot give men intellect, and do not pretend to make any man "*a reasoning geometrical investigator.*"

Your Letter, as a reply to mine of the 24th inst., appears to me "*impertinent,*" and you must not expect me to reply to "*impertinent*" communications.

I remain,

Faithfully yours,

J S.

THOMAS S. BARRETT, Esq.

When I wrote the Letters which appear in the "*British Association in Jeopardy,*" I did not know so much of the existing race of "*recognised Mathematicians*" as I now know of them. At that period, though I thought them mistaken as to the arithmetical value of π, I gave them credit for intellectual capacity, and as a class, looked upon them as above chicanery and sophistry. On these points, I now find that I was mistaken. Surely, for a man to be a Mathematician, it is not necessary that he must be "*recognised*" as such by the whole world! Well, then, there is no flaw in the *argument* on page 14 of the "*British Association in Jeopardy,*" whatever Mr. Barrett may say; and the *argument* has been convincing to several

Mathematicians. Had I to write page 14 of that work now, I could certainly improve it.

Euclid shews us how to construct an equilateral and equiangular pentagon in a given circle. (Prop. 11: Book 4.) Now, it is conceivable—even if we suppose it to be impracticable by pure Geometry—that a regular polygon of 25 sides may be inscribed in a circle, and straight lines drawn from the angles of the polygon to the centre of the circle. Well, then, since Mathematicians adopt 360° to denote the circumference of a circle; it follows, that $\frac{360°}{25}$ $= 14° \, 24'$ is the value of an angle at the centre of the circle, contained by any two radii. The circular measure of this angle, to a circle of radius unity, is $\frac{14° \, 24' \times \pi}{180} = \frac{864 \text{ minutes} \times \pi}{180 \times 60}$. The circular measure of a right angle is $\frac{90° \times \pi}{180} = \frac{\pi}{2}$, and is therefore equal to the semi-circumference of a circle of diameter unity; or, one-fourth part of the circumference of a circle of radius unity. Hence, $\frac{2\pi}{4}$ = the arc AyC, when AB = 1; and it follows, that the arc AyC $= 4\left(\frac{2\pi}{4}\right)$, when AB = 4. Now, $\frac{2\pi}{4} \div \frac{8\pi}{100}$ is a constant quantity $= 6{\cdot}25$, whatever value we may put upon π. For example: By hypothesis, let $\pi = 3$. Then: $\frac{2\pi}{4} = \frac{6}{4} = 1{\cdot}5 = \frac{\pi}{2}$. $\frac{8\pi}{100} = \frac{24}{100} = {\cdot}24$; therefore, $\frac{1{\cdot}5}{{\cdot}24} = 6{\cdot}25$. Again: By hypothesis, let $\pi = 3{\cdot}16$. Then: $\frac{2\pi}{4} = \frac{6{\cdot}32}{4} = 1{\cdot}58 = \frac{\pi}{2}$. $\frac{8\pi}{100} = \frac{25{\cdot}28}{100}$ $= {\cdot}2528$; therefore, $\frac{1{\cdot}58}{{\cdot}2528} = 6{\cdot}25$. Well, then, $\frac{2\pi}{4} \div \frac{8\pi}{100}$ is a constant quantity $= 6{\cdot}25$, whatever be the value of π. This is a premiss, and logical reasoning upon it, leads to the incontrovertible conclusion, that $\frac{6{\cdot}25}{2} = 3{\cdot}125$, is the true circumference of a circle of diameter unity.

Now, $4\,(14° \, 24') = 57° \, 36'$, and this is the value of an angle at the centre of a circle, subtended by an arc equal to radius, expressed in degrees, when 360° is adopted as the circumference of a circle. $6\,(57° \, 36') = 345° \, 36'$ is the perimeter of a regular inscribed hexagon to

a circle of circumference $= 360°$: and $\frac{\pi}{3}$ $(345° \; 36') = 360°$, whatever be the value of π. Now, since 6 times radius is equal to the perimeter of a regular inscribed hexagon, in every circle; it follows, that

$$\frac{\text{The perimeter of every regular hexagon}}{\text{The circumference of its circumscribing circle}} \text{ is a constant quantity;}$$

therefore, $\frac{345° \; 36'}{360°} = \frac{20736 \text{ minutes}}{21600 \text{ minutes}} = \cdot 96$, and is equal to $\frac{24}{25}$ (1); and it follows, that $\frac{24}{25}$ (circumference) $=$ the perimeter of a regular inscribed hexagon, in every circle.

Now, let the circumference of a circle $= 1$. Then: $\frac{24}{25}$ $(1) = \cdot 96 =$ the perimeter of a regular inscribed hexagon: $\frac{\cdot 96}{6} = \cdot 16$ radius: $\frac{\cdot 16}{2} = \cdot 08 =$ semi-radius. So far, "*π does not appear, need not appear.*" We can prove, in many ways, that circumference $\times$ semi-radius $= \pi (r^2)$, and that this equation $=$ area in every circle. These facts I take to be admitted, for I have never met with a Mathematician who disputed them. Well, then, it follows, not as an *assumption*, but as a *legitimate and logical deduction*, that $1 \times \cdot 08 = \cdot 08 =$ area of a circle of circumference $= 1$. But, you may say: What has this to do with π? I do not think *you* will dispute, that $\pi (r^2) =$ area in every circle; but this is "*no help to finding π's value,*" *per se.* "*π still lies lurking in his den,*" and it only remains for me to bring him out, and expose him. Well, then, the radius of a circle of circumference unity $= \cdot 16$: and $r^2 = \cdot 16^2 = \cdot 0256 =$ area of a square on the radius of the circle. *We know,* that the area of the circle $= \cdot 08$; and it follows, not as an *assumption*, but as a *logical and legitimate deduction,* that the arithmetical quantity that will multiply by $\cdot 0256$ and produce $\cdot 08$, *must be the true* value of π. Well, then, $3 \cdot 125 \times \cdot 0256 = \cdot 08 =$ area of a circle of circumference unity, and every other value of π fails. What, then, in the name of common sense, can the value of π be, but $3 \cdot 125$?

Now, $\frac{2\pi}{4} = \frac{6 \cdot 25}{4} = 1.5625 = \frac{\pi}{2}$. $\frac{8\pi}{100} = \frac{25}{100} = \cdot 25 =$ semi-radius of a circle of diameter unity, and $\frac{1 \cdot 5625}{\cdot 25} = 6 \cdot 25 = 2\pi$.

But, $4\pi (s\, r^2) = 12 \cdot 5 \; (\cdot 25^2) = 12 \cdot 5 \times \cdot 0625 = \cdot 78125 = \frac{\pi}{4}$; and it fol-

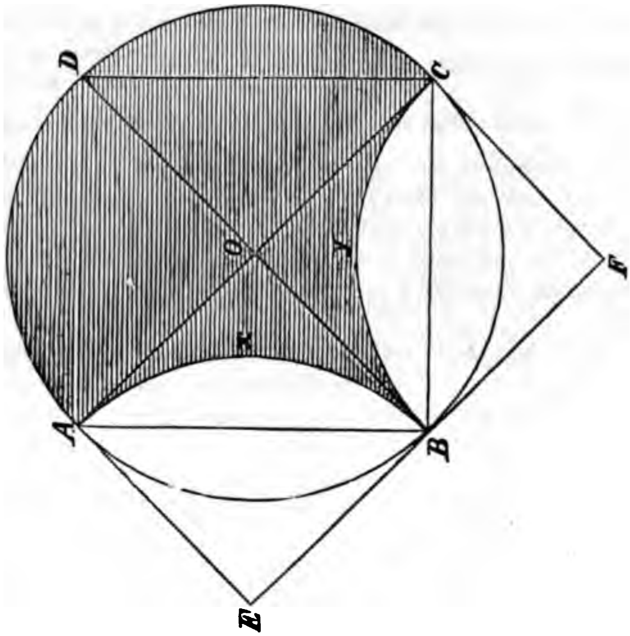
lows, that $12\frac{1}{2}$ times the area of a square on the semi-radius = area in every circle. I have already given you a method of finding the area of a circle from the area of its inscribed square. The area of an inscribed square to a circle of diameter unity = ·5; therefore, $\{(\cdot 5 + \frac{1}{4}\cdot 5) + \frac{1}{4}(\cdot 5 + \frac{1}{4}\cdot 5)\} = (\cdot 625 + \frac{1}{4}\cdot 625) = (\cdot 625 + \cdot 15625) = \cdot 78125 = \frac{\pi}{4}$ = area of a circle of diameter unity.

Well, then, I have proved that $57^\circ\ 36'$ is the value of an angle at the centre of a circle, subtended by an arc equal to radius, when circumference = 360°. Now, $\frac{57^\circ\ 36' \times \pi}{180} = \frac{3456 \text{ minntes} \times 3\cdot 125}{180 \times 60} = \frac{10800}{10800} = 1$ = radius of a circle of which the circumference = 2π. It appears to me that Mr. Barrett does not know where a "*premiss*" ends, and where the reasoning upon it begins, and I have not thought it worth my while to give him the foregoing information, for his individual benefit. If he be a Mathematician of "*intellectual capacity*," I have done quite sufficient to suggest it to him.

This brings me to the close of your Letter of the 30th August, where you observe:—"*But my chief motive in thus troubling you, is to answer the question that concludes your Letter. I have never heard of the Geometrical figure called a pelacoid*," and therefore can know nothing of its properties. In a Postscript to my Letter of the 11th inst., in reply to yours (not controversial) of the 10th, I said:—"I think you will find that "*pelacoid*" should be "*pelecoid*." I employed the word, on the authority of the gentleman who directed my attention to it, at the late meeting of the British Association; not thinking of, or examining, the derivation of the word. You will find that the Greek Lexicographers, Liddell and Scott, give as a secondary meaning of πελευς a "mathematical figure." The Rev. Professor Whitworth, in his Letter of the 3rd September, admits that he never heard of a "*pelacoid;*" and supposes the name to have been designed by some one ignorant of Greek, and applied to some figure "*developed from his own consciousness.*" This is a curious expression, and I cannot make out its meaning. I infer, that a "*pelecoid*," if not its properties, was known to Mathematicians long ago. Mr. Whitworth observed at the close

of his Letter, "*Perhaps you can send me its equation: then I can trace it.*" I sent him its equation, as you know; but whether he succeeded in tracing the figure or not, I cannot tell, for I have not heard from him since. Perhaps we shall hear something about it in the next number of the *Mathematical Journal*, which he edits. I will now give you two methods of constructing a "*pelecoid.*"

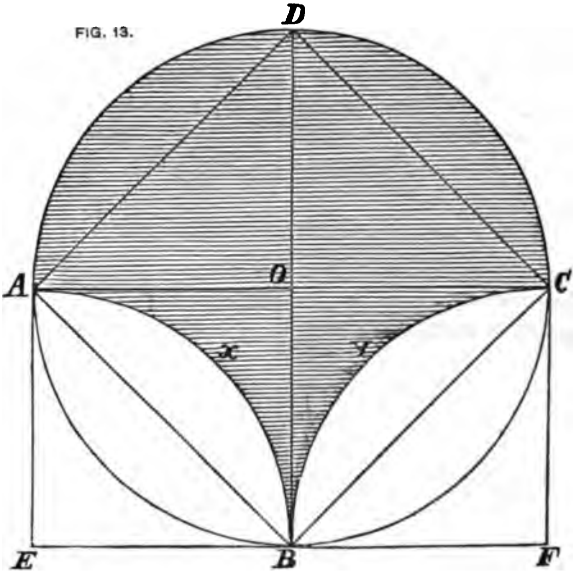
FIG. 12.



On the straight line A B describe the square A B C D, and draw the diagonals A C and D B, intersecting and bisecting each other at the point O. With O as centre and O A as radius, describe the circle. From A and C draw straight lines tangental to the circle, to meet two other straight lines drawn from B tangental to the circle, to meet at the points E and F. With E as centre and E A or E B as interval, describe the arc A x B; and with F as centre and F B or F C as interval, describe the arc B y C. The coloured part of the figure is the "*pelecoid.*"

It is self-evident, that the circle is a circumscribing circle to the square A B C D, and that O B E A and O B F C are squares on O B a radius of the circle. It is also self-evident, that the arcs A x B, B y C, BC, and B A are equal. It follows, that if P denote the area of the "*pelecoid*," and A denote the area of the square A B C D, then, P = A.

FIG. 13.



Draw two straight lines at right angles intersecting and bisecting each other at the point O, and with O as centre and any radius, describe a circle. It is self-evident that A C and D B are diameters of the circle. Join the extremities of these diameters, and so construct the square A B C D. From A draw a straight line tangental to the circle, to meet another straight line drawn from B also tangental to the circle, at the point E: and similarly, from C draw a straight line tangental to the circle, to meet another straight line drawn from B also tangental to the circle, at the point F. Again the coloured part of the figure denotes the "*pelecoid*," and is in the form of an ancient axe or hatchet. In a modern Greek and English Lexicon I have before me (Dunbar), the meaning of the word "*pelecoid*" is given "*like an axe.*"

The *charm* of this geometrical figure is the simplicity of its construction. In conversation with a Mathematician, you can have no difficulty in roughly constructing the figure, and in getting him to admit the construction step by step. Having got a Mathematician to admit the construction, it is amusing—if he attempt to advocate Orthodoxy—to see the state of "bewilderment" and confusion into which he may be thrown by means of it. I saw several instances of this at Exeter. It was an interesting incident to me, at the late meeting of the "*British Association*," to see the way the "*pelecoid*" was handled by the gentleman who drew my attention to it, to prove that *Smith is right*. I had the satisfaction to hear several Mathematicians admit, that *Smith must be right.*

29th September.

This morning's post brought me another communication from Mr. Barrett, of which the following is a copy:—

LANGLEY HOUSE, GROVE LANE,
CAMBERWELL, *28th Sept.*, 1869.

JAMES SMITH, Esq.,

My dear Sir,

Don't be angry with me. The reason I did not reply to your Letter dated 24th, was simply because I thought you must still misunderstand my stricture on your argument in "*The British Association in Jeopardy*," and that therefore the best course to adopt was for me to repeat what I had said, in, if possible, clearer terms; and to refrain from noticing your remarks, which seemed quite wide of the question. I thought this would be the best way of avoiding confusion; and I still think we had better dispose of my objection to your argument in "*The British Association in Jeopardy*," before proceeding to other topics.

But, possibly, you are now aware of the correctness of my stricture on that argument—for, indeed, who could help seeing it when pointed out? I will therefore now reply to your remarks received this morning.

The first question in your Letter, you, yourself, answer in the last sentence of the paragraph—namely, in the middle of your third page. You then ask, why Mathematicians don't make the radius of a circle 2 or 4 or 6? Simply because 1 is the most convenient.

I agree with you that the word "*one*" is only a name—that any other name "*would smell as sweet*" if universally adopted. I am, however, innocently ignorant of what I could have said to make you say "*one is one*," and that appears to be all I "*know about it*."

I agree with you and the "*little boy*" that twice one makes two.

I agree with you that the circular measure of a right angle is $\frac{2\pi}{4}$.

I agree with you that $\frac{\pi}{3}$ expresses the ratio between the circumference of a circle, and the perimeter of its regular inscribed hexagon.

But I do *not* agree with you that $\frac{\text{The perimeter of hexagon}}{\text{The circumference of circle}}$ $=\cdot96$. I make it $\cdot9549\ldots$, consequently, I disagree with the remainder of your paragraph.

I *can* tell you the measure of the difference between the perimeter and the circumference; but you would not agree with my reply, because our views of π differ.

Hoping I have answered everything,

I remain, dear Sir,

Faithfully yours,

THOMAS S. BARRETT.

The following is a copy of my reply:—

BARKELEY HOUSE, SEAFORTH,
29th September, 1869.

MY DEAR SIR,

Admitting it to be proved, that $\pi = 3\cdot14159265$, &c.—the very point in dispute—no doubt $\frac{\text{The perimeter of hexagon}}{\text{The circumference of circle}}$

would be ·9549... as you put it, the dots indicating the &c. Can you divide &c. by 2, and find the quotient? Pray answer this question?

Don't you think our correspondence on the vexed question of π's value had better terminate? Is it not self-evident, that further correspondence between us, would simply be, like playing a game at "*cross questions and crooked answers?*"

You have left two of my questions in previous letters unanswered. What is the area of a circle, when the circumference is 1?

I remain, dear Sir,

Faithfully yours,

J. S.

THOMAS S. BARRETT, Esq.

It appears to me, that Mr. Barrett is a "*pseudomath*," that is, according to De Morgan's interpretation of the word, "*a person who handles Mathematics as the monkey handled the razor*," and so, "cuts his own throat."

My correspondent, "Eminent Mathematician," said: "*If the circumference of a circle be* FINITE, *away goes the diameter into decimals without end: and if the diameter of a circle be* FINITE, *away goes the circumference into decimals without end.*" Mr. Barrett makes it out, that if the circumference of a circle be finite, "away goes" the perimeter of its inscribed regular hexagon into decimals without end. "*This appears to me not only to "bang banagher" but to bang spiritualism itself.*"

Suppose Mr. Barrett to put the following question to the "*little boy*":—

What is the difference between 6 times 1, and 6 times the square root of 1? Would he expect to get the following answer from the "*little boy*."? $6(\sqrt{1}) = \sqrt{6^2 \times 1} = \sqrt{36 \times 1} = \sqrt{36} = 6$, and is just equal to 6 times 1. If the radius of a circle be 1, is not the perimeter of its inscribed regular hexagon equal to 6 times 1 $= 6$?

Mr. Barrett says, he can "tell the measure of difference between the perimeter and the circumference." He means to say

he can tell the measure of difference between the perimeter of a regular hexagon, and the circumference of its circumscribing circle. This I defy either Mr. Barrett or any other Mathematician to do, until π's true value is found. The measure of difference will not be the same if we assume $\pi = 3{\cdot}14159$, as when we assume $\pi = 3{\cdot}14159265$, &c.

Mr. Barrett fails to perceive that all the admissions he makes, are involved in the argument to which he refers on page 14 of "*The British Association in Jeopardy, &c.*"

I shall now bring this long communication to a close, and with it our *controversial* correspondence, in which I think everything has been said that can be said, on both sides. If this epistle fail to convince you, "*I can't help it;*" but I am happy to think that our correspondence will terminate in "*undisturbed good will.*" With best wishes for your health and happiness.

Believe me, my dear Sir,

Most truly yours,

JAMES SMITH.

THE REV. GEO. B. GIBBONS, B.A.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

LANEAST, LAUNCESTON,

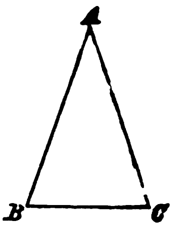
2nd October, 1869.

MY DEAR SIR,

I have just received your very large packet of Papers, dated 7th September and onwards, and though our *controversy* is ended, I feel sure you would like to have them acknowledged.

I would correct one small error of yours. I said I had avoided Logarithms *in finding* the chord B C. When A = 15°, and A C =

B C = 1, B C exceeds ·261, whereas you employ them both for "numbers" and trigonometrical functions.



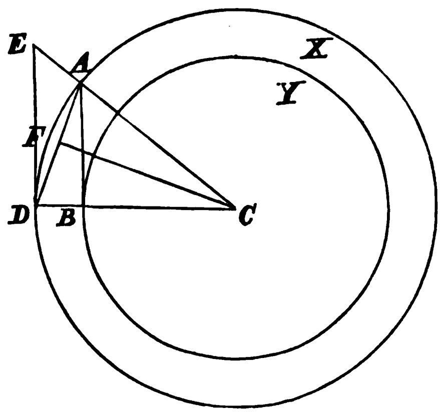
The Letter of Mr. R. F. Glaister is evidently satirical—not half so neatly done as Professor Wilson's—but the plan is bad, however cleverly executed.

I am much obliged by the great trouble you have taken in communicating to me, such full particulars of your other correspondents' Letters, and as we agree cordially on the fitness of ending all debates about π, I have only to reciprocate your good wishes for my health and happiness;

And remain, dear Sir,

Yours truly,

G. B. GIBBONS.



The geometrical figure represented by the diagram in the margin, is a facsimile of that on page 114, with the following additions. Produce C A—the hypothenuse of the right-angled triangle A B C—to meet a straight line drawn from the angle D tangental to the circle X at the point E, and so construct the right-angled triangle E D C. Bisect A D in F, and join C F, dividing the isosceles triangle A D C into two similar and equal right-angled triangles C F A and C F D.

It is self-evident, that A B C and E D C are similar right-angled triangles, and C B is to B D in the ratio of 4 to 1, by construction; and it follows, that C A is to A E in the ratio of 4 to 1, by construction. Hence: By analogy or proportion, C B : B D : : C A : A E: and by alternation, C B : C A : : B D : A E. So far, all is pure Geometry: but, at this point steps in, that "*indispensable instrument of science, Arithmetic,*" and asserts its authority; and without it, not one thing more can we prove. But by the aid of Arithmetic, we can prove, the ***harmony*** that exists between Geometry and Mathematics. For example: Let C B = 4. Then: In the analogy or proportion, C B : B D : : C A : A E; B D = 1 : C A = 5: and A E = 1·25: and 4 : 1 : : 5 : 1·25: and the product of the extremes, is equal to the product of the means; that is, $4 \times 1{\cdot}25 = 1 \times 5$. But further: By the aid of Arithmetic, we can prove, that $(AB^2 + BC^2 + AC^2) = 3\frac{1}{8}(BC^2)$; or in other words, we can prove, that the sum of the squares of the three sides of the triangle A B C = $3\frac{1}{8}$ times the square of the middle side B C. We can also prove, that $(ED^2 + DC^2 + EC^2) = 3\frac{1}{8}(DC^2)$; or in other words, we can prove, that the sum of the squares of the three sides of the triangle E D C = $3\frac{1}{8}$ times the square of the middle side D C.*

I have said on page 114, that the difference between the areas of the two circles, is exactly equal to the area of a circle of which A B is the radius: and any Mathe-

* On page 114, lines 8 and 9 from top, I have said, $3\frac{1}{8}(CD^2) = (3{\cdot}125 \times 25) = 78{\cdot}125$ = area of the circle E D C, when A B = 3. This is a lapsus. I should have said, $3\frac{1}{8}(CD^2) = (3{\cdot}125 \times 25) = 78{\cdot}125$ = area of the circle, when A B = 3. E D C does not denote the circle. I cannot conceive, how I came to overlook this slip in correcting for the press.

J. S.

matician may readily convince himself of this fact, by means of any hypothetical value of the symbol π, intermediate between 3 and 4, so that it be *finite* and *determinate.* Now, it is self-evident that the angle A C D at the centre of the circles, is common to the triangles A B C and E D C; hence, E C the *secant* of the angle E C D $= 2 (3\frac{1}{8}) = 6{\cdot}25$, when A B a tangent of the angle C $= 3$.

It is self-evident, that C A = C D, for they are radii of the same circle. Let C A and C D = 1. Find A D.

A B $= \frac{3}{5}$ (C A), by construction; therefore, $\frac{3}{5}$ (C A) $= \frac{3}{5}(1) = \frac{3 \times 1}{5} = \frac{3}{5} = {\cdot}6 =$ A B: But, B C $= \frac{4}{5}$ (C A), by construction; therefore, $\frac{4}{5}$ (C A) $= \frac{4}{5}(1) = \frac{4 \times 1}{5} = \frac{4}{5} = {\cdot}8 =$ B C. But, C D = C A; therefore, C D — B C $= 1 - {\cdot}8 = {\cdot}2 =$ B D; and A B D is a right-angled triangle, by construction; therefore, by Euclid: Prop. 47: Book 1: $(\text{A B}^2 + \text{B D}^2) = ({\cdot}6^2 + {\cdot}2^2) = ({\cdot}36 + {\cdot}04) = {\cdot}4 = \text{A D}^2$; therefore, A D $= \sqrt{{\cdot}4}$.

Now, A D is bisected in F, and C F is joined, dividing the isosceles triangle ADC into two similar and equal right-angled triangles C F A and C F D. The angle F C A = the angle F C D: and A F = D F $= \frac{1}{2}(\sqrt{{\cdot}4}) = \sqrt{{\cdot}1} = {\cdot}3162277\ldots$, and ·3162277 is the trigonometrical sine of the angles F C A and F C D. But, the angle D A B = half the angle A C D; and it follows, that the angles D A B, F C A, and F C D are equal angles. I have demonstrated, in my Letter to Mr. Gibbons of the 2nd September (page 111), that D A B is an angle of 18° 26′, and A C D an angle of 36° 52′.

In Plane Trigonometry, if a triangle be right angled, the side subtending the right angle is called the *hypothenuse.*

The cosine of an angle is the sine of the complement of that angle. The cotangent of an angle is the tangent of the complement of that angle. The cosecant of an angle is the secant of the complement of that angle. In an angle of 45° the sine and cosine are equal. If an angle be less than 45° the cosine is greater than the sine. If an angle be greater than 45° and less than 90°, the sine is greater than the cosine. These facts result from the *greater* angle being opposite the *greater* side.

Given :—The angles E C D and A C B = 36° 52′.

The trigonometrical functions of these angles are :—

Sine	·6
Cosine	·8
Tangent	·75
Cotangent	$1\frac{1}{3}$
Secant	1·25
Cosecant	$1\frac{2}{3}$
Versed sine	·2
Coversed sine	·4

According to Hutton's Logarithmic Tables :—

Sine	·5999549
Cosine	·8000338
Tangent	·7499119
Cotangent	1·3334900
Secant	1·2499471
Cosecant	1·6667920
Versed sine	·1999662
Coversed sine	·4000451

The time will come—it may not be in my day—when some *honest* "recognised" Mathematician will arise.

who, having been trained in the school of Orthodoxy, will think, and fancy he can convince others, that James Smith was a non-reasoning geometrical investigator, and profoundly ignorant of Mathematics; and who will, in his attempt at proofs, stumble upon the fact, that 8 circumferences of a circle = 25 diameters; making 8 to 25 the true ratio of diameter to circumference, in every circle; and $\frac{25}{8} = 3{\cdot}125$, the true arithmetical value of the circumference of a circle of diameter unity.

Because the sides that contain the right angle in the triangles E D C and A B C are in the ratio of 3 to 4, by construction, the sum of E D and E C = 2 (D C): and the sum of A B and A C = 2 (B C). Where is the Mathematician who can controvert these geometrical truths? And where is the HONEST Mathematician to be found, who will dispute my *right* to adopt these geometrical data as premisses, to find the ratio of diameter to circumference in a circle?

Now, if n denote $\{(CB + BD) + \frac{1}{4}(CB + BD)\}$, then, n = the line E C, the hypothenuse of the right-angled triangle E D C: and when A B = 3, which makes B C = 4, and A C = 5, then, $n = 6{\cdot}25$. Hence: $\frac{n}{6{\cdot}25} = \frac{EC}{6{\cdot}25} = BD$, and $\frac{EC}{\frac{1}{8}(6{\cdot}25)} = \frac{6{\cdot}25}{{\cdot}78125} = 8$ = the sum of A B and A C = 2 (B C); making $\frac{6{\cdot}25}{2} = 3{\cdot}125$ the true arithmetical value of the circumference of a circle of diameter unity; and $\frac{3{\cdot}125}{4} = {\cdot}78125$ the area of a circle of diameter unity.

Let m denote the area of an inscribed square, and let n denote the area of a circumscribed square, to any circle. Then: $m : n :: BC : (AB + AC)$; and $m : n :: DC : (ED + EC)$; and because the area of a circumscribed

square to any circle is the double of the area of an inscribed square to the same circle, $m : n :: 2 : 4$; therefore, m is to n in the same ratio as B C to (A B + A C), or, as D C to (E D + E C).

Let B C be represented by any *finite* and determinate arithmetical quantity, say 8. Then: $\frac{5}{4}$ (B C) $= \frac{5 \times 8}{4} = \frac{40}{4} = 10 =$ D C: $\frac{3}{4}$ (D C) $= \frac{3 \times 10}{4} = \frac{30}{4} = 7{\cdot}5 =$ ED: and E D C is a right-angled triangle, by construction; therefore, by Euclid: Prop. 47: Book I: $(ED^2 + DC^2) = (7{\cdot}5^2 + 10^2) = 56{\cdot}25 + 100 = 156{\cdot}25 = EC^2$; therefore, $\sqrt{156{\cdot}25} = 12{\cdot}5 =$ E C. Hence: $\frac{EC}{6{\cdot}25} = \frac{12{\cdot}5}{6{\cdot}25} = \frac{1}{5}$ (D C) $= 2 =$ D B: and the sum of the squares of the three sides of the triangle E D C $= 3\frac{1}{8}$ times the square of D C the middle side: that is, $(56{\cdot}25 + 100 + 156{\cdot}25) = 3\frac{1}{8}\,(10^2) = 312{\cdot}5$: and this equation = area of the circle X, when the radii D C, and A C = 10.

Let B C $= \sqrt{8}$. Then: $\frac{5}{4}$ (B C) $= \frac{5}{4}\,(\sqrt{8}) = \sqrt{\frac{5^2}{4^2} \times 8} = \sqrt{\frac{25}{16} \times 8} = \sqrt{1{\cdot}5625 \times 8} = \sqrt{12{\cdot}5} =$ D C: $\frac{3}{4}$ (D C) $= \frac{3}{4}\,(\sqrt{12{\cdot}5}) = \sqrt{\frac{3^2}{4^2} \times 12{\cdot}5} = \sqrt{\frac{9}{16} \times 12{\cdot}5} = \sqrt{{\cdot}5625 \times 12{\cdot}5} = \sqrt{7{\cdot}03125} =$ E D: and E D C is a right-angled triangle, by construction; therefore, by Euclid: Prop. 47: Book. I: $(ED^2 + DC^2) = (\overline{\sqrt{12{\cdot}5}}^{\,2} + \overline{\sqrt{7{\cdot}03125}}^{\,2}) = \sqrt{(12{\cdot}5 + 7{\cdot}03125)} = \sqrt{(19.53125)} = EC^2$; therefore, E C $= \sqrt{19{\cdot}53125}$; and the sum of the squares of the three sides of the triangle E D C $= 3\frac{1}{8}$ times the square of D C the middle side: that is, $(12{\cdot}5 + 7{\cdot}03125 + 19{\cdot}53125) = 39{\cdot}0625$; and this equation

=area of the circle X, when the radii D C and A C $= \sqrt{12 \cdot 5}$.
But, $\sqrt{39 \cdot 0625} = 6 \cdot 25$: and $\frac{E\,C}{6 \cdot 25} = \frac{\sqrt{19 \cdot 53125}}{6 \cdot 25} = \sqrt{\left(\frac{19 \cdot 53125}{6 \cdot 25^2}\right)}$
$= \sqrt{\left(\frac{19 \cdot 53125}{39 \cdot 0625}\right)} = \sqrt{\cdot 5} =$ D B: and 5 (D B) $= 5(\sqrt{\cdot 5})$
$= \sqrt{5^2 \times \cdot 5} = \sqrt{25 \times \cdot 5} = \sqrt{12 \cdot 5} =$ D C and A C.

It is axiomatic, if not self-evident, that from a given value of E C we can find A B and B D, and prove that the angle D A B = half the angle E C D.

Mathematicians not having been able to find the exact arithmetical value of the circumference of a circle of diameter unity, have adopted the symbol π to denote it. Hence, $\frac{\pi}{1}$ expresses the ratio between the circumference and diameter in every circle, whatever be the value of π. Because 3 times diameter = the perimeter of a regular inscribed hexagon in every circle, and because 6 times radius $= 6\,(\frac{1}{2}) = (6 \times \cdot 5) = 3 =$ the perimeter of a regular inscribed hexagon to a circle of diameter unity; it follows, that $\frac{\pi}{3}$ expresses the ratio between the circumference of every circle and the perimeter of its inscribed regular hexagon, whatever be the value of π.

Now, by hypothesis, let $\pi = 3 \cdot 126$. Then: $\frac{8 \times 3}{8 \times 3 \cdot 126}$ $= \frac{24}{25 \cdot 008}$. But, $\frac{24 \times 3 \cdot 126}{25 \cdot 008 \times 3} = \frac{75 \cdot 024}{75 \cdot 024} = 1 =$ unity; and would make 8 circumferences of a circle *greater* than 25 diameters. Again: By hypothesis, let $\pi = 3 \cdot 124$. Then: $\frac{8 \times 3}{8 \times 3 \cdot 124} = \frac{24}{24 \cdot 992}$. But, $\frac{24 \times 3 \cdot 124}{24 \cdot 992 \times 3} = \frac{74 \cdot 976}{74 \cdot 976} = 1 =$ unity, and would make 8 circumferences of a circle *less* than 25

diameters. Again: By hypothesis, let $\pi = \frac{3{\cdot}126 + 3{\cdot}124}{2} = \frac{6{\cdot}25}{2} = 3{\cdot}125$. Then: $\frac{8 \times 3}{8 \times 3{\cdot}125} = \frac{24}{25}$. But, $\frac{24 \times 3{\cdot}125}{25 \times 3} = \frac{75}{75} = 1 =$ unity;* and on this hypothesis, 8 circumferences of a circle $= 25$ diameters exactly. No such reasoning—says the Mathematician—is admissible in the enquiry as to π's true arithmetical value.

In the year 1867, I was brought into contact with a recognised Mathematician, through the intervention of a friend: our mutual friend acting as the medium of communication between us, and the gentleman is personally unknown to me at this moment. I quote the following from one of his Papers, dated April 8, 1868:—"*I am perfectly sure that Mr. Smith will admit, that if π cannot be shewn to be a determinate quantity, and shewn by "a priori" reasoning, that is, without reference to its arithmetical value—that process of reasoning by which he some time ago said he arrived at his conviction that $\pi = 3\frac{1}{8}$ cannot be valid.*" How are we to know whether π be arithmetically determinate or indeterminate, until by "*some process of reasoning*," we have found *an* arithmetical value of it? How can we find *an* arithmetical value of π, without the aid of arithmetic? If Mathematicians will write nonsense, "*I can't help it.*"

It stands to common sense, whether we assume the circumference of a circle to be represented by unity, or by 360°, that, if, in either case, we can find the arith-

* The reader should compare these facts with the facts to which I have directed attention on pages 101, 102, and 103.

J. S.

metical value of the perimeter of a regular inscribed hexagon, the "*same process of reasoning*" will enable us to find the perimeter of the hexagon in both cases.

Now, since $\frac{\pi}{3}$ expresses the ratio between the circumference of every circle and the perimeter of its inscribed regular hexagon; it follows, that the converse of this must be true; and consequently, $\frac{3}{\pi}$ must express the ratio between the perimeter of every regular hexagon and the circumference of its circumscribing circle; and it follows, that $\frac{\text{Perimeter of hexagon}}{\text{Circumference of circle}}$ must be a constant quantity. Well, then, $\frac{24}{25}(1) = {\cdot}96$, and $\frac{24}{25}(360°) = 345°\ 36'$; therefore, $\frac{{\cdot}96}{1}$ and $\frac{345°\ 36'}{360°}$ are equivalent ratios, and both express the ratio between the perimeter of every regular hexagon and the circumference of its circumscribing circle.

Let c denote the circumference of a circle, and let p denote the perimeter of its inscribed regular hexagon.

THEOREM.

Given: $c = 360°$: Find p, and prove that $\frac{p}{c}$ is an indeterminate arithmetical ratio.

I, of course, maintain that $\frac{p}{c}$ is a definite arithmetical ratio, and when that unscrupulous mathematical critic, Professor de Morgan, can solve this theorem, he will be able to prove that "*James Smith, Esq., of Liverpool, is nailed by himself to the barn-door, as the delegate of miscalculated and disorganised failure.*" (See *Athenæum*, July 25, 1868: Article, *Our Library Table.*)

Now:—

$\frac{24}{25}(c) = \frac{24}{25}(360^\circ) = 345^\circ\ 36' = p$.

$\therefore p : c :: 345^\circ\ 36' : 360^\circ$: and $\frac{p}{c}$ is *a definite arithmetical ratio.*

Again:—

$\frac{c}{3\cdot125} = \frac{360^\circ}{3\cdot125} = 115^\circ\ 12' =$ diameter: $\frac{d}{2} = \frac{115^\circ\ 12'}{2}$
$= 57^\circ\ 36' =$ radius: $6\,(\text{radius}) = 6\,(57^\circ\ 36') = 345^\circ\ 36' = p$.

$\therefore p : c :: 345^\circ\ 36' : 360^\circ$; and $345^\circ\ 36' : 360^\circ : 3 : 3\cdot125$.

Hence:—

$\frac{24}{25}$, $\frac{345^\circ\ 36'}{360^\circ}$, and $\frac{3}{3\cdot125}$ are *definite and equivalent arithmetical ratios;* and it follows, that all these ratios express the true ratio between the perimeter of every regular hexagon and the circumference of its circumscribing circle; and it also follows, that the measure of the difference between the perimeter of every regular hexagon and the circumference of its circumscribing circle, is exactly equal to $\frac{1}{25}$th part of the circumference of the circle; therefore, $\frac{25}{24}(p) = c$.

Such men as Whitworth, De Morgan, and Glaister, may have the audacity to *assert* that these conclusions are NONSENSE; but, to prove the truth of their assertion, they must *demonstrate,* that the perimeter of a regular inscribed hexagon to a circle of diameter unity, is not $6\,(\frac{1}{2}) = 3$.

It would be a waste of time to give myself any further trouble with the existing race of "*recognised Mathematicians:*" *argument with them is indeed hopeless.* Had ancient Geometers and Mathematicians been privileged to know the foregoing facts, we should not have had Mathematicians in the nineteenth century of the

Christian era, groping in the dark to find the arithmetical value of the circumference of a circle of diameter unity. The symbol π is not to be found in any Treatise on Elementary Geometry: and ere this, it would have ceased to hold a place in modern Treatises on Plane Trigonometry.

THOMAS S. BARRETT, ESQ., *to* JAMES SMITH.

LANGLEY HOUSE,
GROVE LANE, CAMBERWELL, S.E.,
September 30*th*, 1869.

MY DEAR SIR,

I will answer your questions in your former Letters, which I have not yet answered.

In that dated September 21*st*,

(1.) You say, $4\left(\frac{\pi}{2}\right)^2 = \pi^2$.

(1.) I reply: I agree with you.

(2.) You say 3·125 × ·0256 = ·8.

(2.) I make it ·08.

(3.) You ask, can I find the area of a circle of which the circumference = 1?

(3.) My answer is: Yes. (See answer 9.)

In your note dated September 24*th*,

(4.) You ask: Is not $\frac{2\pi}{4}$ the circular measure of a right angle?

(4.) I answer: Yes.

(5.) You ask: whether I had attended the late meeting of the British Association?

(5.) Answer: I had not that pleasure.

In your Letter received this morning,

(6.) You ask, whether I can divide an unknown quantity by 2, and find the quotient. The unknown amount you have particularly in your mind, is that which follows the first four places of ·9549... indicated by the dots.

(6.) My answer is as follows:—If you mean, can I divide such a number, and find the quotient true to a *greater* number of places

than the number of figures known in the dividend, the answer is, "No." Thus, supposing the fraction ·9549... is known true to only 4 places, then I cannot find a quotient true to a *greater* number of places than 4. But I can find the quotient true to the *same number* of places, namely: ·4775.*

(7.) You ask, whether I do not think our correspondence on the question of the value of π had better terminate?

(7.) My answer is: That is for you to decide. As all I am doing is to answer your questions, it follows, that if you wish to close the correspondence, all you have to do is to stop writing to me.

(8.) You ask, whether further correspondence will not be like playing at "cross questions and crooked answers?"

(8.) I reply: Since the questions are all yours, and the answers all mine, I cannot admit the latter are crooked. On the contrary, I try to make them as straightforward as possible.

(9.) You ask me what the area of a circle is when the circumference is 1?

(9.) I reply : $\frac{1}{4\pi}$.

I remain, dear Sir,
Faithfully yours,
THOMAS S. BARRETT.

JAMES SMITH *to* THOMAS S. BARRETT, ESQ.

BARKELEY HOUSE, SEAFORTH,
1st October, 1869.

SIR,

You have now answered my questions, and in a *certain sense* correctly.

With reference to my Letter of the 21st September, among other things, you observe:—"You (I) say, $3·125 \times ·0256 = ·8$. I make it ·08." In a communication I posted yesterday, intended for publication, and in which I gave a copy of my Letter to you of 21st September, I detected the lapsus and corrected it. Any Mathematician would have seen that it was a lapsus, and would have

* The reader will observe, that Mr. Barrett makes $\frac{·9549}{2}$ not ·47745 but ·4775.

corrected it accordingly, and no "*man of honour*" would have thought of raising a quibble upon it.

The closing paragraph of your Letter runs thus:—"You ask me what the area of a circle is when the circumference is 1:—I reply: $\frac{1}{4\pi}$." It is no doubt true, that the algebraical expression $\frac{1}{4\pi}$ denotes the area of a circle, when the circumference is 1. In putting my question, I should have said: What is the *arithmetical* value of the area of a circle, when the circumference is 1? If you were to say you did not understand this to be my meaning, I should not believe you; and if you were to say, you can find the arithmetical value of the area of a circle when the circumference is 1, with an *indeterminate* value of π, you would say that which is *untrue*, and which you know to be *untrue*.

Well, then, $3{\cdot}125\ ({\cdot}16^2) = 3{\cdot}125 \times {\cdot}0256 = {\cdot}08$: and $12{\cdot}5\ ({\cdot}08) = 1$.

I remain, Sir,
Faithfully yours,
JAMES SMITH.

THOMAS S. BARRETT, ESQ., *to* JAMES SMITH.

LANGLEY HOUSE,
GROVE LANE, CAMBERWELL, S.E.,
October 2nd, 1869.

MY DEAR SIR,

I was quite innocent of any other motive (when I wrote that I made $3{\cdot}125 \times {\cdot}0256 = {\cdot}08$), than that of pointing out to you a slip of your pen, which I thought you would probably like to be made aware of.

With regard to the area of a circle when the circumference is 1, if you are right in saying that $\pi = 3{\cdot}125$, then you are likewise right when you make the area of such a circle to be ·08.* If, on the other hand, $\pi = 3{\cdot}14159265$......, then, the area = ·07957747......

I remain, my dear Sir,
Faithfully yours,
THOMAS S. BARRETT.

* In my Letter of the 21st September, to Mr. Barrett, I had inadvertently written $3{\cdot}125 \times {\cdot}0256 = {\cdot}8$, instead of ·08. J. S.

The symbol π is adopted by Mathematicians to denote the circumference of a circle of diameter unity; and our unit of length being 1, it follows, that $\frac{1}{\pi}$ denotes the ratio of diameter to circumference in a circle, whatever be the value of π.

Mr. Barrett *admits*, in his Letter of September 30th, that the algebraical expression $\frac{1}{4\pi}$ denotes the area of the circle, when the circumference is 1; and in his Letter of October 2nd, he *admits*, that if I am right in saying that $\pi = 3{\cdot}125$, then, I am likewise right when I make the area of a circle of circumference unity, to be ·08; but immediately adds: — "*If, on the other hand,* $\pi = 3{\cdot}14159265...$, *then, the area* = ·07957747..."

Mark what follows!! This argument of Mr. Barrett would make the area of a circle of circumference unity, less than ·08, when π is greater than 3·125. Does not a line in the form of the circumference of a circle, enclose a larger superficies than it can be made to enclose in any other form whatever? How, then, in the name of common sense, can the arithmetical value of the circumference of a circle of diameter unity, be either greater or less than 3·125? Impossible! And it follows, that 8 circumferences = 25 diameters, in every circle.

By hypothesis, let the circumference of a circle be greater than 3·125, say 3·1416. Then: $\frac{3{\cdot}1416}{(4\ 3\frac{1}{8})} = \frac{3{\cdot}1416}{12{\cdot}5} = {\cdot}251328$ = semi-radius of the circle: $2\,(s\,r) = 2\,({\cdot}251328) = {\cdot}502656$ = radius: and $3\frac{1}{8}\,(r^2) = 3{\cdot}125\,({\cdot}502656^2)$; therefore, $3{\cdot}125 \times {\cdot}252663054336 = 3{\cdot}1416 \times {\cdot}251328$; and this equation $= {\cdot}7895720448$ = area of a circle when the circumference = 3·1416, and is greater than $\frac{1}{4}\,(3{\cdot}1416)$; and "*upsets*" the *admitted* fact,

that $\frac{\pi}{4}$ = area of a circle of diameter unity. Again: By hypothesis, let the circumference of a circle be less than 3·125, say 3·12. Then: $\frac{3·12}{4(3\frac{1}{8})} = \frac{3·12}{12·5} =$ ·2496 = semi-radius of the circle; 2 (sr) = 2 (·2496) = ·4992 = radius: and, $3\frac{1}{8}$ (r^2) = 3·125 (·4992^2); therefore, (3·125 × ·24920064) = (3·12 × ·2496); and this equation = ·778752, and is less than $\frac{1}{4}$ (3·12), and again "*upsets*" the admitted fact, that $\frac{\pi}{4}$ = area of a circle of diameter unity. Both examples work out with arithmetical exactness, but the former hypothesis would make $\frac{\pi}{4}$ greater than the area of a circle of diameter unity; and the latter hypothesis would make $\frac{\pi}{4}$ less than the area of a circle of diameter unity; and both examples are ***reductio ad absurdum*** demonstrations of the orthodox *assertion*, that the circumference of a circle of diameter unity, can only be expressed arithmetically by an infinite series.

Well, then, Mr. Barrett *assumes* π = 3·141592653..., and on this assumption makes the area of a circle of circumference unity = ·07957747..., that is, less than ·08, and so "*upsets*" the orthodox ratio of diameter to circumference in a circle. If Mr. Barrett has not the mental capacity to see this, "*I can't help it, but the fact remains notwithstanding.*"

Now, I repeat, that the symbol π is adopted by Mathematicians to denote the circumference of a circle of diameter unity.

Let the diameter of a circle be represented by the MYSTIC number 4. Then: The circumference and area of the circle are represented by the same arithmetical symbols,

whatever be the arithmetical value of the circumference of a circle of diameter unity. Again: Let the circumference of a circle be represented by the MYSTIC number 4. Then: The diameter and area are represented by the same arithmetical symbols, whatever be the arithmetical value of the circumference of a circle of diameter unity. These facts may be demonstrated by means of any hypothetical value of π, intermediate between 3 and 4, so that it be *finite* and *determinate.*

I quote the following from a Letter of one of my correspondents, dated July 18th, 1860.* "The fallacy which pervades your argument in the paper respecting the squaring of the circle, is, that you treat linear and square units as if they were identical. It is very true, that one, and the square of one, are expressed by the same symbol, but they are very different things. As for instance, if you have an estate bounded by a fence one mile long on each side of a square, it is true to say that the side of the square is 1, it is also true to say that the area is 1, but you speak of one *linear* unit as to the fence, and one *square* unit as to the estate; and it would be absurd to say that the fence of one side of the estate is equal to the estate itself; equally absurd is it to say, that the *side* of any square is equal to its *area,* or any area." If the side of a square be denoted by the *linear* unit 1, is not the area of the square one square unit? If the side of a square be denoted by $\sqrt{1}$, is not the area of the square one *square* unit? Are not 1 and $\sqrt{1}$ symbols of the same arithmetical value? Is it not self-evident, that Eminent Mathematician means it

* See *The Quadrature of the Circle, Correspondence between an* Eminent Mathematician *and* James Smith, Esq., Page 2.

to be inferred, that because 1 may represent a *linear* unit, and also represent a *square* unit, that this *undoubted fact* can be of no help to us in finding the true circumference of a circle of diameter unity? If this were a correct inference, would it not follow, that no number of *linear* units multiplied by a number of *square* units, could give the number of *square* units enclosed by the circumference of a circle, whether inscribed or circumscribed to a square? Are not the circumference and semi-radius of every circle represented—if represented arithmetically—by *linear* units? Is not the area of a square on the semi-radius of a circle represented—if represented arithmetically—by *square* units? Does not the symbol π denote a line? If c denote the circumference, and $s\,r$ denote the semi-radius of a circle, is not $c \times s\,r = 4\,\pi\,(s\,r^2)$, whatever be the value of π? Does not this equation = area in every circle? How could these facts be possible, if Eminent Mathematician's inference were a *logical* inference from indisputable premisses? Now, is not a *furlong* a measure of length? Is not a *furlong* one-eighth part of a mile? If an estate be bounded by a fence on each side of a square, and the length of one side of the fence be one *linear* furlong, is it not as certain, that the area of the estate is one *square* furlong, as that the area of the estate is one *sqnare* mile, when the length of the fence on one side of the estate is one *linear* mile? Are not the arithmetical symbols 1, 1^2, and $\sqrt{1}$ of the same arithmetical value? Is not $2(\cdot 5) = 1$? Is not $2(\sqrt{\cdot 5})$ greater than 1? Is not $2(4) = 8$? Is not $2(\sqrt{4})$ less than 8? Hence it is, that Algebra, *per se*, can be "*no help to us*," in finding the true arithmetical value

of the circumference of a circle of diameter unity; or the area of a circle of circumference unity.

Mathematicians maintain, that the foregoing facts "*are no help to us in finding* π*'s arithmetical value*," and this is implied in Eminent Mathematician's argument. Now, Mr. Barrett *admits*, that if $\pi = 3{\cdot}125$, the area of a circle of circumference unity $= {\cdot}08$; but strangely enough, *assumes* $\pi = 3{\cdot}14159265\ldots$, and then computes the area of a circle of circumference unity to be ${\cdot}07957747\ldots$, that is, less than ${\cdot}08$; thus, making the area of a circle of circumference unity less than ${\cdot}08$ with a value of π greater than $3{\cdot}125$. Could absurdity go further? If Mathematicians will write nonsense, "*I can't help it.*"

If the circumference of a circle be represented by unity, the semi-radius and area of the circle are represented by the same arithmetical symbols.

Proof:

Mr. Barrett *admits*, that the area of a circle of circumference unity $= \frac{1}{4\pi}$. Now, $\frac{1}{4(3\frac{1}{8})} = \frac{1}{12{\cdot}5} = {\cdot}08$ $=$ semi-radius: $2\,(s\,r) = 2 \times {\cdot}08 = {\cdot}16 =$ radius: and $3\frac{1}{8}\,(r^2) = 3{\cdot}125\,({\cdot}16^2) = 3{\cdot}125 \times {\cdot}0256 = {\cdot}08 =$ area of the circle.

Does not $2\,(\pi) \times r = 4\,(\pi) \times s\,r$? Does not this equation $=$ circumference in every circle? Does not $\frac{1}{4\pi} =$ area of a circle of circumference unity? Well, then, when that "*unscrupulous critic*," Professor de Morgan, can make the area of a circle of circumference unity greater than ${\cdot}08$, with a value of π greater than $3{\cdot}125$, he will be able to prove the following assertion,

which I quote from his Budget of Paradoxes, No. 26,* "*I shall show that he* (James Smith) *has convicted himself of ignorance and folly, with an honesty and candour worthy of a better value of* π."

PROSPECT HOTEL, HARROGATE,
6th October, 1869.

MY DEAR SIR,

Your Letter of the 2nd inst. has been forwarded to me here, and came into my hands this morning.

Had I supposed you had no "*other motive*" than that of "*pointing out*" *a slip of the pen*, when I fell into the lapsus of making $3{\cdot}125 \times {\cdot}0256 = {\cdot}8$, instead of ·08, I should have been sorry to charge you with raising a "*quibble*" upon it.

The last paragraph of your Letter runs thus:—"*With regard to the area of a circle when the circumference is* 1, *if you are right in saying that* π *equals* 3·125, *then you are likewise right when you make the area of such a circle to be* ·08. *If, on the other hand,* $\pi = 3{\cdot}14159265\ldots$, *then, the area* $= {\cdot}07957747\ldots$." The *ifs* in this paragraph convince me, that your mind is in a state of dubiety as to π's true arithmetical value.

I will try once more to convince you, that the value of the symbol π can be nothing else but 3·125, and with this effort, so far as I am concerned, our *controversial* correspondence must terminate.

Have you admitted: That,

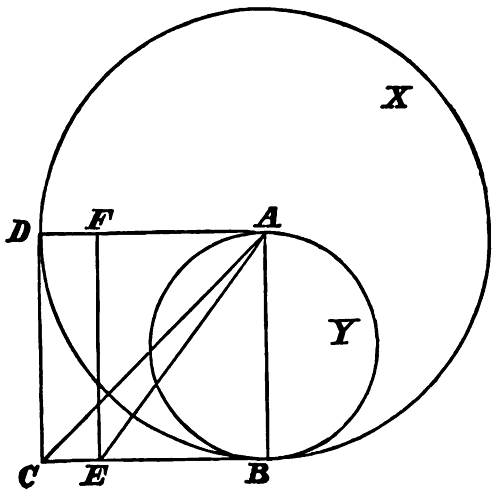
$$\frac{\text{The perimeter of every regular hexagon}}{\text{The circumference of its circumscribing circle}}$$

is a constant quantity? You have admitted that the area of a circle is denoted by $\frac{1}{4\pi}$, when the circumference is 1. You have also admitted that $\frac{\pi}{3}$ expresses the ratio between the circumference of every circle, and the perimeter of its inscribed regular hexagon. Now, it is axiomatic, if not self-evident, that 6(radius × semi-radius) = area of an inscribed regular dodecagon to every circle; and it follows, that $\frac{\pi}{3}$ expresses the ratio between the area of every

* See *Athenæum*, June 24, 1865.

circle, and the area of its inscribed regular dodecagon. Now, 6 times radius = the perimeter of a regular inscribed hexagon to every circle ; and it follows, that 6 times 1 = 6 = the perimeter of a regular inscribed hexagon to a circle of radius unity : and you may readily convince yourself, by means of any *finite* and *determinate* value of π, that $\left(\frac{\pi}{2} \div \frac{8\,\pi}{100}\right)$ is a constant quantity = 6·25 ; and it follows, that $\frac{6{\cdot}25}{6}$ expresses the ratio between the circumference of a circle and the perimeter of its inscribed regular hexagon. Now, $\frac{6}{6{\cdot}25}$ = ·96 = the perimeter of a regular inscribed hexagon to a circle of circumference unity. Hence: 6·25 × ·96 = 6 × 1 = 6 = the perimeter of a regular inscribed hexagon to a circle of radius unity; and $\left(\frac{6{\cdot}25 \times {\cdot}96}{6 \times 1}\right) = \frac{6}{6} = 1$ = radius ; and it follows : First : That $\frac{6{\cdot}25}{6}$ and $\frac{1}{{\cdot}96}$ are equivalent ratios, and both express the ratio between the circumference of every circle, and the perimeter of its inscribed regular hexagon. Second : That the measure of the difference between the circumference of any circle, and the perimeter of its inscribed regular hexagon, is exactly equal to $\frac{1}{25}$th part of the circumference of the circle.

I construct the geometrical figure in the margin in the following way:—I draw the straight line A B, and with A as centre and A B as radius describe the circle X : and with A B as diameter, describe the circle Y. On A B I describe the square A B C D, cut off from



the side C B a part C E, equal to $\frac{1}{4}$ (C B), draw E F parallel to D C and A B, and join A E and A C, and so construct the right-angled triangle A B E, and the oblique-angled triangle A E C.

The following things are self-evident:—First: The triangles A B E and E F A are similar and equal right-angled triangles, and have the sides that contain the right angle in the ratio of 4 to 3, by construction. Second: F E B A is a parallelogram, and is divided by the diagonal A E into the two similar and equal right-angled triangles A B E and E F A. Third: A B C D is a rectangular parallelogram of which all the sides are equal, and is divided by the diagonal A C into the two similar and equal right-angled isosceles triangles A B C and A D C.

Hence:

By Euclid: Prop. 12: Book 2.

$$AE^2 + EC^2 + 2(EC \times EB) = AC^2.$$

A moment's reflection will—or ought to—satisfy you, that in finding the areas of geometrical figures, the science of Mathematics is involved, and consequently, Euclid no where treats of areas; nor could he, without travelling out of the domain of pure geometry. You may readily convince yourself: First: That the perimeter of the triangle A B E = the perimeter of a regular inscribed hexagon to the circle Y. Second: That the area of the circle Y : the area of an inscribed regular dodecagon : : the circumference of the circle Y : the perimeter of an inscribed regular hexagon. Third: That the area of the parallelogram F E B A is exactly equal to the area of an inscribed regular dodecagon to the circle Y.

From these facts, it follows, that $3\frac{1}{8}$ (AB^2) = the sum of the squares of the three sides of the right-angled triangle A B E, or, E F A, and this equation = area of the circle X; and makes 8 circumferences = 25 diameters in every circle, and $\frac{25}{8} = 3{\cdot}125$ the true arithmetical value of π. "*If you can't see this, I can't help it, but the fact remains notwithstanding.*"

I remain, my dear Sir,

Faithfully yours,

JAMES SMITH.

THOMAS S. BARRETT, ESQ., *to* JAMES SMITH.

LANGLEY HOUSE, GROVE LANE,
CAMBERWELL, LONDON, S.E.,
October 6th, 1869.

MY DEAR SIR,

As it is now several days since I wrote last to you, and as I have received no further communication from you, I conclude you do not care to ask me any further questions.

I will therefore go back to the subject on which I commenced the correspondence.

You will remember you were so good as to present me with a copy of your "*British Association in Jeopardy*." This act I interpreted into a challenge to criticise or refute the reasonings contained in your book.

I have read, I think, the whole of your work, and each argument as I read it seemed to contain an error or an assumption which rendered the conclusion "*not proven*."

The one on page 14 (which I have already alluded to) would be a perfect demonstration of your position, if only it were clear that its starting point, viz., that $\left(\dfrac{\text{Circular measure } 90^\circ}{\text{Circular measure } \frac{360^\circ}{25}} = 2\pi\right)$ is correct. Consequently, a person who comes to you asking for your proof that $\pi = 3{\cdot}125$ goes away disatisfied, if you give him the argument on page 14, ***and nothing more***. He wants a proof for the starting point as much as for the conclusion.

Will you give me the proof of the starting point? Or, if you prefer to do so, will you give me what you consider the ***simplest*** and ***most convincing***, ***complete*** proof that $\pi = 3{\cdot}125$.

If you do this or the other, what you say shall receive a ***strict*** and ***impartial*** consideration at my hands. If I find your argument convincing, I will candidly tell you so, and become henceforth a disciple; if on the other hand I find what I conceive error in it, will point it out to you fairly, and I hope respectfully. I do not think, because gentlemen disagree on a point in science, they should feel angry towards each other. On the contrary, they should be all the more cool.

Very truly yours,
THOMAS S. BARRETT.

THOMAS S. BARRETT, ESQ., *to* JAMES SMITH.

LANGLEY HOUSE, GROVE LANE,
CAMBERWELL, LONDON,
October 7th, 1869.

MY DEAR SIR,

I have this morning received your reply to my Letter of the 2nd, and am much obliged thereby. I have carefully read the two arguments contained in it; and there are only two points therein which I do not see.

First: I do not see how "$\frac{6\cdot25}{6}$ expresses the ratio between the circumference of a circle and the perimeter of its inscribed regular hexagon," follows from $\left(\frac{\pi}{2} \div \frac{8\pi}{100}\right) = 6\cdot25$. And, in the second argument, I do not see how it is proved that the area of circle X, equals the equation: $3\frac{1}{8}(A\,B)^2$ = the sum of the squares of the sides of the triangle A B E. If you prove these two points to me, I shall be convinced that $\pi = 3\cdot125$.

I remain, my dear Sir,
Faithfully yours,
THOMAS S. BARRETT.

P.S.—I wrote a Letter to you yesterday; but should not have done so had I received yours first.

JAMES SMITH *to* THOMAS S. BARRETT, ESQ.

PROSPECT HOTEL, HARROGATE,
8th October, 1869.

MY DEAR SIR,

Your favor of the 6th instant has been forwarded to me here, and with that of yesterday came into my hands this morning. From these communications I conclude that we are agreed upon all points but two, and that on these two points you wish for further information.

On the first point you say:—"I do not see how $\frac{6\cdot25}{6}$ expresses the ratio between the circumference of a circle and the perimeter of its inscribed regular hexagon, follows from $\left(\frac{\pi}{2} \div \frac{8\,\pi}{100}\right) = 6\cdot25$."

With reference to this, I may observe: Mathematicians adopt as a "*starting point*" in their search after π, a circle of radius 1; and $2\,\pi$ (radius) = circumference of a circle of radius 1, whatever be the value of π; and it follows, that 6 times radius = 6, and 6 is the perimeter of a regular inscribed hexagon to a circle of radius unity. Hence: $\frac{2\,\pi}{6}$ expresses the ratio between the circumference of every circle, and the perimeter of its inscribed regular hexagon, whatever be the value of π: and $\left(\frac{\pi}{2} \div \frac{8\,\pi}{100}\right)$ is a constant quantity = 6·25, whatever be the value of the symbol π; the latter fact can be proved by means of any hypothetical value of π, so that it be *finite* and *determinate;* π may be 3, or it may be 4, or it may be any finite arithmetical quantity intermediate between 3 and 4. One example will suffice. By hypothesis, let $\pi = 3\cdot2$. Then: $\left(\frac{3\cdot2}{2} \div \frac{8\,(3\cdot2)}{100}\right) = \left(\frac{3\cdot2}{2} \div \frac{25\cdot6}{100}\right) = \frac{1\cdot6}{\cdot256} = 6\cdot25.$

You have not disputed, that:—

$$\frac{\text{The perimeter of every regular hexagon}}{\text{The circumference of its circumscribing circle}}$$

is a constant quantity. Now, $\frac{6}{6\cdot25} = \cdot96$: and it follows, that $\frac{6}{6\cdot25}$ and $\frac{\cdot96}{1}$ are equivalent ratios, and both express the ratio between the perimeter of every regular hexagon, and the circumference of its circumscribing circle. It has been more than once said to me, "*Stick to Algebra.*" Now, if $\left(\frac{\pi}{2} \div \frac{8\,\pi}{100}\right) = x$: what is the arithmetical value of x? I have shewn you how you may prove that $x = 6\cdot25$, by means of any hypothetical value of π, so that it be *finite* and *determinate;* but if you go to work to find the value of x, with an *indeterminate* value of π, say 3·14159265..., where will you be? Certainly not—as Professor de Morgan would say—"*in π glory.*"

these facts fail to convince you: First: of the absurdity of the orthodox notion that π can only be expressed arithmetically by an infinite series: and, Second: that $\frac{x}{2} = \frac{6\cdot25}{2} = 3\cdot125$ is the true arithmetical value of π: "*I can't help it,*" and must be content to leave you in the enjoyment of your own opinions.

On the second point you say:—"I do not see how it is proved that the area of circle X = the equation: $3\frac{1}{8}$ (A B^2) = the sum of the squares of the triangle A B E."

With reference to this point I may observe:—It is not more certain that a square has a perimeter and four sides, and that these four sides enclose a superficies or area: than that a circle has a circumference and diameter, and that the circumference encloses a superficies or area. It is self-evident that A B = the radius of the circle X = half the diameter of the circle; and it stands to common sense, that from circumference, radius, or area given, Mathematics should enable us to find the other two with arithmetical exactness.

THEOREM.

Let the area of the circle X be represented by any arithmetical quantity, say 60. Find the value of A B, and prove that $3\frac{1}{8}$(A B^2) = the sum of the squares of the sides of the triangle A B E, and is equal to the given area of the circle X.

The following is the solution of this theorem:—

$\sqrt{\frac{\text{area}}{\pi}} = \sqrt{\frac{60}{3\cdot125}} = \sqrt{19\cdot2} = \text{A B}.$ But, A B : B E : : 4 : 3, by construction, therefore, $\frac{3}{4}$(A B) = $\frac{3}{4}(\sqrt{19\cdot2}) = \sqrt{\frac{3^2}{4^2} \times 19\cdot2} = \sqrt{\frac{9}{16} \times 19\cdot2} = \sqrt{\cdot5625 \times 19\cdot2} = \sqrt{10\cdot8} = \text{B E}.$

And by Euclid: Prop. 47: Book 1: $\text{A B}^2 + \text{B E}^2 = \sqrt{19\cdot2}^2 + \sqrt{10\cdot8}^2 = 19\cdot2 + 10\cdot8 = 30 = \text{A E}^2$; therefore, $(\text{A B}^2 + \text{B E}^2 + \text{A E}^2) = (19\cdot2 + 10\cdot8 + 30) = 60 =$ the given area of the circle X. Q.E.D.

Now, my dear Sir, since neither you nor any other Mathematician can solve this theorem, with a false value of π, it follows, that 3·125 must be the true arithmetical value of π; and it also follows,

that the measure of the difference between the circumference of any circle and the perimeter of its inscribed regular hexagon, is exactly equal to $\frac{1}{25}$th part of the circumference of the circle.

Are not these facts in perfect harmony with what you have yourself admitted, viz.:—that $\frac{1}{4\pi}$ denotes the area of a circle when the circumference is 1?

I remain, my dear Sir,
Faithfully yours,
JAMES SMITH.

JAMES SMITH *to* THOMAS S. BARRETT, ESQ.

PROSPECT HOTEL, HARROGATE,
11th October, 1869.

MY DEAR SIR,

In my Letter of the 8th instant, I gave you the fullest and clearest information on the two points, upon which your mind appeared to be in doubt. I have no reply to that communication. Why? You have hitherto answered my Letters by return of post. Am I to infer, that you find yourself incapable of controverting my conclusions, and yet, that you are unwilling to admit them?

You cannot *controvert*, and do not *dispute*, that the algebraical expression $\left(\frac{\pi}{2} \div \frac{8\,\pi}{100}\right)$ denotes a constant quantity = 6·25, whatever be the value of π; and you *admit* that $\frac{1}{4\,\pi}$ denotes the area of a circle when the circumference is represented by the symbol 1. Who would think of disputing that 2 (6·25) = 12·5? Is not $\frac{100}{8}$ = 12·5? Are not $\frac{100}{8}$, $\frac{1}{8}(100)$, and $\frac{100}{8}$ (1), arithmetical expressions of the same value.

Now, take the algebraical expression $\left(\frac{1}{12{\cdot}5} \div \frac{8\,\pi}{100}\right)$. Surely, it will not be disputed that 12·5 is contained 8 times in 100; and $\frac{8}{100}$ = ·08, and is equal to the semi-radius of a circle of circumference

unity. Again: Take the algebraical expression $\left(\frac{1}{12{\cdot}5} \times \frac{8\pi}{8}\right)$, and, by hypothesis, let $\pi = \frac{12{\cdot}5}{4} = 3{\cdot}125$. Then: $\left(\frac{1 \times 8\pi}{12{\cdot}5 \times 8}\right) = \frac{8\pi}{100} = \frac{8 \times 3{\cdot}125}{100} = \frac{25}{100} = {\cdot}25$, and is equal to the area of a square on the radius of a circle of diameter unity. In the algebraical expression $\left(\frac{1}{12{\cdot}5} \times \frac{8\pi}{8}\right)$, the 8's go out, and $\left(\frac{1}{12{\cdot}5} \times \pi\right)$ remains; and it follows, that $\left(\frac{1}{12{\cdot}5} \times \pi\right) = ({\cdot}08 \times 3{\cdot}125) = {\cdot}25 =$ semi-radius of a circle of diameter unity. But, $\frac{1}{\frac{1}{4}(12{\cdot}5)} = \frac{1}{3{\cdot}125} =$ ${\cdot}32 =$ the diameter of a circle, when the circumference is 1: $\frac{{\cdot}32}{2} =$ ${\cdot}16 =$ radius: $\frac{{\cdot}32}{4} = {\cdot}08 =$ semi-radius: and $3\frac{1}{8}({\cdot}16^2) = \frac{12{\cdot}5\,({\cdot}08^2)}{100}$ $= \frac{12{\cdot}5 \times {\cdot}64}{100} = {\cdot}08 =$ area of a circle of circumference unity; and it follows, that $\pi = 3{\cdot}125$. Hence: $\frac{\pi\,(8\,\pi)}{4\pi\,(100)} = \frac{78{\cdot}125}{1250} = {\cdot}0625 =$ area of a square on the semi-radius of a circle of diameter unity, and is equal to $\frac{2\,\pi}{100}$.*

Well, then, my good Sir, I defy you or any other Mathematician, to make $\frac{2\,\pi}{100} =$ area of a square on the semi-radius of a circle of diameter unity, either with an *indeterminate* value of π, or any other value of π but $\frac{25}{8} = 3{\cdot}125$. I might ring other changes, but it is not worth while: it would but be, like an attempt to "*paint the lily*," or "*perfume the rose.*"

* When writing from home I usually kept machine copies of my Letters. This I could not do while at Harrogate. In my communications addressed to Mr. Barrett from Harrogate, I first penned a rough copy, and in writing a fair copy made corrections when I found it necessary. I find my rough copy full of interlineations, and I cannot be certain that this paragraph is exactly as I gave it in my Letter. Be this as it may, it is certain from my rough copy, that the paragraph, as given above, is what I intended to say, in my Letter to Mr. Barrett.

J. S.

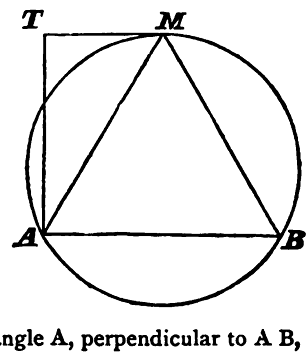
Since I brought out my work on "*The Geometry of the Circle,*" a series of articles have appeared in a Liverpool journal, called the *Liverpool Leader*, entitled "*Curiosities of Mathematics,*" and are to be continued. These articles bear internal evidence that they are from the pen of the Rev. Professor Whitworth, formerly Professor of Mathematics in Queen's College, Liverpool. No. 2 of the series appeared in the *Leader* of the 28th August last. (See *Appendix* C). In this article there is introduced a somewhat elaborate diagram, representing a geometrical figure, which includes a regular dodecagon inscribed in a circle. The construction of the figure is given quite correctly, and Mr. Whitworth adopts the following datum as a premiss. "Let A B C D E F G H J K L M be a regular dodecagon, of which every side = 2:" and then, by a certain process of reasoning, *fancies* he proves that π is greater than 3·126.

Having completed his method of construction, he says:—"Then, since all the exterior angles of a rectilinear figure are together equal to four right angles, each exterior angle of a XII.gon, must be $\frac{1}{12}$th of four right angles, or $\frac{1}{3}$ of a right angle.

Therefore, T A M = $\frac{1}{3}$ of a right angle.
Therefore, A M Z = $\frac{1}{3}$ of a right angle.
And, M A Z = $\frac{2}{3}$ of a right angle.

And the triangle M A Z is, therefore, half of an equilateral triangle."

In this quotation lies the "*lurking*" fallacy, by which Mr. Whitworth is led to the *absurd* conclusion, that π is greater than 3·126.



I construct the geometrical figure in the margin in the following way. On the straight line A B, I describe the equilateral triangle M A B, and about it circumscribe the circle. I will not insult you by supposing that you cannot find the centre of the triangle, and then construct the circle. I then draw a straight line from the angle A, perpendicular to A B, to meet another straight line drawn from the angle M tangental to the circle, at the point T, and so construct the right-angled triangle A T M.

It is axiomatic if not self-evident, that the triangle A T M is equal to half the equilateral triangle M A B ; and it follows, that A M is to M T in the ratio of 2 to 1 ; that is, that in the triangle A T M, the hypothenuse is to the shortest side, in the ratio of 2 to 1, by construction. It follows, that T A M is an angle of 30°, and A M T an angle of 60°, and that the three interior angles of the right-angled triangle A T M are together equal to two right angles. But, the angles M, A, and B in the equilateral triangle M A B are subtended by arcs of 120° ; therefore, M, A, and B are angles of 120° ; and it follows, that the angles M A B and A M T are together equal to two right angles: and it also follows, that the angle M A B is not equal to $\frac{2}{3}$ of a right angle, but equal to $\frac{1}{3}$ of four right angles. Hence: Euclid's Theorem: Prop. 32: Book 1, which makes the three interior angles of a plane triangle equal to two right angles, is not ***universally true*** : an equilateral triangle, having all its angles touching the circumference of a circle, being an exception.

Poor Whitworth! He fails to perceive, that if the sides of a regular inscribed dodecagon to a circle = 2, the radius of the circle cannot be 1 ; and conversely, if the radius of a circle be 1, the sides of a regular inscribed dodecagon cannot be *finite* and *determinate.*

You may say my arguments fail to convince you of the truth of my conclusions, and that you cannot become a "*disciple*" of mine. This I can't help, but you must not expect me to waste further time in efforts to convince you.

I remain, my dear Sir,

Faithfully yours,

JAMES SMITH.

Let c denote the circumference of a circle: let d denote the diameter: let r denote the radius: let $s\ r$ denote the semi-radius: let a denote the area: and let x denote the area of a regular dodecagon inscribed in the circle.

Let the sides of the dodecagon be represented by the arithmetical symbol 2, as the Rev. Professor Whitworth

puts it, in his attempt to prove π greater than 3·126. Then: $12(2) = 24 =$ the perimeter of the dodecagon. Add $\frac{1}{24}$th part $= \frac{24}{24} = 1$. Then: $24 + 1 = 25 = c$: $\frac{c}{3·125} = \frac{25}{3·125} = 8 = d$: $\frac{d}{2} = \frac{8}{2} = 4 = r$: and $3\frac{1}{8}(r^2) = 3·125(4^2) = 3·125 \times 16 = 50 = a$; therefore, $\frac{24}{25}(a) = 6(r \times sr)$, and this equation $= 48 = x$, and is equal to three-fourth parts of the area of a circumscribing square to the circle; and it follows, that x is equal to the area of a square on the side of an equilateral triangle inscribed in the circle, when all the angles of the equilateral triangle touch the circumference of the circle.

THEOREM.

From a given value of c find x.

Let $c = 100$. Then: $\frac{24}{25}(c) = \frac{24 \times 100}{25} = \frac{2400}{25} = 96$: and $\frac{1}{3}(96) = \frac{96}{3} = 32 = d$: $\frac{d}{2} = \frac{32}{2} = 16 = r$: $\frac{d}{4} = \frac{32}{4} = 8 = sr$; therefore, $3\frac{1}{8}(r^2) = c \times sr)$, and this equation $= 800 = a$: and it follows, that $\frac{24}{25}(a) = 6(r \times sr)$, and this equation $= 768 = x$.

Can the Rev. Professor Whitworth find the value of x from a given value of c? Certainly not, unless he ***plough with my heifer.*** How will the Reverend and learned gentleman explain this?

Let x denote the area of a regular dodecagon: let y denote the area of a circumscribed circle to the dodecagon: let z denote the area of an inscribed square to the circle: and let r denote the radius, and sr the semi-radius of the circle.

Then:

$(6 \times r \times sr) = x$: and, $\frac{25}{24}(6 \times r \times sr) = y$: and if r be represented by a whole number, or by the square root of a whole number, then, $\frac{6(r \times sr)}{24}$ is a *finite* and *determinate* arithmetical quantity. For example:—Let $r =$ 20. Then: $\frac{20}{2} = 10 = sr$; $\frac{6(20 \times 10)}{24} = 1200$; and $(1200 + 50) = 1250 = y$. Again: Let $r = \sqrt{7}$. Then: $\frac{\sqrt{7}}{2} = \sqrt{\frac{7}{2^2}} = \sqrt{1{\cdot}75} = sr$: $\frac{6(\sqrt{7} \times \sqrt{1{\cdot}75})}{24} = \frac{6(\sqrt{7 \times 1{\cdot}75})}{24}$ $= \frac{6(\sqrt{12{\cdot}25})}{24} = \frac{\sqrt{6^2 \times 12{\cdot}25}}{24} = \frac{\sqrt{36 \times 12{\cdot}25}}{24} = \frac{\sqrt{441}}{24} = \frac{7}{24}\left(\frac{6}{2}\right)$ $= \frac{7 \times 3}{24} = \frac{21}{24} = {\cdot}875$, and is a *finite* and *determinate* arithmetical quantity. Well, then, it cannot be controverted, that $\frac{6(r \times sr)}{24}$ is a *finite* and *determinate* arithmetical quantity, whether r be represented by a whole number, or by the square root of a whole number. How happens this? It happens, because $\frac{24}{8} = 3 =$ the perimeter of a regular inscribed hexagon to a circle of diameter unity; and $\frac{24}{6} =$ the MYSTIC number 4; and it follows, that $4(r \times sr) = z$. Hence: $\{(z + \frac{1}{4}z) + \frac{1}{4}(z + \frac{1}{4}z)\} = y$: and conversely, $\{(y - \frac{1}{5}y) - \frac{1}{5}(y - \frac{1}{5}y)\} = z$; and it follows, that $\frac{24}{25}(y) = x$.

THEOREM.

From a given value of x find y.

Unless x be represented by an arithmetical quantity, divisible by 3 or some multiple of 3 without a remainder, it is impossible to solve this theorem with arithmetical exactness. For example: Let $x = 100$. Then $\frac{25}{24}(x) =$

$\frac{25 \times 100}{24} = \frac{2500}{24} = 104 \cdot 1666$ with 6 to infinity $= y$. But, let $x = \frac{24}{25}(100) = 96$. Then: $\frac{25}{24}(x) = \frac{25 \times 96}{24} = \frac{2400}{24} = 100 = y$.

According to Mathematicians, if the *circumference* of a circle be *finite*, away goes the *diameter* into decimals without end: and if the *diameter* of a circle be *finite*, away goes the *circumference* into *decimals* without end: and whether *circumference* and *diameter* be *finite* or *indeterminate*,—according to Mathematicians,—away goes the *area* into decimals without end.

The day will come, when Mathematicians will think it worth *their* while to make a study of the relations that exist between the square roots of numbers and *commensurable* quantities; and will then discover, that one of Euclid's greatest faults was, his attempt to make his fifth book, *on proportion*, alike applicable to *commensurables* and *incommensurables*.

T. S. BARRETT, ESQ., *to* JAMES SMITH.

LANGLEY HOUSE, GROVE LANE,
CAMBERWELL, *October* 11*th*, 1869.

MY DEAR SIR,

In answer to your favour of the 8th instant, which has duly been received and considered. You say (near the top of page 2 of your letter) that x (by which you mean 2π) $= \left(\frac{6}{2\pi} \div \frac{8\pi}{100}\right)$. But I think this must be a slip of your pen; and that you intended to write $\left(\frac{\pi}{2} \div \frac{8\pi}{100}\right) = x$. Well, then, I confess I still do not see how you get $\left(\frac{\pi}{2} \div \frac{8\pi}{100}\right) = 2\pi$. It comes out $6 \cdot 25$, certainly; but $6 \cdot 25$ is the very value of 2π about which we are discussing. You say $\left(\frac{\pi}{2} \div \frac{8\pi}{100}\right) = 6 \cdot 25$, whatever value is given to π, provided that

value be a finite and determinate one. I go further than you here, and assert that $\left(\frac{\pi}{2} \div \frac{8\,\pi}{100}\right) = 6{\cdot}25$, whatever be the value of π, whether determinate or indeterminate.

With respect to what you say in support of your statement that the area of the circle X is equal to the sum of the squares of A B, B E, and A E, you prove it to be so on the datum that $\pi = 3{\cdot}125$, which is the very point in dispute.

Your previous letter urges that $\pi = 3{\cdot}125$, because the area of the circle X is equal to the sum of those three squares. And when I ask you how you prove that the area of the circle is equal to the sum of the squares, you prove it from the value of π, the point in dispute. Thus: $\pi = 3{\cdot}125$, because area of circle = sum of squares. Area of circle = sum of squares because $\pi = 3{\cdot}125$. Is not this reasoning in a circle?

I remain, my dear Sir,
Faithfully yours,
THOMAS S. BARRETT.

It is hardly conceivable that any Mathematician could have penned such NONSENSE as the closing part of the first paragraph of the foregoing Letter, viz.—"*I go further than you here, and assert that* $\left(\frac{\pi}{2} \div \frac{8\,\pi}{100}\right) = 6{\cdot}25$, *whatever be the value of* π, *whether determinate or indeterminate.*"

JAMES SMITH *to* THOMAS S. BARRETT, ESQ.

PROSPECT HOTEL, HARROGATE,
12th October, 1869.

SIR,

Your Letter of yesterday is to hand. I admit the lapsus to which you refer. $\left(\frac{6}{2\,\pi} \div \frac{8\,\pi}{100}\right)$, should have been $\left(\frac{\pi}{2} \div \frac{8\,\pi}{100}\right)$.*

* In making this admission, I have assumed, that in my Letter of the 8th I must have inadvertently written $\left(\frac{6}{2\,\pi} \div \frac{8\,\pi}{100}\right)$ instead of $\left(\frac{\pi}{2} \div \frac{8\,\pi}{100}\right)$.

J. S.

But, had you been a "*reasoning geometrical investigator*," and a sincere enquirer after scientific truth, you would have discovered, that $\left(\frac{6}{2\pi} \div \frac{8\pi}{100}\right)$ = the perimeter of a regular inscribed hexagon to a circle, when the circumference = 4.

Your Letter concludes as follows:—"Your previous Letter urges that $\pi = 3{\cdot}125$, because the area of the circle X is equal to the sum of those three squares. And when I ask you to prove that the area of the circle is equal to the sum of the squares, you prove it from the value of π, the point in dispute. Thus: $\pi = 3{\cdot}125$ because area of circle = sum of squares. Area of circle = sum of squares because $\pi = 3{\cdot}125$. Is not this reasoning in a circle?" This is sheer mathematical impertinence. You should have controverted my solution of the theorem given in my Letter of the 8th instant, and favoured me with your own solution.

THEOREM.

Let the circumference of the circle X be represented by any arithmetical quantity, say 777. Find the arithmetical value of A B the radius of the circle X, and prove that $3\frac{1}{8}$ (A B^2) = the sum of the squares of the sides of the right-angled triangle A B E.

This theorem is founded on the geometrical figure in my Letter of the 6th instant. If you are an *honest* Mathematician, you will favour me with the solution of this theorem. You can of course decline, either to solve it, or attempt to solve it; but, if so, you will please cease to play the part of a mathematical "*quibbler*" with me.

Faithfully yours,
JAMES SMITH.

THOMAS S. BARRETT, ESQ. *to* JAMES SMITH.

LANGLEY HOUSE, GROVE LANE,
CAMBERWELL, *October 12th*, 1869.

MY DEAR SIR,

You have probably received this morning my reply to your last Letter, and have found that my silence has not been owing to inability to reply to your arguments.

Do not ever conclude again from my silence, that I am unwilling to confess to conversion. If I thought you had proved your point, I should *immediately* admit it, why should I not? I should be very glad to find that $\pi = 3 \cdot 125$; for what numberless advantages a determinate value of π would possess over $3 \cdot 14159 \ldots$!

With regard to the latter part of your Letter received this morning, have you not made some mistake therein? You surely don't intend to maintain that each angle of an equilateral triangle is greater than a right angle; and $120°$ is greater than a right-angle, you know.

I remain, my dear Sir,
Faithfully yours,
T. S. BARRETT.

JAMES SMITH *to* THOMAS S. BARRETT, ESQ.

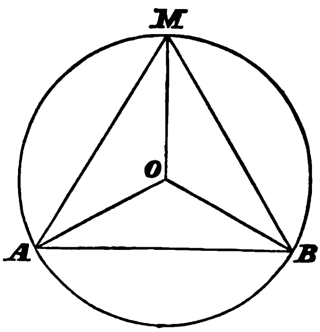
PROSPECT HOTEL, HARROGATE,
13th October, 1869.

SIR,

Your Letter of yesterday is to hand. I am no "*mathematical quibbler.*" My object is truth, and I pursue it where-ever it may lead me, regardless of consequences; knowing that it can never lead me either into error, or absurdity. Well, then, I maintain distinctly, that in an equilateral and equiangular triangle, of which all the angles touch the circumference of a circle, each of the angles is *mathematically* greater than a right angle, and that the sum of the three angles is equal to four right angles. But if,

in an equilateral triangle, one angle is an angle at the centre of a circle, it is an angle of 60°, and is less than a right angle, and the sum of the three angles is equal to two right angles.

In the geometrical figure in the margin, let A B be a straight line. On A B describe the equilateral and equiangular triangle M A B. If straight lines were drawn from each of the angles of the equilateral triangle M A B, so as to bisect the angles and their subtending chords, these lines would intersect each other at the point O, and give the centre of the triangle. It is axiomatic, if not self-evident, that the point O is equidistant from M, A, and B, the angles of the equilateral triangle M A B. With O as centre and O M, O A, or O B as radius, describe the circle, and join O M, O A, and O B. It is self-evident, that O is the centre of the circle, since all the angles of the equilateral triangle M A B touch the circumference of the circle.



Now, O M A, O A B, and O B M, are isosceles triangles, and the angles at the base are equal. But, the angles M O A, A O B, and B O M, at the centre of the circle, are subtended by arcs equal to one-third part of the circumference of the circle, and are therefore angles of 120°, and the sum of the three angles is equal to four right angles; and it follows, that the six angles, O M A, O A M, O A B, O B A, O B M, and O M B touching the circumference of the circle, are equal angles of 60°, and together equal to four right angles. But, each of these angles is equal to half an angle of the equilateral triangle M A B, = 60°; for, the angles of the triangle M A B are subtended by arcs of 120°, and are therefore angles of 120°. But, this makes the sum of the three angles of the triangles O M A, O A B, or O B M, not equal to two right angles, but to 240°; that is, to $1\frac{1}{3}$ times two right angles. Hence: Euclid's Prop. 32: Book 1: is not *universally* true, and of general application.

I might write a dozen sheets, and shew you the consequences that follow; but, if you are a "*reasoning geometrical investigator*," you

can "*trace* these consequences for yourself; and if not, why should I waste my time on a "*bootless*" errand?

Faithfully yours,

JAMES SMITH.

My Letters to Mr. Barrett, written from Harrogate, were intended to be suggestive, rather than demonstrative and exhaustive. Referring to the diagram in the foregoing Letter, the angles A M B, B A M, and A B M are subtended by arcs equal to one-third part of the circumference of the circle, and are therefore angles of 120°. This conclusion rests upon a fact, which I have never heard disputed by any Mathematician, viz.:—If an angle be an angle at the centre of a circle, the angle and its subtending arc are equal. Euclid nowhere proves this, nor could he, without the aid of "*that indispensable instrument of science, Arithmetic.*" Euclid nowhere puts an arithmetical value upon a right angle, nor could he, "*without travelling out of the domain of pure Geometry.*"

Now, Mathematicians have adopted 90° as the measure of a right angle, and I do not think they could have adopted a better; and it follows, that $4 (90°) = 360°$ is the measure of the circumference of a circle; since the angles contained by two diameters of a circle, whether drawn at right angles or not drawn at right angles, are together equal to four right angles. Hence: Whether 3, 4, 6, 8, 9, 24, or any other number of straight lines be drawn from the centre of a circle to its circumference, the sum of all the angles contained by these lines, cannot be either greater or less than four right angles; and it follows, that

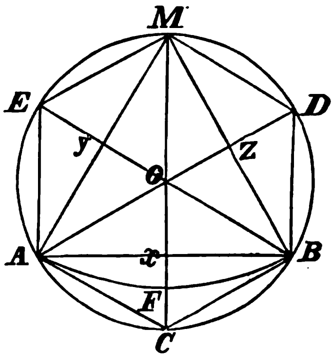
the sum of all the angles is equal to the circumference of the circle; therefore, the sum of the three angles M O A, A O B, and B O M is equal to the circumference of the circle; and it follows, that the angles at the centre of a circle are equal to the arcs by which they are subtended.

It may be said, referring to the diagram, that the angle A M B is not an angle at the centre of the circle. Granted! But we can readily make it an angle at the centre of a circle by describing another circle, with M as centre, and M A or M B as radius. If this circle be constructed, the angle A M B will be subtended by two arcs: *One*, equal to one-third part of the circumference of the circle in the diagram; *and the other*, equal to one-sixth part of another circle. In the one case, A M B will be an angle of 120°, and in the other case, an angle of 60°. Hence: $3\,(120°) = 6\,(60°)$, and this equation = four right angles = 360°; that is to say, equals the number of degrees into which the circumference of a circle is divided for practical purposes.

It would appear, that it had never entered into the mind of any Mathematician, that an equilateral triangle, having one angle at the centre of a circle, is derived from an equilateral triangle, having all its angles *touching* the circumference of the same circle. At any rate, Mathematicians have never turned this fact to any practical account; and yet, it is of the utmost importance in any enquiry into the properties of angles and triangles.

It is true, we may, *mechanically*, that is, with rule and compasses, construct a regular polygon of six sides within a circle, but no merely *mechanical* construction

can be admitted to be a purely *geometrical* construction. By Prop. 1 : Book 1 : Euclid shews us how to construct an equilateral triangle : By Prop. 5 : Book 4 : he shews us how to describe a circle about a given triangle : and by Props. 9 and 10 : Book 1 : and his 2nd Postulate, Euclid shews us how to bisect a given angle, and its subtending chord and arc. Hence : From a given equilateral triangle *only*, can we construct a regular inscribed six-sided polygon to a circle, by pure Geometry.



The geometrical figure in the margin is a facsimile of that in my Letter of the 13th October to Mr. Barrett, with the following additions. Produce M O, A O, and B O to meet and terminate in the circumference of the circle at the points C, D, and E. Join C B, B D, D M, M E, E A, and A C, and so, construct the regular hexagon M E A C B D, which is obviously an inscribed regular hexagon to the circle; because the angles of the equilateral triangle M A B are bisected, and their subtending chords are also bisected, by the lines M C, B E, and A D, at the points *x*, *y*, and *z*.

Now, the angles C O B, B O D, D O M, M O E, E O A, and A O C, are equal angles at the centre of the circle ; and because the measure of a right angle is fixed at 90°, it follows, that each of these angles is equal to • of a right angle = 60°. But, O C B, O B D, O D M, O M E, O E A, and O A C, are equilateral triangles ; and it follows, that the angles of all these triangles are similar

and equal to the angles of the triangle M A B. Well, then, the angle A M B in the triangle M A B is subtended by the two arcs A C B and A F B: the arc A C B being equal to $\frac{1}{3}$ of the circumference of the circle, in the diagram; and the arc A F B being equal to $\frac{1}{6}$ of the circumference of a circle of which M A and M B are radii. Now, assuming the angle A M B to be equal to $\frac{2}{3}$ of a right angle, that is, an angle of 60°, it follows, that an angle of 60° may be subtended by an arc of 120°; but, surely it will not be contended by any Geometer and Mathematician that because an angle of 60° may, under certain circumstances, be subtended by an arc of 120°, that the angle A O B at the centre of the circle is *not* an angle of 120°, having fixed 90° as the measure of a right angle! If any Mathematician could prove this he would upset Euclid's Theorem: Prop. 20: Book 3. It appears to me, that Mathematicians have never thought of applying this Proposition of Euclid to the Geometry of a circle. In my long correspondence with the Rev. Geo. B. Gibbons, I proved, that equal chords may be subtended by unequal arcs, and equal arcs subtend unequal chords; and it follows, that a circle and square of equal superficial area can be constructed, isolated, and exhibited; and this I have proved in many ways in my work on "*The Geometry of the Circle;*" but, as was said by the late Baden Powell—an eminent and well known "*recognised* Mathematician"—"*no amount of attestation by innumerable and honest witnesses would ever convince any one versed in mathematical science, that any man had squared the circle.*" The attempt to find an entrance into the mind of a "*recognised Mathematician,*" by any amount of logical reasoning, has hitherto been to me a hopeless

task; but "*recognised Mathematicians*" are beginning to discover Euclid's faults, and we may now hope for a better state of things.

By Euclid: Prop. 5: Book 1:—"*In an isosceles triangle* (A B C) *the internal angles at the base are equal, and when the equal sides* (A B, B C) *are produced, the external angles at the base are also equal.*" This Theorem of Euclid is unquestionably true, whether the isosceles triangle (A B C) be equiangular or not equiangular: but it may be demonstrated with far less circumlocution than it is proved by Euclid's Proposition. It is not to be wondered at, that this Theorem of Euclid should have obtained the name of "*pons asinorum.*"

Mr. J. M. Wilson, in his "*Treatise on Elementary Geometry,*" demonstrates, by Theorem 9, that:—"*An isosceles triangle has the angles at the base equal:*" and conversely, he demonstrates, by Theorem 10, that:—"*A triangle which has the angles at the base equal is isosceles.*" Now, in a triangle of which all the angles and sides are equal, any side may be the base, and it follows, that the angles at the base are equal. Hence: Euclid's 21st definition is defective, which excludes an equilateral and equiangular triangle, and leaves it to be inferred, that a triangle is *not* isosceles unless two of its sides *only* are equal.

Let three straight lines be drawn from the centre of a circle to meet and terminate in the circumference, so that the angle contained by any two of these lines shall be greater than a right angle: if the extremities of these lines be joined, the figure will be an inscribed polygon of three sides to the circle, and the sum of the angles at the centre of the circle contained by these three lines

will be equal to four right angles. Again: Let six straight lines be drawn from the centre of a circle to meet and terminate in the circumference, so that the angle contained by any two of these lines shall be equal to the angle contained by any other two of them; if the extremities of these lines be joined, the figure will be a regular hexagon or regular six-sided inscribed polygon to the circle; and the sum of the six angles at the centre of the circle, will be equal to four right angles. In either case the angles at the centre of the circle will be equal to the arcs by which they are subtended. Hence: Euclid's 19th definition is positively fallacious. J. Radford Young, a living "*recognised Mathematician*" saw this, for, in his "*Elements of Euclid*," he observes:—"*The term polygon, however, is often employed as a general name for rectilineal figures of all kinds, without regard to the number of sides; so that the rectilineal figures defined above* (definitions XVII. and XVIII.), *may, without impropriety, be called polygons of three and of four sides respectively.*"

This brings to light some remarkable facts, with reference to Mathematics as applied to Geometry. If a regular polygon of any number of sides greater than 4 be inscribed in a circle, the ratio between the sides of the polygon and their subtending arcs is constant, and $\frac{3}{\pi}$ expresses this ratio, whatever be the value of π; but this ratio does not hold good for a polygon of 3 or of 4 sides inscribed in a circle. But, if the circumference of a circle be divided into any number of equal arcs, and from one of these arcs $\frac{1}{33}$th part be deducted, and the remainder be multiplied by the sum of the arcs, the product is con-

stant, and equal to the perimeter of a regular hexagon or six-sided polygon inscribed in the circle : and this is true, whether the inscribed polygon to the circle, be a polygon of 3 sides or of 4 sides, or of any other number of sides. Any one conversant with the simple rules of arithmetic "*can follow out and test this.*"

Geometry is an exact science, and Geometers by excluding arithmetical considerations from their study of Geometry, are led into all sorts of blunders ; and I am glad to find, that "*recognised* Mathematicians" are beginning to discover that Euclid is indeed defective, as a text-book for teaching Geometry to beginners.

THOMAS S. BARRETT, ESQ., *to* JAMES SMITH.

LANGLEY HOUSE, GROVE LANE,
CAMBERWELL, *October* 13, 1869.

MY DEAR SIR,

I am in receipt of yours of yesterday in reply to mine of the day before, and am obliged to you for it.

As I believe I touched on the point in mine of yesterday, I perhaps need scarcely again assure you in all earnestness and candour that I *am* a sincere inquirer after the truth, and that I *am* at least an "*honest* Mathematician." Truth, and truth only, is all I seek or argue for. I do not care about victory ; and, as I said in my note yesterday, I should be only too glad to learn that $\pi = 3{\cdot}125$. If at present my *head* leans towards 3·14159, &c., my *heart* desires 3·125. But if you knew me personally you would not so often charge me with intentional quibbling and other dishonest practices. I am not afraid to own the truth when I am convinced of it. On the contrary, I am generally thought by my acquaintances to be too outspoken. Therefore, for the future, please do not think me dishonest

when I give an argument that you think impertinent or irrelevant; but put the best construction possible on my words; and the best motives, pray, attribute to me.

You ask me to find the arithmetical value of the radius of a circle whose circumference is 777. How can I do this to your satisfaction while I remain unconvinced of your value of π? And then you ask me to demonstrate that $3\frac{1}{8}$ $(A\ B)^2$ = the sum of the squares of the three sides of the triangle A B E. In answer to this, I need only say, that I am quite satisfied with the demonstration that you yourself gave in the letter which contained the diagram. It could not be proved in a better manner.

I remain, my dear Sir,
Yours faithfully,
(*really* faithfully),
THOMAS S. BARRETT.

The closing part of the foregoing Letter is a piece of *gross mathematical quibbling.* Does Mr. Barrett mean to say that I have *really* solved the theorem to which he refers? Certainly not! What does he mean? He means to say and admit that I have solved the theorem by assuming the value of π, "*the point in dispute.*" This I deny, and maintain on the contrary, that I have proved in many ways, that $\frac{25}{8} = 3{\cdot}125$ is the true and exact arithmetical value of the circumference of a circle of diameter unity. If Mathematics will not enable "*recognised Mathematicians*" to solve the theorem given to Mr. Barrett in my Letter of the 12th October, what do they mean by telling us that Mathematics is an exact science? High Mathematics enable us to shew some curious properties of numbers, but from the days of Newton to the present time, high Mathematics have never enabled any Mathe-

matician to find the radius of a circle from a given circumference, and never will. Nor will it ever be in the power of any Mathematician to find by high Mathematics, the area of a circle from a given circumference; or find the circumference of a circle from a given area. "*That indispensable instrument of science, Arithmetic*," is alone competent to enable Mathematicians to find the area of a circle from a given circumference, and the circumference of a circle from a given area.

JAMES SMITH *to* T. S. BARRETT, ESQ.

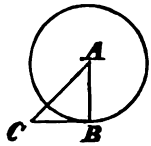
PROSPECT HOTEL, HARROGATE,
14th October, 1869.

MY DEAR SIR,

I am in receipt of your favour of yesterday. I shall *assume* that every word of that communication that precedes the following paragraph, is true. "*You ask me to find the arithmetical value of the radius of a circle whose circumference is 777. How can I do this to your satisfaction while I remain unconvinced of your value of* π?"

The question, my dear Sir, is not whether you can solve the theorem to *my* satisfaction, but whether you can solve the theorem in any way. For my present argument, it matters not, how you have arrived at the conviction, that $\pi = 3{\cdot}14159$, &c., or how I have arrived at the conviction, that $\pi = 3{\cdot}125$: the question remains:— Is either or neither of these values, the true arithmetical value of π?

I will make one more effort to satisfy you that the value of the symbol π can be nothing else but $3{\cdot}125$, and if this fail to convince you, I must put an end to further *controversial* correspondence.



In the geometrical figure in the margin, let A B C be a right-angled triangle, of which the sides A B and B C that contain the right angle, are in the ratio of 4 to 3, and A B, the radius of the circle, by construction.

Now, circumference × semi-radius = $\pi\,(r^2)$, and this equation = area of the circle, whatever be the value of π: and, $\left(\frac{\text{circumference}}{4\,\pi}\right)$ = semi-radius in every circle, whatever be the value of π. No Mathematician can controvert these plain and simple truths, and I am sure you will at once admit them.

Well, then, let the circumference of the circle be represented by the arithmetical quantity 777. Then : $\left(\frac{777}{4\,\pi}\right) = \left(\frac{777}{12{\cdot}5}\right) = 62{\cdot}16$ = semi-radius of the circle: and, circumference × semi-radius = $777 \times 62{\cdot}16 = 48298{\cdot}32$ = area of the circle. But, $2\,(62{\cdot}16) = 124{\cdot}32$ = A B, the radius of the circle. $\frac{3}{4}(\text{A B}) = \frac{3 \times 124{\cdot}32}{4} = \frac{372{\cdot}96}{4} = 93{\cdot}24$ = B C : and $\frac{5}{4}(\text{A B}) = \frac{5 \times 124{\cdot}32}{4} = \frac{621{\cdot}60}{4} = 155{\cdot}4$ = A C : therefore, $3\frac{1}{8}\,(\text{A B}^2) = (\text{A B}^2 + \text{B C}^2 + \text{A C}^2)$; that is, $3\frac{1}{8}(124{\cdot}32^2) = (124{\cdot}32^2 + 93{\cdot}24^2 + 155{\cdot}4^2)$, or, $3{\cdot}125\,(15455{\cdot}4624) = (15455{\cdot}4624 + 8693{\cdot}6976 + 24149{\cdot}16) = 48298{\cdot}32$ = area of the circle: for, $\sqrt{\frac{48298{\cdot}32}{3{\cdot}125}} = \sqrt{15455{\cdot}4624} = 124{\cdot}32$ = A B the radius of the circle ; and it follows, that $3\frac{1}{8}\,(\text{A B}^2)$ = the sum of the squares of the three sides of the triangle A B C, and equals area of the circle. Now, Sir, if you can find a value of π, either greater or less than 3·125, by which you can obtain these results, you will prove that Professor de Morgan is right in calling me a "*Pseudomath ;*" that is, as he explains the meaning of the word, *a person who handles Mathematics as the monkey handled the razor.*" *

I am, my dear Sir,

Faithfully yours,

JAMES SMITH.

* See *Athenæum*, January 4th, 1868. Article : Pseudomath, Philomath, and Graphomath, by A. de Morgan

THOMAS S. BARRETT, ESQ. *to* JAMES SMITH.

LANGLEY HOUSE, GROVE LANE,
CAMBERWELL, *October* 14*th*, 1869.

MY DEAR SIR,

Seeing that you have distinctly stated your belief that the angles in an equilateral triangle may occasionally be each greater than a right angle; and seeing that you maintain that each angle of the equilateral triangle in the diagram in your Letter is greater than a right angle, a conclusion contradicted by your eyesight. I am persuaded that further discussion between us will be perfectly useless.

Thanking you for your several Letters, and your attempts to convert me to your views.

I remain,
My dear Sir,
Yours faithfully,
THOMAS S. BARRETT.

The statement that under certain circumstances the angles of an equilateral triangle may be subtended by arcs of 120°, affrighted Mr. Barrett from his propriety, "*upset his equilibrium*," and obviously put him into a perfect state of "*bewilderment*:" but, when he recovers his bewildered senses, he will discover that if he were right, the angles of an equilateral triangle could not, under any circumstances, be either greater or less than 60°, and so "*upset*" Euclid's Theorem: Prop. 20: Book 3. What especial reason is there for fixing 90° as the measure of a right angle? Can any Mathematician prove the impossibility of fixing 180° as the measure of a right angle? Certainly not!

Well, then, if the measure of a right angle were fixed at 180°, would not the angles of an equilateral triangle be angles of 120°, and equal to $\frac{4}{3}$ of a right angle? The only difference would be, that the sum of the angles would be 360°, and each of the angles be subtended by an arc of 240°, making the circumference of a circumscribing circle to the triangle = 720°. These facts are consistent with Euclid's Theorem: Prop. 20: Book 3: which is, indeed, irrefragable; and never was—and never will be—disputed by any *Geometer and Mathematician.*

JAMES SMITH *to* T. S. BARRETT, ESQ.

PROSPECT HOTEL, HARROGATE,
15*th October*, 1869.

SIR,

Your Letter of yesterday is to hand. I admit that geometrically a right angle is the angle contained by two straight lines drawn at right angles to each other. Thus: Two diameters of a circle drawn at right angles to each other, produce four right angles at the centre of the circle; and each of these angles is subtended by an arc equal to one-fourth part of the circumference of the circle; and I admit that 90° is the measure of a right angle. An angle of 45° at the centre of a circle, is subtended by an arc equal to one-eight part of the circumference of the circle. An angle of 30° at the centre of a circle, is subtended by an arc equal to one-twelfth part of the circumference of the circle. And, an angle of 15° at the centre of a circle, is subtended by an arc equal to one twenty-fourth part of the circumference of the circle, and so on *ad infinitum.* Hence: Although the angles of an equilateral triangle inscribed in a circle, with all the angles touching the circumference, are *geometrically* less than a right angle, they are *mathematically* greater; and because they are subtended by arcs equal to one-third

part of the circumference of the circle, must be angles of 120°. Can you shew me where Euclid fixes 90° as the measure of a right angle? Can you tell me who it was that *arbitrarily* fixed 90° as the measure of a right angle? Can you demonstrate that this is a proved premiss? I trow not! Well, then, if you were a "*reasoning geometrical investigator*," and an "*honest Mathematician*," you would perceive that these facts furnish another proof to the many I have already given, in my work on the "*Geometry of the Circle*," that "*Mathematics, as applied by Mathematicians to Geometry*," is "*a mockery, a delusion, and a snare.*"

With reference to the diagram in my Letter of the 13th instant, my "*eyesight*" carries me from the angles of the equilateral triangle, to their subtending arcs, and when you speak of my arriving at a conclusion contradicted by my *eyesight*, you charge me with that which is untrue, and which you must know to be untrue.

I had no hope of you from the beginning, and I am now convinced, that you are one of that numerous class, whose heads are so stuffed with "*crammed erudition*," that "*there is not a cranny left for reasoning to get in at.*"

You need not be afraid of any further attempts on my part, to convert you to my views.

Faithfully yours,

JAMES SMITH.

T. S. BARRETT, ESQ., *to* JAMES SMITH.

LANGLEY HOUSE, GROVE LANE,
CAMBERWELL, *October 16th*, 1869.

MY DEAR SIR,

You are unjust to accuse me of saying what I know to be untrue. Not knowing that there was any distinction in your mind between an angle being "*geometrically*" greater than another, and its being "*mathematically*" greater, it was natural for me to conclude that your conclusion, that the angle of an equilateral triangle may be greater than a right angle, was a conclusion contra-

dicted by your eyesight. In the same way as the conclusion of Romanists concerning transubstantiation contradicts *their* eyesight. I will not argue against your distinction between "geometrically greater" and "mathematically greater," nor against anything contained in your Letter received yesterday, since you agree with me that our minds are so differently constituted that further discussion between us will be quite useless.

I remain, my dear Sir,
Faithfully yours,
THOMAS S. BARRETT.

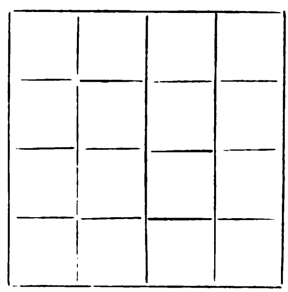
The foregoing Letter of Mr. Barrett is a perversion—whether intentional or not intentional—of the statements in my Letter to him of the 15th October.

Now, on pages 228 and 229, I have shewn "*that the angles at the centre of a circle are equal to the arcs by which they are subtended.*" I might, and perhaps should have said, when expressed in degrees, but I did not consider this necessary, as I thought my meaning could not be misconstrued; for, any one conversant with Elementary Trigonometry knows, that the angles at the centre of a circle are proportional to the arcs on which they stand. But, are not the angles at the circumference of a circle proportional to the arcs on which they stand? It is no doubt true, that the proportions are not alike in both cases, the ratio in the former case being as 1 to 1, and in the latter case as 1 to 2, but, nevertheless, in both cases the angles are proportional to the arcs on which they stand.

I sent a few of the preceding pages (in proof) to a well-known "*recognised Mathematician.*" He wrote me

a Letter, in which he dissented from that part of the last paragraph which I have put in italics: and his Letter was accompanied by a Paper, from which I quote the following:—"Thus, if O (see Diagram, page 227,) be the centre of circle circumscribing the equilateral triangle M A B, the angle A O B contains 120 degrees, and the arc A B contains 120 degrees of arc. The angle A M B not being at the centre, does not contain the same number of degrees, but only half the number (60 degrees), and any angle at the circumference of a circle will contain (Euclid: Book 3) half as many degrees as there are degrees of arc in the circumference on which it stands."

"The angle A O B is an angle of 120 degrees, and the arc A B is an arc of 120 degrees of arc. But we cannot say that the angle equals the arc, as there cannot be equality (or even ratio) between two magnitudes of different kinds.



It is for a like reason that we cannot say that the area and periphery of a square of side = 4 are equal. Each is sixteen something, but when we ask the question:—Sixteen what? we get a different answer in each case. If any one writes, Periphery = 16, Area = 16, they can only be regarded as an abbreviated way of expressing: Periphery = 16 times the unit of length; Area = 16 times the unit of area: and as there is no equality between the unit of area and the unit of length, it does not follow that Periphery = Area. I give this as an illustration of the error which appears to me to have crept into the argument on page 239." I presume

a "*recognised Mathematician*" would consider it an outrage on common sense to say, that *Perimeter* and *Area* of a square are expressed arithmetically by the same symbols, when the sides of the square = 4. Can a central angle of 30° be subtended by an arc either greater or less than 30°? Is not the ratio of angle to arc as 1 to 1?

Now, with half the side of the square as radius, inscribe a circle within the square. Let the sides of the square = 4. Then: The units of area in the circle, are exactly the same as the units of length in the periphery of the circle. But, the units of length in the periphery or circumference of the circle may be *linear* inches, feet, yards, or miles, and the units of area in the circle will then be *square* inches, feet, yards, or miles, as the case may be. To say that we cannot compare the circumference and area of any circle, would at once render practical geometry an impossibility. But, have not "*recognised Mathematicians*" made a comparison between the circumference and area of a circle? If not, how have they made the discovery that $\frac{\pi}{4}$ denotes the area of a circle of diameter unity? Verily! Verily! the inconsistencies and absurdities of "*recognised Mathematicians*" are perfectly marvellous.

The "*recognised Mathematician*" referred to in the previous paragraphs, gave me, in the Paper which accompanied his Letter, the following definition of a Degree. "Degree is the 90th part of a right angle. It is therefore *an angle*, and consequently, may be made use of as a standard, by comparison with which other angles may be measured." I admit, and adopt, this definition of

a degree. He also gave me the definition of a degree of arc, viz. :—" A degree of arc is that part of the circumference of a circle which subtends a degree (of angle) at the centre. It is easily seen to be the 360th part of the whole circumference. It follows that the number of degrees in an angle *at the centre of a circle*, will be equal to the number of degrees of arc in the circumference which subtends it." I also admit, and adopt, this definition of a degree of arc.

Well, then, a Degree has to do *directly* with a curvilinear figure, and only *indirectly* with a rectilinear figure. But there are cases, in which the ratio of a chord to its subtending arc can be expressed in degrees. I will give one instance. A chord of $14^\circ\ 24'$ is subtended by an arc of 15°, and by analogy or proportion, $14^\circ\ 24' : 15^\circ :: 3 : 3{\cdot}125$: and because 3 is the perimeter of a regular inscribed hexagon to a circle of diameter unity, it follows, that $3{\cdot}125$ is the circumference of a circle of diameter unity: and it also follows, that $\frac{24}{25}$ (circumference) = the perimeter of a regular inscribed hexagon, in every circle.

Now, we cannot get the square of a *curved* line, as a *curved* line, and although $30^2 = 900$, it would no doubt be very absurd if I were to say, that the square of 30 *linear* degrees equals 900 *square* degrees. It is, however, conceivable that a *curved* line may be equal in length to a given straight line. Nay, this is not only conceivable, but demonstrable. For example: Let the circumference of a circle be 240, it may be 240 feet, yards, or miles. A quadrant of the circumference of this circle will be 60 feet, yards, or miles, as the case may be, and it cannot be denied, that the length of a straight line may be 60 feet, yards, or miles. But mark what follows! If we put an arithme-

tical value upon a degree, say 60, it may be 60 feet, yards, or miles. Has not the length of an astronomical and nautical degree been actually fixed at 60 miles? Hence: If p denote the perimeter and a denote the area of a square, and the sides of the square $= 4$; then, although p and a denote magnitudes of different kinds (the one denoting *linear* units and the other *square* units) the ratio of p to a is as 1 to 1. But, if the sides of the square $= 8$, then, $p = 32$, and $a = 64$, and the ratio of p to a is as 1 to 2. If the sides of the square $= 16$, then $a = 256$, and the ratio of p to a is as 1 to 4, and so on *ad infinitum*. Well, then, $(30 \times 60)^2 = 900\,(60^2)$, and this equation $= 3240000$ square units, so that in Mathematics we may draw comparisons between magnitudes of different kinds, and get both ratios and equations from them. Now, a quadrant of the circumference of a circle $= 90$ degrees; and (90 degrees $\times$ 60 minutes) $= 5400$ minutes. Multiply by 4. Then: $(5400 \times 4) = 21600$ minutes $=$ circumference of the circle; and $\frac{24}{25}$ (21600 minutes) $= \frac{24 \times 21600 \text{ minutes}}{25} = \frac{518400 \text{ minutes}}{25} = 20736$ minutes. Divide by 60. Then: $\frac{20736 \text{ minutes}}{60} = 345^\circ\ 36'$ $=$ the perimeter of a regular inscribed hexagon to a circle of circumference $= 360^\circ$: and by analogy or proportion: $345^\circ\ 36' : 360^\circ :: 3 : 3{\cdot}125$, and we arrive at the same conclusion as before, by an entirely different process of reasoning.

Take the diagram on page 227. Produce M O to meet and terminate in the line A B at a point D. I shall assume this simple addition to have been made to the diagram. Then: A B will be bisected in D, and M D A and O D A will be right-angled triangles. But, O M A is an isosceles

triangle, therefore, the angles O M A and O A M at the base are equal, and assuming the angle M in the equilateral triangle M A B to be an angle of 60°, O M A and O A M are angles of 30°. But, the angle M A B is bisected by the line A O, therefore, O A D is an angle of 30°, and it follows, that the angle O A D = the angle D M A. Now, D A is the *geometrical* sine of the angle D M A; and O D is the *geometrical* sine of the angle O A D. But, D A and O D are not lines of equal length, and I put the question:—Can equal angles have unequal sines? I answer! *Geometrically*, they may, but, certainly not, *trigonometrically*.

If in a right-angled triangle, the lesser of the acute angles be an angle of 30°, the *geometrical, trigonometrical*, and *natural* sine, is arithmetically the same. This fact is traceable to the properties of an equilateral and equiangular triangle. But, according to *recognised* Mathematicians, the *geometrical, trigonometrical*, and *natural* sines are alike in all equal angles, which is not true, and this is one of the blunders into which Mathematicians have floundered.

The geometrical figure on page 230, includes eighteen similar right-angled triangles, each of which is the half of an equilateral triangle; and it follows, that in all these triangles, the ratio between the hypothenuse and shortest side is as 2 to 1. The lesser of the acute angles in all these triangles is an angle of 30°, and the *geometrical, trigonometrical*, and *natural* sine is arithmetically the same to a circle of radius unity $= \frac{1}{2} = \cdot 5$. But none of these triangles are commensurable right-angled triangles, and the ratio between the sides that contain the right angle cannot be ascertained with arithmetical exactness; and it follows, that Mathematics do not enable us to find

the arithmetical values of the two parts into which the diameters M C, D A, and E B are divided, at x, z, and y. But it does not follow, that there is not an exact ratio between the diameter and circumference of the circle!

Take the diagram on page 227. From the point O the centre of the circle, draw two straight lines O C and O D parallel to M A and M B, to meet and terminate in the circumference of the circle, and join C D. Then: O C D and M A B will be similar equilateral triangles, and the angles C O D and A M B will be equal angles. Can equal angles be subtended by unequal arcs, *trigonometrically?*

The careful reader of my Letter of the 28th July, 1869, to Mr. Gibbons (page 40), *if* "*a reasoning geometrical investigator*," will discover that Mr. Gibbons' arguments and reasonings would *upset* one of the first principles of Elementary Trigonometry, viz.—"*The angles at the centre of a circle are proportional to the arcs on which they stand.*" This Letter should be taken in connection with what I have demonstrated by means of the diagram on page 16; and also with what I have said from pages 62 to 69.

I have been driven to make startling assertions, and so, play the part of a "*Will o' the Wisp*," in the hope of leading Mathematicians to see the quagmire of geometrical inconsistency and absurdity into which they have fallen.

JAMES SMITH *to* T. S. BARRETT, ESQ.

PROSPECT HOTEL, HARROGATE,
18th October, 1869.

SIR,

I am in receipt of your favour of the 16th inst. Sunday intervening, I could not reply until to-day. You and I are like

two persons gazing upon a distant horizon. One sees a something which he takes to be a cloud: the other takes it to be a mountain. Either or neither may be right, but both cannot be.

At present your HEAD "*leans towards*" $\pi = 3{\cdot}14159$, &c., while your HEART "*desires*" it to be $3{\cdot}125$. My head makes $\pi = 3{\cdot}125$, and my head and my heart are in harmony. You observe:—"*Further discussion between us will be quite useless.*" On this point we are agreed, and it follows, that we must leave it to future Geometers and Mathematicians to settle the difference between us, and decide how far I have been "*unjust*" towards you.

With many thanks for your numerous communications,

I remain, Sir, faithfully yours,

JAMES SMITH.

T. S. BARRETT, ESQ. *to* JAMES SMITH.

LANGLEY HOUSE, GROVE LANE,
CAMBERWELL, *October* 19*th*, 1869.

MY DEAR SIR,

I am in receipt of yours of yesterday. Pardon me for again troubling you, but there is one thing which I meant to have said in my Letters to you. You have frequently spoken of the absurdity of the notion that π is an indeterminate quantity. It is no doubt awkward having to deal with indeterminate amounts; but there is nothing absurd in π being indeterminate. It is no more absurd that π should be indeterminate, than that the ratio between the diagonal and a side of a square should be so.

In your note received this morning, you speak of future Mathematicians and Geometers deciding between us. If by this you intend to hint that you mean to print our correspondence, I hope you will do me the favour of sending me a copy, that I may put it among my other curiosities.

I remain, My dear Sir,

Faithfully yours,

THOMAS S. BARRETT.

Mr. Barrett admits that $\frac{1}{4\pi}$ denotes the area of a circle of circumference unity: and he also admits that if $\pi = 3{\cdot}125$, the area of a circle of circumference unity $= {\cdot}08$. Taking these two admissions—and they are inseparably connected—we can demonstrate, that the circumference of a circle of diameter unity $= \frac{25}{8} = 3{\cdot}125$, and can be nothing else.

Because $\frac{\text{circumference}}{2\pi} =$ radius, in every circle; it follows, that $\frac{\text{circumference}}{4\pi} =$ semi-radius, in every circle, whatever be the value of π; and it will not be disputed, that the symbol π denotes the circumference of a circle of diameter unity.

Let c denote the circumference of a circle and be represented by unity: let d denote the diameter of the circle: and, let $s\,r$ denote the semi-radius of the circle.

Then: Our unit of length being 1, $\frac{1}{4(3{\cdot}125)} = \frac{1}{12{\cdot}5} = {\cdot}08 = s\,r$: $4(s\,r) = (4 \times {\cdot}08) = {\cdot}32 = d$: and, $3{\cdot}125\,(d) = (3{\cdot}125 \times {\cdot}32) = 1 =$ unity $= c$. But, $2\,(s\,r) = (2 \times {\cdot}08) = {\cdot}16 =$ radius of a circle of circumference unity, and $3\frac{1}{8}({\cdot}16^2) = 3{\cdot}125 \times {\cdot}0256 = {\cdot}08$; and it follows, that semi-radius and area of a circle of circumference unity, are represented by the same arithmetical symbols. This is *unique*, and can be predicated *only* of a circle of circumference unity, and work out with arithmetical exactness. But, $\frac{c}{12{\cdot}5} = s\,r$, whatever be the value of c. For example: Let $c = 360$ *linear* yards. Then: $\frac{360}{12{\cdot}5} = 28{\cdot}8$ *linear* yards $= s\,r$: $2(s\,r) = 2 \times 28{\cdot}8 = 57{\cdot}6$ *linear* yards $=$ radius: and it follows, that $c \times s\,r = 3{\cdot}125\,(r^2)$;

that is, $360 \times 28{\cdot}8 = 3{\cdot}125\ (57{\cdot}6^2)$; or, $(360 \times 28{\cdot}8) \doteq (3{\cdot}125 \times 3317{\cdot}76)$, and this equation $= 10368$ *square* yards = area of the circle, when the circumference = 360 *linear* yards; and it cannot be controverted, that $c \times s\,r =$ area in every circle.

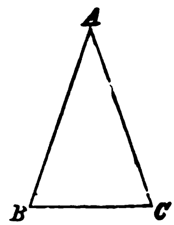
When Mathematicians can find a value of π greater than 3·125, and make the area of a circle of circumference = 360 *linear* yards, either equal to, or greater than 10368 *square* yards, they will be able to prove that James Smith is a "*Pseudomath*," and "***handles Mathematics as the monkey handled the razor.***"

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
1st November, 1869.

MY DEAR SIR,

Your favour of the 2nd October was forwarded to me when at Harrogate. I did not acknowledge its receipt, and, indeed, I should not have thought it necessary to reply to it, but for the following paragraph.



"*I would correct one small error of yours. I said I had avoided Logarithms in finding the chord B C, when A = 15°. A C = B C = 1. B C exceeds* ·261, *whenever you employ them both for numbers and trigonomettical functions.*"

Now, my dear Sir, the error is yours, not mine. A B may equal A C, and both may = 1, our unit of length being 1: but it does not follow, that A = 15°.

Let A be an angle of 14° 24′, and let A B = A C = 1. I have shewn you that on this hypothesis (and your assumption that A = 15° is but an hypothesis) B C = 13 degrees and $\frac{824}{1000}$th parts of a

degree; and you have never attempted to controvert my proof. When A denotes an angle of $14° 24'$, the circular measure of the angle A is ·25, and the ratio between B C and the circular measure of the angle A is as ·24 to ·25. You forget that the trigonometrical functions of angles are not lengths, but ratios of one length to another.

I do not intend to re-open our controversy. Enough has been said on both sides, and we must leave it to future Mathematicians to decide between us.

Believe me, my dear Sir,
Very truly yours,
JAMES SMITH.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

LANEAST, LAUNCESTON,
3rd November, 1869.

MY DEAR SIR,

Thanks for your Letter and the Liverpool " Leader."

I used the angle $15°$ in my calculation, and the value of the chord comes out greater than ·261.

But you are quite right in treating our controversy as closed.

Believe me,
Yours truly,
G. B. GIBBONS.

R. F. GLAISTER, ESQ. *to* JAMES SMITH.

11, LYNDHURST ROAD,
PECKHAM, SURREY, S.E.,
December 12*th*, 1869.

DEAR SIR,

It was far from my intention to have again troubled you, but I happened quite by chance the other day to again look at your pamphlet, "The British Association in Jeopardy, &c.," and more especially at what you evidently consider your great coup, pages 56, 57, 60, 61.

I think if you will look carefully at your reasoning, you will find it merely amounts to this: You divide your circumference into 360 = arcs or degrees. You divide your diameter into 115·2 lines or linear units.

Your "$8 \times c = 25 \times d$" is simply this:—

8 times the number of degrees in the circumference = 25 times the number of lines in the diameter, or 2880 = 2880, an absolute identity. But you do not make the *slightest* step towards finding the ratio of the *curved line* or arc called degree to the *straight line* unity, of which your diameter is made to contain 115·2.

If I make the diameter = 720,000,000, I might say the diameter is two million times the circumference, if I only take 360 for such circumference. And so it is if I merely consider the *numbers*. 25,000 *ants* are more *numerous* than 10 *men*. How does this help you to find the number of times an ant's mop is contained in that of a man?

Yours truly,
R. F. GLAISTER.

P.S.—I must repeat (without any offence meant) what Wilson says: Your conclusions are as sound as your premises, with which they are in fact identical. R. F. G.

JAMES SMITH *to* R. F. GLAISTER, ESQ.

BARKELEY HOUSE, SEAFORTH,
13th December, 1869.

DEAR SIR,

I am in receipt of your Letter of yesterday's date, and in reply may direct your attention to the following facts.

In Euclid's Elements of Plane Geometry, we get the following definition of a polygon. "*A polygon is a rectlinear figure having more than four sides.*" Well, then, an inscribed polygon to a circle may have 25 equal sides, just as certainly as it may have 24 equal sides. Now, if we draw two diameters of a circle at right angles, the four angles at the centre of the circle will be angles of 90°, which makes the circumference = 360°. The measure of a right angle has been fixed by Mathematicians at 90°. I had nothing to do with fixing it, and simply adopt it, but I know that if we made 100° or 20° the measure of a right angle, the only difference would be, that in the one case the circumference of the circle would be 400°, and in the other 80°, and on either hypothesis we might make our computations; but having once fixed 90° as the measure of a right angle, we must stick to it in all our enquiries into the properties of angles, triangles, polygons, and circles. Euclid nowhere puts an arithmetical value upon a right angle, and fixing the measure of a right angle, is an application of Mathematics to the "*exact science*" of Geometry. One would think that no rational man would assert that Geometry is a mere branch of Mathematics, and yet, "*recognised Mathematicians*" make this assertion without any hesitation."

Now, $\frac{24}{25}(360^\circ) = 345^\circ\ 36'$, and $\frac{345^\circ\ 36'}{360^\circ}$ is a ratio. Divide the two terms of this ratio by 24, and we get the equivalent ratio $\frac{14^\circ\ 24'}{15^\circ}$. The fraction $\frac{24}{25}$ is a ratio. Divide the two terms of this ratio by 8, and we get the equivalent ratio $\frac{3}{3 \cdot 125}$: and by analogy or proportion, $3 : 3{\cdot}125 :: 345^\circ\ 36' : 360^\circ$. But, $\frac{345^\circ\ 36'}{360^\circ} = {\cdot}96$:

$\frac{14^\circ\ 24'}{15^\circ} = \cdot 96$: $\frac{24}{25} = \cdot 96$: and $\frac{3}{3\cdot 125} = \cdot 96$. Now, $\frac{24}{25}(1) = \cdot 96$, and $\frac{\cdot 96}{1}$ is a ratio. Hence: $\frac{345^\circ\ 36'}{360^\circ}$, $\frac{14^\circ\ 24'}{15^\circ}$, $\frac{24}{25}$, $\frac{3}{3\cdot 125}$, and $\frac{\cdot 96}{1}$ are equivalent ratios, and all express the ratio between the perimeter of every regular hexagon, and the circumference of its circumscribing circle.

Well, then, $\frac{24}{25}(1) = \cdot 96 =$ the perimeter of a regular inscribed hexagon to a circle of circumference unity : $\frac{\cdot 96}{6} = \cdot 16 =$ radius of the circle, when the circumference is 1 : $\frac{\cdot 16}{2} = \cdot 08 =$ semi-radius of the circle when the circumference is 1 ; and $3\frac{1}{8}(r^2) =$ circumference $\times$ semi-radius ; or, $3\cdot 125\,(\cdot 16^2) = c \times sr$; that is, $3\cdot 125 \times \cdot 0256 = 1 \times \cdot 08$, and this equation $= \cdot 08 =$ area of the circle when the circumference is represented by unity. Can you, Sir, tell me the area of a circle when the circumference is 360? Certainly not without "*ploughing with my heifer.*" Well, then, when you can prove that $\frac{345^\circ\ 36'}{360^\circ}$, $\frac{14^\circ\ 24'}{15^\circ}$, $\frac{24}{25}$, $\frac{\cdot 96}{1}$ and $\frac{3}{3\cdot 125}$ are not equivalent ratios, you will be able to demonstrate that 8 circumferences of a circle are not equal to 25 diameters, but not till then.

When a Mathematician undertakes to prove me wrong, I *assume*, and fairly *assume*, that he considers himself possessed of superior intellect, and consequently, also possessed of superior powers of reflection. I may attempt, and have attempted, to bring these qualities of the mind into play with many "*recognised Mathematicians:*" more I cannot do, but in your case and that of all other "*recognised Mathematicians,*" that I have come into contact with, I have failed; (you must not understand that no Mathematician agrees with me), nevertheless, $\frac{3}{3\cdot 125}$ expresses the ratio between the perimeter of every regular hexagon and the circumference of its circumscribing circle; and $\frac{1}{3\cdot 125}$ expresses the ratio between the diameter and circumference in every circle. "*If you can't see these facts, I can't help it, but the facts remain notwithstanding.*"

Referring to the diagram on page 47 of *The British Association in Jeopardy, &c.*, $b\,c$ is a side of a regular inscribed hexagon to the circle, and the chords $b\,d$, $d\,e$, $e\,f$, and $f\,c$, are sides of a regular polygon of 24 sides inscribed in the circle, by construction. $\frac{360^\circ}{24} = 15^\circ$ = the arcs $b\,d$, $d\,e$, $e\,f$, and $f\,c$. $\frac{24}{25}(15^\circ) = 14^\circ\ 24'$ = the chords $b\,d$, $d\,e$, $e\,f$, and $f\,c$. $\frac{360^\circ}{6} = 60^\circ$ = the arc $b\,d\,e\,f\,c$, and $\frac{24}{25}(60^\circ) = 57^\circ\ 36'$ = the chord $b\,c$. Hence: (6 chord $b\,c$) = 24 (chord $b\,d$), that is, $6(57^\circ\ 36') = 24(14^\circ\ 24')$, and this equation $= 345^\circ\ 36'$ = the perimeter of a regular inscribed hexagon to the circle, and *apparently* makes the chord $b\,c$ equal to the sum of the four chords $b\,d$, de, $e\,f$, and $f\,c$.

Now, Sir, make what you can of this. It is one of the "*Curiosities of Mathematics.*" I have explained this mathematical curiosity over and over again to "*recognised Mathematicians,*" but I cannot give men, any more than I can give "*ants,*" more intellect or greater powers of reflection than they possess, and if you can't trace this *mathematical curiosity* to its cause, "*I can't help it,*" but I think that with this communication our controversy had better terminate.

Yours truly,

JAMES SMITH.

P.S.—I send you herewith a *Liverpool Journal,* in which you will find an article headed "*Curiosities of Mathematics,*" of which the Reverend Professor Whitworth is to my mind the writer.

Mr. Glaister revised his two elaborate Papers (see pages xxiii and xxx in the Address to the Reader), and with the corrected proofs he sent me the following notes:—

11, LYNDHURST ROAD,
PECKHAM, SURREY, S.E.,
November 12th, 1869.

MY DEAR SIR,

Thanks for your courtesy in sending me the enclosed, which I have received and return. I post at half-past ten a.m., having received at nine.

Yours truly,
R. F. GLAISTER.

11, LYNDHURST ROAD,
PECKHAM, SURREY, S.E.,
19th November, 1869.

DEAR SIR,

I return your proof sheets, with many thanks for your kind attention.

Yours truly,
R. F. GLAISTER.

R. F. GLAISTER, Esq. *to* JAMES SMITH.

LYNDHURST ROAD,
PECKHAM, SURREY, S.E.,
December 15th, 1869.

DEAR SIR,

I thank you for your Letter and the accompanying "*Liverpool Leader*" Newspaper. The proof there is quite sufficient to convince any one. It is very clear, very neat, and free from complicated diagrams, whether it be Whitworth's, De Morgan's, or that of "*any other man.*" (See Appendix C : Letter of 28th August.)

Geometry *is* a branch of Mathematics, Mathematics = the doctrine of Number + the doctrine of Figure—strictly speaking all Mathematics is or are (which ever verb you like) Arithmetic or Geometry. This Arithmetic in its extended form becomes what we ordinarily call common Arithmetic + Algebra. Algebra again is common Algebra + Differential Calculus, &c., &c. We might divide and sub-divide

without end. The question of the quadrature of the circle is not one of *Mixed* Mathematics. The following is a question of *Mixed* Mathematics:—"To find the line of quickest descent from one point, A, to another, B—the two points not being in the same vertical line." Here we have to apply Mathematics to *Mechanics*, the principles of the latter science being mainly derived from observation.

Again, from the optical and experimental truth, that the sine of the angle of incidence has a certain ratio to that of refraction, we arrive at great results by the application of Mathematics. Optics, therefore, is a branch of Mixed Mathematics.

In your last Letter, you repeat what you have already said in your Pamphlet. You *assume* all you have to prove. You really prove nothing. If $a = b$, then $b = a$, is the be-all and the end-all of your so-called quadrature.

I am, Sir,

Yours truly,

R. F. GLAISTER.

JAMES SMITH *to* R. F. GLAISTER, ESQ.

BARKELEY HOUSE, SEAFORTH,
16th December, 1869.

SIR,

Your Letter of yesterday's date is to hand.

If the sides of a regular dodecagon be represented by the Arithmetical symbol 2, what is the Arithmetical value of the circumference of its circumscribing circle?

Your knowledge of Mathematics, will surely enable you to answer this plain and simple question. If you decline to answer it, you must not think me discourteous if I decline to print any more of your *impertinent* effusions, although you have made the request that I should print all you write me.

Yours truly,

JAMES SMITH.

R. F. GLAISTER, ESQ., *to* JAMES SMITH.

11, LYNDHURST ROAD,
PECKHAM, SURREY, S.E.,
December 17*th*, 1869.

SIR,

If $\frac{355}{113}$ be taken as π, it is merely an approximation true to 6 decimal places, the circumference of your required circle will be $= \frac{1420}{113} \times \frac{\sqrt{2}}{\sqrt{3}-1}$; or in a more computable form, $\frac{710}{113} \times (\sqrt{6}+\sqrt{2})$, say 24·2, &c. I shall not trouble you with an extended computation. Your knowledge of Common Arithmetic will surely enable you to manage this little matter of multiplication, division, and evolution. $\frac{355}{113}$ as an *approximate* and convenient π, was found by Metius about the end of the 16th century, but its discoverer was far too sensible a man to consider it as an exact π. It is true to 6 places of decimals and a little in excess of the true value. Well, the above is my approximate answer to your problem, it *cannot* be exactly answered. I obtain it thus—If r=radius, $2\,r \sin \frac{180^\circ}{12}$ = side of dodecagon $= 2\,r.\ \text{Sin.}\ 15^\circ = 2\,r\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$. But you make side $= 2 \mathrel{.\dot{}.} 2 = 2\,r\,\frac{\sqrt{3}-1}{2\sqrt{2}} \mathrel{.\dot{}.} 2\,r$ or diameter $= 4\sqrt{2} \div \sqrt{3}-1 =$ $\frac{4\sqrt{2}}{\sqrt{3}-1} = \frac{4(\sqrt{6}+\sqrt{2})}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{4(\sqrt{6}+\sqrt{2})}{2} = 2(\sqrt{6} + \sqrt{2})$, which multiplied by π gives the circumference.

There is the answer to your "plain and simple question."

I revise and send back your proof sheet. Your courtesy is coextensive with your mathematical knowledge.*

I am, Sir,
Your obedient Servant,
R. F. GLAISTER.

P.S.—This is my very last Letter to you.

R. F. G.

* For a fac-simile of the original of this remarkable Letter, see Appendix B.

So ended my Correspondence with R. F. Glaister, Esq., and I shall leave readers to form their own opinion of him.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

WERRINGTON, LAUNCESTON,
CORNWALL, 21*st February*, 1870.

MY DEAR SIR,

I have just received the Liverpool *Leader*, of 19th inst., sent by you, and beg to thank you for it.

I do not know Professor Whitworth's demonstration. But if he assumes the side = 2 exactly, and *also the radius* = 4 *exactly*, I certainly do differ from him. But I suspect he has *not* assigned 4 for the radius. The chord is "indirectly" connected with the angle and thereby the arc, by this property at any rate, that the arc is *longer* than the chord.

Yours very truly,
G. B. GIBBONS.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
23*rd February*, 1870.

MY DEAR SIR,

Mr. Whitworth in his demonstration denotes the sides of a regular inscribed dodecagon to a circle by the arithmetical symbol 2; but he does not assume the radius of the circle = 4 exactly. This, however, does not affect the question at issue. The chord subtending a central angle of 30° is *arithmetically indeterminate*. Mr. Whitworth assumes it to be *arithmetically finite* and *determinate*. Thus, Mr. Whitworth and you differ from each other. Does the nature of the case admit of Mr. Whitworth's datum or premiss?

Passing by these matters, however, the area of a circle is 10368 *square* yards, when the circumference is 360 *linear* yards. To controvert this fact, I may set—not only you and Mr. Whitworth, but—all the Mathematicians in the world at defiance.

I send you herewith Mr. Whitworth's fancied proof that π is greater than 3·126. The fallacy that lies at the root of it is, the attempt to measure *directly*, a curvilinear figure by means of a rectilinear figure. This can only be done *indirectly*.

Yours very truly,

JAMES SMITH.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

WERRINGTON, LAUNCESTON,
25th February, 1870.

MY DEAR SIR,

I had written you a letter at once, on receiving yours this morning, and had put into a P.S. a suggestion that a circle turned in a lathe would suffice to decide in so great a difference as 3·125 or (nearly) 3·1416 for the value of π. Afterwards, I turned to Mr. Whitworth's paper, where I found previously the same suggestion. I did not propose a round table, but a circle of 12 inches radius, carefully turned for the purpose. The difference in the circumference would be $\frac{1}{5}$th of an inch, a quantity too large, to be accounted for by the measured expansion or contraction of a slip of paper passed round the circumference. So I really think the experiment might be made.

If s = side of a dodecagon in a circle, r = radius.

It is a matter of pure arithmetic and geometry to shew that—

$$s = (\cdot 5176 \ldots)\, r$$

Or to six decimals.

$$s = (\cdot 517638 \ldots)\, r.$$

Consequently, if r is taken finite, s is indeterminate, and *vice versâ*.

I do not differ at all from Mr. Whitworth, if he does not fix radius at 4. He is at liberty to assign any value he likes to s (say $s = 2$), but then his *radius* is not expressible in finite terms. Since any investigation of the value of π leads us to the extraction of a square root, in which the quantity is not an exact square, π must come out in an indeterminate decimal. All you do, is, not to treat π as an unknown quantity whose value is to be discovered, but to *assume* a value for it, and then shew correctly what would be the results of such a value.

Yours very truly,
G. B. GIBBONS.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
SATURDAY EVENING,
26th February, 1870.

MY DEAR SIR,

Your Letter of yesterday is to hand.

The *Liverpool Leader* is open to you, and if you send a copy of your Letter for insertion in that Journal, I will reply to it through the same medium.*

Yours very truly,
JAMES SMITH.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

WERRINGTON, LAUNCESTON,
7th March, 1870.

MY DEAR SIR,

I have to thank you for the *Liverpool Leader* of Saturday.

I see the Editor shuts up the controversy. I cannot blame him, for it would be endless. Yet I had prepared a Letter to him proving by three distinct methods, that π *must* be greater than 3·13.

It is best however to close the debate.

* For Mr. Gibbons' Letter to the *Liverpool Leader*, see Appendix C.

On turning to Mr. Whitworth's proof that π must exceed 3·126, I find (as I expected) that he makes, r not 4, but $\sqrt{6} + \sqrt{2}$ exactly as I do.

For, let $C = 30°$, $AB = 2$, and therefore $AD = 1$.

$$2 \text{ Sin. } 15° = \frac{\sqrt{3} - 1}{\sqrt{2}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{2}{\sqrt{6} + \sqrt{2}}.$$

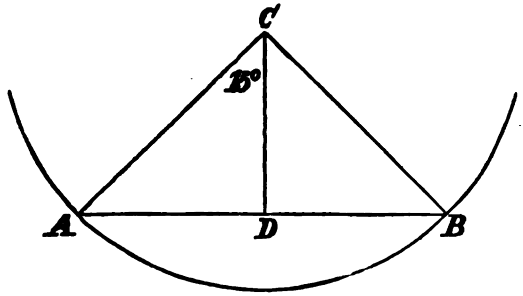
$$\text{Or, Sin. } 15° = \frac{1}{\sqrt{6} + \sqrt{2}}.$$

$$\text{But, Sin. } 15° = \frac{1}{r} \left(\text{or } \frac{AD}{AC}\right)$$

that is, $r = \sqrt{6} + \sqrt{2}$, as Whitworth has put it.

Yours very truly,

G. B. Gibbons.



P.S.—I have always had the misfortune to disagree with you on these matters, but I should have been astonished if I had differed from Professor Whitworth.

The Rev. Geo. B. Gibbons *to* James Smith.

Werrington, Launceston.

14th March, 1870.

My Dear Sir,

I have to thank you for the *Liverpool Leader* just received.

There is no difference of views between Mr. Whitworth and

myself. We agree. If the chord is finite = 2, the radius is *not* finite, (it is $\sqrt{6} + \sqrt{2} = 3 \cdot 8637\ldots$), and *vice versâ*, if the radius is made finite, the chord will not be so.

Yours very truly,
G. B. GIBBONS.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
TUESDAY EVENING,
15th March, 1870.

MY DEAR SIR,

I have just received your favour of yesterday's date.

I have no doubt you agree with Todhunter that $2\pi r =$ the circumference of a circle, when r denotes the radius. This being so, and admitted, it follows, that a central angle of 30° is subtended by a side of a regular dodecagon. But, the sides of a regular dodecagon are arithmetically *indeterminate*, whatever be the arithmetical value of the circumference of its circumscribing circle.

Will you be kind enough to give me a *distinct* answer to the following plain and simple question?

Can a geometrical something *known* to be arithmetically *indeterminate*, be rationally and legitimately adopted as a datum or premiss, in a geometrical enquiry *involving* Mathematics?

Yours very truly,
JAMES SMITH.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

WERRINGTON, LAUNCESTON,
17th March, 1870.

MY DEAR SIR,

You are quite right in saying that the side of a regular dodecagon in a circle is arithmetically indeterminate, when the radius is determinate, for the calculation of such side involves cos. 30° $= \frac{\sqrt{3}}{2}$, which cannot be expressed in finite numbers.

Your expression "*known* to be indeterminate," in the question you put me, implies that this inexactness has been discovered by calculation.

Indeed, all Geometry, Algebra, and Trigonometry, deals with lines and angles and their measurement, without assigning *any* numerical values. The results are *general*, and particular values are *afterwards* assigned: so in Euclid—no particular numerical value is supposed in his discussion of lines, in their ratio to each other.

To reply to your question closely, (I need not copy it out), my answer is:—

It can.

Ex. cos. $30^\circ = \frac{\sqrt{3}}{2}$: diagonal of square (side 1) $= \sqrt{2}$. These are known to be arithmetically indeterminate, and yet they may be rationally and legitimately adopted as data or premises in a Geometrical enquiry involving Mathematics.

But perhaps I do not rightly interpret your meaning of "data" or premises. If you mean *axioms*—(or the self-evident truths from which we *must* start), you could not call the preceding by that term. Cos. $30^\circ = \frac{\sqrt{3}}{2}$, and the diagonal of a square $= \sqrt{2}$, must be *results* of reasoning and calculation; but the mere fact of a something being *known to be arithmetically indeterminate*, does not hinder its legitimate employment in the way you name, otherwise how could you ever use $\sqrt{2}$ or $\sqrt{3}$, which are indeterminate?

Yours truly,

G. B. GIBBONS.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
FRIDAY EVENING,
18th March, 1870.

MY DEAR SIR,

I am in receipt of your favour of yesterday's date.

If Mr. Whitworth had adopted as a datum or premiss a side of a regular dodecagon, (which subtends a central angle of 30°) repre-

senting the side of the dodecagon by the arithmetical expression $\frac{\sqrt{3}}{2}$, to prove that the arithmetical value of the symbol π is greater than 3·126, he would have adopted as a datum or premiss what is rational. But, in adopting as a datum or premiss a side of a regular dodecagon, and assuming it to be represented by the *finite* and *determinate* arithmetical symbol 2, he *assumes* what is "*discovered by calculation*" to be arithmetically impossible, and therefore mathematically irrational.

I will not say more in this communication, but the *Liverpool Leader* is still open to you on very moderate terms, and if you send a copy of your Letter for insertion in that Journal, I will reply to it at greater length through the same medium.

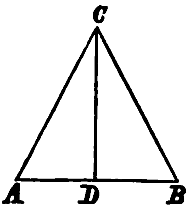
Yours very truly,
JAMES SMITH.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

WERRINGTON, LAUNCESTON,
21st March, 1870.

MY DEAR SIR,

In the isosceles triangle adopted by Mr. Whitworth, he could assign a value for either A B or A C whichever he pleased.



For, $A B = 2 A D.$

$= 2 A C \text{ Sin. } 15°.$

$= \frac{2 A C}{\sqrt{6} + \sqrt{2}}$

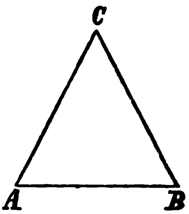
If he puts $A B = 2$, obviously $A C = \sqrt{6} + \sqrt{2}$.* It is more usual to assign a value that can be expressed exactly, though the

* Why is it *obvious*, that $A C = \sqrt{6} + \sqrt{2}$? I deny the truth of this assertion, for, obviously, we cannot make $\sqrt{6} + \sqrt{2}$ either a whole number, or the square root of a *finite* and *determinite* arithmetical quantity. J. S.

value of the *other* quantity may thereby turn out to be not so expressible. In drawing a square, it would be more usual to make its side 1, and thence deduce the diagonal $\sqrt{2}$; than to define the square by its diagonal $\sqrt{2}$, and thence deduce 1 for its side.

I see no use in prolonging a controversy in the *Liverpool Leader*. Your own books have published most fairly and correctly my case, to wit this :—

That without involving π at all, or introducing a circle, I find by Elementary Trigonometry, that if C = 15° in the isosceles triangle :—



$$AB = (\cdot 261\ldots)\, AC$$

$$\text{or, } \frac{AB}{AC} = \frac{\cdot 261}{1}$$

and here the solution of the triangle is finished.*

I have repeatedly examined my work, I used no tables, and have tested the result in various ways. That result would be untouched, if we had never attempted to investigate the ratio of a circle's circumference to its diameter.

But after completing this work, I can with centre C and distance C A or C B draw a circle, and A B is the chord of 15°, a chord of $\frac{\pi}{12}$ in such circle.

Confessedly unable to measure accurately this arc by the chord, (or indeed *any* curved line by any straight one), I can at least see that the arc is *longer* than the chord.

That is, $\frac{\pi}{12}$ is greater than $\cdot 261$.

π is greater than $3\cdot 132$.

I cannot conceive any proof plainer than this, and since you deny it, I have exhausted all my available or hopeful arguments. I

*I deny Mr. Gibbons' assertion :—"*Here the solution of the triangle is finished.*" I also deny that $\frac{AB}{AC} = \frac{\cdot 261\ldots}{1}$. I maintain that $\frac{AB}{AC} = \frac{\cdot 2604166}{1}$ and $\frac{\cdot 2604166}{2} = \cdot 1302083$ is the trigonometrical sine of half the angle C. Mr. Gibbons arrives at his conclusion by a *misapplication* of the 47th Proposition of the first book of Euclid, and so, (apparently without knowing it) attempts to measure a curvilinear figure by means of a rectilinear figure. J. S.

can offer indeed several other independent proofs, (from the higher mathematics), but none clearer or simpler than this. If A C were 1000 feet long, A B would be more than 261 feet, and semi-circumference more than 3132 feet.

Yours very truly,
G. B. GIBBONS.

P.S.—Mr. Whitworth's assigning 2 for the side of his dodecagon, gives a radius of the circle $\sqrt{6} + \sqrt{2}$. If you were to assign 1 as the side of a square, its diagonal would be $\sqrt{2}$, the cases being precisely similar.

Should you say, in the latter case, that the assumption was arithmetically impossible, and therefore mathematically irrational?

I confess, I often cannot make out what you mean. Thus $\sqrt{2}$ is "irrational" (for that is the *technical* phrase for a surd quantity), but to take 2 for the side of any polygon is as legitimate surely, as to take 1 for the side of an isosceles right-angled triangle.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
TUESDAY EVENING,
22nd March, 1870.

MY DEAR SIR,

I have to acknowledge the receipt of your favour of yesterday's date.

It is obvious that further discussion between us on the arithmetical value of the symbol π, with any chance of agreement, *is indeed hopeless*, and I shall not trouble you with any more Letters on the subject.

You must not think me discourteous, if for the future, I decline to acknowledge the receipt of any communication from you on this interesting and important question.

Believe me, my dear Sir,
Very truly yours,
JAMES SMITH.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

WERRINGTON, LAUNCESTON,
23rd March, 1870.

MY DEAR SIR,

I duly received the *Liverpool Leader* of Saturday last.

You may safely challenge all the Mathematicians of Europe in the terms you adopt, for they will, one and all, confess that *for a given circumference* a larger value for π would give a *smaller* value for the area. If circumference = 360, and π were 3·125, area would be 10368 ; but increasing π while keeping the same circumference would diminish the area of the circle.

The well known forms are

$$\text{First: } A = \pi r^2.$$

$$\text{Second: } C = 2\pi r.$$

and taking the value of r from c and substituting it in A, we get—.

$$\text{Third: } A = \frac{\pi C^2}{4\pi^2} = \frac{C^2}{4\pi}.$$

Noticing equations 1 and 3 we see, that A varies directly as π, when r is given, and A varies inversely as π, when C is given.

Numerical application:

$$\text{Put } C = 360. \quad A = \frac{C^2}{4\pi}$$

$$= \frac{129600}{4\pi} = \frac{32400}{\pi}.$$

Hence, if

$$\pi = 3, \quad A = 10800.$$

$$\pi = 3\tfrac{1}{8}, \quad A = 10368.$$

$$\pi = 4, \quad A = 8100.$$

or, the larger π is, the less A is, as T. S. B. rightly observes.

If we could *prove* that the perimeter of a regular hexagon, increased by its $\frac{1}{24}$ part exactly, equalled the circumference of the circle, π would be found at once, but it is futile merely to *assume* this without any proof.

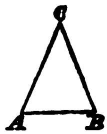
Side of hexagon = radius = 1.

Hence, perimeter of hexagon = 6.

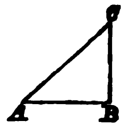
But, $6 + \frac{1}{24}6 = 6{\cdot}25$.

Very true ! but equally true, that $6 + \frac{1}{25}6 = 6{\cdot}24$.

And we might just as well take 6·24 as 6·25 for the circle's circumference, so far as the hexagon is concerned. It is mere guess, arbitrary assumption, with not even a show of proof.



I have already adverted to Mr. Whitworth. He takes the side of a dodecagon A B = 2: this will lead to A C = $\sqrt{6} + \sqrt{2}$.



Suppose an equilateral right-angled triangle. I take A B = 1, this leads to A C = $\sqrt{2}$. It often happens, that one line being taken of finite (exact) value, another line, connected with it, *thereby* assumes an indeterminate value. Your own books would supply many such instances.

Yours truly,
G. B. GIBBONS.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

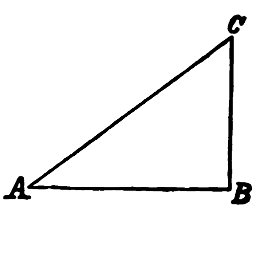
BARKELEY HOUSE, SEAFORTH,
THURSDAY NIGHT,
24th March, 1870.

MY DEAR SIR,

I am in receipt of your favour of yesterday.

I had resolved to bring our controversy to an end, and without intending to say or do anything offensive, I should not have replied to this communication, but for a glaring absurdity in the last paragraph.

In that paragraph you say:—"Suppose an equilateral right-angled triangle. I take A B = 1, this leads to A C = $\sqrt{2}$."



Now, my dear Sir, an equilateral right-angled triangle is a geometrical impossiblity.

From any two consecutive numbers we can find a commensurable right-angled triangle. If the consecutive numbers be 1 and 2, the sides of the triangle will be 3,

4, and 5. If the consecutive numbers be 2 and 3, the sides of the triangle will be 5, 12, and 13. If the consecutive numbers be 3 and 4, the sides of the triangle will be 7, 24, and 25, and so on *ad infinitum*. Hence, all the sides of a right-angled triangle may be represented by the square roots of whole numbers, as certainly as that the two terms of a ratio may be multiplied or divided by the same arithmetical quantity, without altering the ratio itself.

In my work on "The Geometry of the Circle," I have given a theory for finding commensurable right-angled triangles. I made the acquaintance of a high class Mathematician at the Meeting of the British Association in Norwich, and directed his attention to the theory for finding commensurable right-angled triangles; and he did me the honour to confess, that I had taught him more in ten minutes, than he had ever learned either at school or college.

Believe me,

My dear Sir,

Very truly yours,

JAMES SMITH.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
25th March, 1870.

MY DEAR SIR,

In the last paragraph of your Letter of the 23rd instant, you adopt as a datum or premiss a geometrical impossibility, and then profess to shew what it leads to.

From a given point A draw two straight lines A B and A C of equal length, and at right angles to each other. Join B C, and so construct a right-angled isosceles triangle A B C. The angles at the base will be together equal to the angle at the apex; or, in other words, each of the angles at the base will be equal to half the angle at the apex; but the angle at the apex being greater than an angle

of 60°, we cannot draw straight lines from the angles at the base, perpendicular to their opposite sides.

Think, my dear Sir, of what this *leads to.*

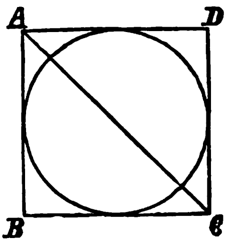
Yours very truly,
JAMES SMITH.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
26th March, 1870.

MY DEAR SIR,

On the straight line A B describe the square A B C D. Draw the diagonal A C, and in the square inscribe a circle.



It is self-evident, that A C a diagonal of the square divides the square into two equal parts, and it is not more certain that A B C D is a circumscribing square to the circle, than that the surface or area contained by two sides of the square and the diagonal A C, is equal to the surface or area of an inscribed square to the circle. Hence: If x denote the area contained by two sides of a square and the diagonal; O denote the area of the circle; and y denote the area of the square A B C D circumscribed about the circle, then, $(x + \frac{1}{4}x) + \frac{1}{4}(x + \frac{1}{4}x) = \text{O}$: and $\frac{\text{O}}{\frac{1}{4}(3{\cdot}125)} = \frac{\text{O}}{{\cdot}78125} = y$: and it is known to, and admitted by, all Geometers and Mathematicians, that the area of a circumscribed square to any circle is double the area of an inscribed square. Mark what follows!

In the analogy or proportion $p : c :: x : a$, when p denotes the perimeter of a regular hexagon, c denotes the circumference of its circumscribing circle, a denotes the area of the circle, and x denotes the area of its inscribed regular dodecagon: the product of the *means* is equal to the product of the *extremes;* that is, $\sqrt{c \times x} = \sqrt{p \times a}$. "$\pi$ does not appear, need not appear;" but from the foregoing facts

it follows, not as an assumption but as a logical deduction, that $\frac{25}{8} = 3{\cdot}125$ is the true arithmetical value of the symbol π, which denotes the circumference of a circle of diameter unity.

When you can find some other value of π than $\frac{25}{8} = 3{\cdot}125$, by which you can produce these *results*, you will put me where Professor de Morgan *fancies* he has fixed me, namely:—" Overhead, in a cloud, on one stick across two others, in a nimbus of $3\frac{1}{8}$ diameters to circumference, in π glory."

I send you with this a copy of a French translation of one of my pamphlets, in which you will discover a theory for finding commensurable right-angled triangles. From the accidental omission of the facts now given you, the demonstration on page 28 is defective.

Yours very truly,

JAMES SMITH.

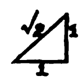
THE REV. G. B. GIBBONS *to* JAMES SMITH.

WERRINGTON, LAUNCESTON,

26th March, 1870.

MY DEAR SIR,

If I wrote an *equilateral* right-angled triangle, of course it was a gross blunder: I meant *isosceles*, as the context shows.



I think you are *quite right* in bringing our controversy to a close.

Yours truly,

G. B. GIBBONS.

JAMES SMITH *to* THE REV. G. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,

29th March, 1870.

MY DEAR SIR,

The conditional IF in your favour of the 26th instant implies *doubt*, and I herewith enclose your Letter of the 23rd, to

convince you that I did not misquote you in my communications of the 24th and 25th instant.

There can be no doubt, that an equilateral right-angled triangle is a geometrical impossibility, and to found an argument "*on the supposition*" that such a triangle exists would be absurd: but in your Letter to me of the 27th July last, you adopted as a premiss the greater of the acute angles in a right-angled triangle, assuming it to be an angle of 16° 16′, that is, less than half a right angle, which is a geometrical impossibility:—was not this absurd?—and from it attempted to prove, by assuming the infallibility of Logarithmic Tables, that the trigonometrical sine of an angle of 8° 8′ is greater than $\sqrt{\cdot 02} = \cdot 1414213\ldots$ to 7 places of decimals.

In my Letter to you of the 5th August, I directed your attention to the gross blunder of assuming the greater of the acute angles in a right-angled triangle to be less than half a right angle; but you have never admitted the blunder.

How, then, could I assume your *supposition* to be a mere lapsus, and venture to substitute the word *isosceles* for *equilateral?* Your Letters, and my replies, must speak for themselves.

You say in your Letter of the 23th inst., that $A = \frac{\pi C^2}{4\pi^2} = \frac{C^2}{4\pi}$. Now, $\frac{\pi C^2}{4\pi^2}$ = diameter of a circle of radius unity, when C^2 denotes the circumference of a circle of radius = 4, and π denotes the circumference of a circle of diameter unity.* Take π out of the denominator in both sides of your equation, and there remains $\frac{\pi C^2}{4^2}$ and $\frac{C^2}{4}$; but it is not true, that $\frac{\pi C^2}{4^2} = \frac{C^2}{4}$. Think, my dear Sir, of what this "*leads to.*"

Yours very truly,

JAMES SMITH.

* "*On the supposition*" that $\pi = 3\cdot125$, $\frac{\pi C^2}{4\pi^2} = \frac{C^2}{4\pi}$ when C^2 denotes the circumference of a circle of radius = 4: but on no other "*supposition*" can we get the equation $\frac{\pi C^2}{4\pi^2} = \frac{C^2}{4\pi}$, or, the proportion, $3\frac{1}{8}(8\pi) : (2\pi)^2 :: 4(2\pi) : 4(\pi)$; that is, $3\frac{1}{8}(25) : 6\cdot25^2 :: 25 : 4(3\cdot125)$.

THE REV. GEO. B. GIBBONS *to* JAMES SMITH.

WERRINGTON, LAUNCESTON,
31*st March*, 1870.

DEAR SIR,

I return this that you may see we are all liable to "clerical" errors. When I said equilateral right-angled triangle, I was thinking of the two *sides* as distinguished from the third, which has a special name "hypothenuse," still the wording was not correct.

Yours truly,
G. B. GIBBONS.

JAMES SMITH *to* THE REV. GEO. B. GIBBONS.

BARKELEY HOUSE, SEAFORTH,
FRIDAY NIGHT,
1*st April*, 1870.

MY DEAR SIR,

Your enclosures under cover of an envelope bearing the Launceston post-mark of yesterday are to hand.

In my Letter of the 29th March (one of those enclosures) you have erased the expression $\frac{\pi C^2}{4^2}$ and substituted $\frac{\pi C^2}{4\pi}$. Take π out of the numerator and denominator of the fraction $\frac{\pi C^2}{4\pi}$, and $\frac{C^2}{4}$ remains.*

Now, let C denote the arithmetical symbol 2.

Then: Although the numerator and denominator are not alike in the fractions $\frac{\pi C^2}{4\pi}$, and $\frac{C^2}{4}$, the numerator divided by the denominator = unity in both fractions, whatever be the value of π. Hence:

*Let the algebraical expression $\frac{\pi (C^2)}{4 (\pi)^2}$ denote a ratio. Take π out of the two terms of the ratio, and there remains $\frac{C^2}{4(\ )^2} = \frac{C^2}{4}$. Is not $\frac{C^2}{4(\ )^2} = \frac{C^2}{4(1^2)}$? Is not algebraical notation susceptible of improvement?

$\frac{\pi(C^2)}{(2C)^2}$ = area of a circle of diameter unity, whatever be the value of π.

I had written so far before I observed your short note on the back of my Letter of the 29th March. I cannot charge myself with ever having hesitated to admit a "*clerical*" error. When I wrote $\frac{\pi C^2}{4^2}$ it was *not* a clerical error as you suppose, but an expression penned advisedly, and intended to suggest to you, that there are "Curiosities of Mathematics" which have hitherto escaped the observation of Mathematicians.

Yours truly,

JAMES SMITH.

P.S.—A Letter of mine will appear in to-morrow's *Liverpool Leader*, but I do not intend to insert any more, or have any further controversy. I shall shortly publish another work, which will include all our correspondence since last July, and in it I shall sum up all I have to say further, on the subject of π's arithmetical value.

So ended my controversy with the Rev. Geo. B. Gibbons. His Letter of the 21st February and subsequent communications, with my replies, should be taken in connection with the Articles and Letters which have appeared in the *Liverpool Leader* on *the Curiosities of Mathematics*. These will be found in Appendix C.

Let c denote the circumference of a circle, and r denote the radius: and let π denote the circumference of a circle of diameter unity.

Let r be represented by the arithmetical symbol and *mystic* number 4. Then: $2\ \pi\,(r) = c$. $\frac{\pi\,(c)}{(4\,\pi)^2}$ = radius of a circle of diameter unity. $\frac{\pi\,(c)}{4\,(\pi^2)}$ = the diameter of a circle

of radius unity. $\frac{\pi(c)}{4(\pi)} = 2\pi$. Hence: $\frac{\pi(c)}{(4\pi)^2} \times \frac{\pi(c)}{4(\pi^2)} =$ unity. But, $\pi = 3$ would make the circumference of a circle of radius $= 4$, exactly equal to the perimeter of its inscribed regular hexagon: and $\pi = 4$ would make the circumference of a circle of radius $= 4$, exactly equal to the perimeter of its circumscribing square; and it follows, and is self-evident, that the arithmetical value of the symbol π must be greater than 3 and less than 4. Mr. Gibbons asserts, that $\pi = 3\frac{1}{8}$ would make the perimeter of a regular hexagon greater than the circumference of its circumscribing circle. (*See his Letter of the 19th July,* 1869, *page* 1.) The formulæ may be demonstrated by means of any hypothetical arithmetical value of the symbol π so that it be *finite* and *determinate.* For example: Let $\pi = 3{\cdot}2$. Then: $2\pi(r) = 6{\cdot}4 \times 4 = 25{\cdot}6 = c$. $\frac{\pi(c)}{(4\pi)^2} = \frac{3{\cdot}2 \times 25{\cdot}6}{12{\cdot}8^2} = \frac{81{\cdot}92}{163{\cdot}84} = {\cdot}5 =$ the radius of a circle of diameter unity: $\frac{\pi(c)}{4(\pi^2)} = \frac{3{\cdot}2 \times 25{\cdot}6}{4(3{\cdot}2^2)} = \frac{81{\cdot}92}{4(10{\cdot}24)} = \frac{81{\cdot}92}{40{\cdot}96} = 2 =$ the diameter of a circle of radius unity; therefore, $\frac{\pi(c)}{(4\pi)^2} \times \frac{\pi(c)}{4(\pi^2)} = {\cdot}5 \times 2 =$ unity, whatever be the arithmetical value of the symbol π.

According to "*recognised Mathematicians*" 3·14159265.... is the arithmetical value of the symbol π, true to 8 places of decimals. "*On the supposition*" that they are right, it follows, that the arithmetical value of the symbol π must be greater than 3·1 and less than 3·2. Recognised Mathematicians will admit, that the arithmetical value of the symbol π is greater than 3 and less than 4: but request them to admit that it is greater than 3·1 and less than 3·2: so far as my experience of that confraternity

goes, they will either decline to notice your request, or positively refuse to make the admission. I courteously, and with the simple intention of narrowing our differences, requested one of my correspondents—a "*recognised Mathematician*"—to make the admission that π must be greater than 3·1 and less than 3·2, and he at once refused, and denied my right to ask him to make any admission.

The arithmetical mean between 3·1 and 3·2 is $\frac{3·1 + 3·2}{2} = \frac{6·3}{2} = 3·15$; and on the shewing of "*recognised Mathematicians*," the arithmetical value of the symbol π must be greater than 3·1 and less than 3·15. The arithmetical mean between 3·1 and 3·15 is $\frac{3·1 + 3·15}{2} = \frac{6·25}{2} = 3·125$.

Now, $\frac{3·125 + 3·1}{2} = \frac{6·225}{2} = 3·1125$; and $\frac{3·125 + 3·15}{2} = \frac{6·275}{2} = 3·1375$; and the arithmetical mean between 3·1125 and 3·1375 is $\frac{3·1125 + 3.1375}{2} = \frac{6·25}{2} = 3·125$. Again: $\frac{3·125 + 3·1125}{2} = \frac{6·2375}{2} = 3·11875$; and $\frac{3·125 + 3·1375}{2} = \frac{6·2625}{2} = 3·13125$; and the arithmetical mean between 3·11875 and 3·13125 is $\frac{3·11875 + 3·13125}{2} = \frac{6·25}{2} = 3·125$. By pursuing this process of "*reasoning and calculation*," we get a pair of numbers at every step, and the arithmetical mean between them is 3·125. If the period alloted to human life admitted of it, we might pursue the calculations until we get a pair of numbers, in which the decimals would reach from London to Windsor, and—as De Morgan would say—"*The Queen will run away on their near approach, as Bishop Hatto did from the rats.*" When, by calculation, we get to the tenth place of decimals, we get the following pair of numbers, namely: 3·1251953125 and

3·1248046875, and the arithmetical mean between this pair of numbers is $\frac{3·1251953125 + 3·1248046875}{2} = \frac{6·25}{2} =$ 3·125.

Now, the arithmetical mean between 3·125 and 3·1248046875 is $\frac{3·125 + 3·1248046875}{2} = \frac{6·2498046875}{2} =$ 3·12490234375; and the arithmetical mean between 3·125 and 3·12490234375 is $\frac{3·125 + 3·12490234375}{2} = \frac{6·24990234375}{2} =$ 3.124951171875, and 3·124951171875 is a closer approximation to 3·125 than 3·1248046875. Again: The arithmetical mean between 3·125 and 3·12490234375 is $\frac{3·125 + 3·12490234375}{2} = \frac{6·24990234375}{2} = 3·124951171875$, and is a still closer approximation to 3·125. By pursuing this process of "*reasoning and calculation*" we should approximate more closely to 3·125 at every step, but by no possibility could we ever reach it. If our decimals reached from London to Windsor, we could still carry them a step further. Will nothing convince the Rev. W. Allen Whitworth, and those who agree with him, of the absurdity of the notion, that the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} +$ &c., stands for the *finite* and *determinate* arithmetical symbol 2?

If a regular dodecagon be inscribed in a circle, with straight lines drawn from the centre of the circle to the angles of the dodecagon, the angles at the centre of the circle will be angles of 30°; and it is not more certain, that $\frac{3}{\pi}$ expresses the ratio between the area of the dodecagon and the area of its circumscribing circle, than that $\frac{3}{\pi}$ expresses the ratio between a side of the dodecagon and its subtending arc. This is a *curiosity in Mathe-*

matics as applied to Geometry, and I have traced it to its cause in pp. 232, 233, and 234. My Letter in the *Liverpool Leader* of the 2nd April, 1870, brings the question at issue between me and "*recognised Mathematicians*" within reach of the capacity of any man of moderate intellect, who is acquainted with the rules of common arithmetic.

My publications on the ratio of diameter to circumference in a circle, brought me Correspondents from all parts of the United Kingdom, but the County of Surrey was the most prolific in furnishing *opponents* of the THEORY, that 8 circumferences = 25 diameters in every circle. A few days ago I received a Note, of which the following is a copy:—

SANDOWN VILLA,
RED HILL, SURREY,
March 29th, 1870.

MY DEAR SIR,

I am obliged to you for the *Liverpool Leader*, but if *your letters were out*, I would not thank you at all for such a thing as that Paper clearly is. I have paid about sixpence for it, so if you write more, please to direct as above. I have left Brighton.

I note your query as to the value of π; but allow me to tell you, that it may be 3·1251 or 3·1259 for anything you have shewn to the contrary; or it may be anything between 3·1251 and 3·1259. Of one thing I am certain, for I long ago proved this in the *Correspondent*, that it is more than 3·125, and you never attempted to *disprove this fact*.

I am glad to see you remain well and vigorous.

Yours faithfully,
R. J. MORRISON.

In this Note, Capt. Morrison makes the following assertion :—"*Of one thing I am certain, for I long ago proved this in the Correspondent, that it* (π) *is more than* 3·125, *and you never attempted to disprove this fact.*" This *assertion* is not true! In August, 1865, I was induced to become a contributor to the *Correspondent*, a London Journal, open to receive communications on all subjects: and during the short existence of that periodical, almost every week contributed an article. My early articles were on the value of π, and I sent copies of the Paper containing these articles to Capt. Morrison. At this time he professed to agree with me as to π's arithmetical value. (*See Zadkiel's Almanac for* 1866). E. L. Garbett and other contributors to the *Correspondent* adopted the same *argument* as Captain Morrison, which I over and over again refuted in that Journal, and pointed out the fallacy that lies at the root of the *argument.* (*For Mr. E. L. Garbett's Letter, and my reply, taken from the Correspondent of January* 27*th, and February* 3*rd,* 1866, *see Appendix D).* Capt. Morrison subsequently became a reader of, and frequent contributor to, the *Correspondent*, principally on Astronomical subjects(he has since recanted the views he then held on Astronomy) ; and in that Journal of April 7, 1866, there is a Letter of his, which will be found in *Appendix D*, in which he attempts for the first time to prove, that if the radius of a circle = 60, the perimeter of an inscribed polygon of 24 sides = 375·91488..., and is therefore greater than the circumference of its circumscribing circle, "*on the supposition*" that π = 3·125. In the same number of the *Correspondent*, there is also a Letter from the Rev. Geo. B. Gibbons, in which he attempts to prove, that if the radius of a circle be represented by

unity, the perimeter of a regular inscribed dodecagon is greater than 3·13260, and therefore greater than the circumference of its circumscribing circle, "*on the supposition*" that $\pi = 3·125$. I replied to both these Letters in the *Correspondent* of April 14, 1866. (*See Appendix E*). In the last issue of that periodical (April 21, 1866) there appears the following communication :—

CIRCLE SQUARING.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

The Letter I last addressed you is referred by Mr. J. Smith to your readers "to draw their own inferences." Of course they will, with or without his permission. If it be not sufficient to take a radius of 60, let him take one of 100. He will then find that *his* theory gives the circumference = 625. The true length of the side of a polygon of 24 sides inscribed in the circle is 26·105, and this multiplied by 24 = 626·52, so that the polygon is equal to 1·52 *more* than the circle! Is not this ridiculous? And can he, in the face of this and similar facts, go on arguing preposterously that the circumference is to the diameter as 3·125 to 1 only? Surely it is time to save your readers this insult to their common sense.

Yours &c.,

R. J. MORRISON.

The *Correspondent* became defunct on the 21st April, 1866, so that I had no opportunity—even if I had been so disposed—of noticing this grand philippic of the gallant Zadkiel through that medium.

No Mathematician in the world can "*upset*" the following analogy or proportion:—The perimeter of a regular hexagon is to the circumference of its circum-

scribing circle, as the area of a regular inscribed dodecagon to the area of the circle.

Now, Captain Morrison *assumes and presumes*, that he can find the true arithmetical value of the sides of a regular inscribed polygon to a circle from a given radius of the circle. If so, surely he can find the area of a regular inscribed dodecagon from a given circumference of the circle. Let him try!

I simply intimated the receipt of Captain Morrison's Note of the 29th March last, by sending him a copy of the *Liverpool Leader* of the following Saturday, correctly addressed. This brought me the following communication:

SANDOWN VILLA,
RED HILL, SURREY,
8th April, 1870.

MY DEAR SIR,

I am confident that you seek and fight only for the TRUTH, and not for victory.

I think you will not object to my shewing you, that you are in error when you declare that the diameter being 8, the circumference will be 25. I engage to prove to you that it, the circumference, will be *more;* indeed, that it will be *considerably more.*

If the diameter be 8 yards, then will the radius be 4 yards. But, $4 \times 3 \times 12 = 144$ inches: on which scale we will, if you please, base our enquiry. Let us imagine *a Polygon of* 90 *sides* inscribed in a circle, the radius of which is 144 *inches.* The circumference, you say, will be 25 yards. But, $25 \times 3 \times 12 = 900$ inches. The angles of the Polygon at the centre will be 4 *degrees.* Now, I shall shew you that the *chords* of each of these angles will be exactly = 10·0510558 inches: which being multiplied into 90, gives us the *whole measure of the Polygon.* Thus, $10·0510558 \times 90 = 904·595022$. These are of course inches. They *exceed the* 900 *inches* that you make the *circumference* of the circle to contain. You will not deny that the Polygon, being inscribed in the circle,

must needs measure *less* than the circle itself. But your measure is itself *short of the measure of the Polygon;* and must be, therefore, still more *short of the measure of the* circumference. Thus it is proved that your 25 yards = 900 inches, is *in error* by MORE than 4·595022 inches.

I now shew you how I demonstrate this fact. There is an axiom in plane trigonometry, that "the chord of an arc is *twice the Sine* of half the arc." Therefore, taking the radius = 144 inches, I say that:

As Radius

: 144 inches ....Log. 2·1583625

: : Sine of 2° ....Log. 8·5248192

: 5·0255279 inches ....Log. 0·7011817

$\times$ 2

10·05110558 = chord of 4° with radius = 144 inches.

$\times$ 90

904·5950220 inches = *Perephery* of the Polygon of 90 sides.*

I know that you object to the use of *Sines,* because you question their accuracy; but in this case, there must be an amazing amount of error, to reduce the *Sine of* 2° to what would reduce the side of the Polygon to 10 *inches;* and even then your measure of the circumference would be FALSE, because *that* must *exceed* the measure of the Polygon. And if there were that amount of error, then would the rule in trigonometry be altogether fallacious: yet I find it true in all other cases; and Geometry shews that it *must be true* from the very nature of the figure in which it occurs, since a line, or radius, drawn from the centre of a circle, at *half the arc of any given chord, must* BISECT THE CHORD.

Yours faithfully,

R. J. MORRISON.

Capt. Morrison *fancies* he proves me in error as to the arithmetical value of the circumference of a circle of given radius, by multiplying the Logarithm of a number into the Log.-Sin. of an angle, which is

* Query: Periphery?

absurd. The four numbers 6, 3, 8, 4, are in proportion, because $\frac{6}{3} = \frac{8}{4}$; and of any two equal fractions, in common Arithmetic, the four terms are in proportion. Thus: $6 : 3 :: 8 : 4$, and the product of the extremes = the product of the means: But, suppose me to say, 6 rats : 4 rats :: 8 geese : 4 geese. Could *absurdity* go further? Certainly not! Mr. Morrison's fancied proof involves an absurdity of a similar description.

Let A B C denote a right-angled triangle: let B be the right angle, and A B and B C the sides that include the right angle. Let the acute angles in the triangle be angles of 81° 52′ and 8° 8′, or, let the acute angles be angles of 53° 8′ and 36° 52′. In either case, I defy the gallant Captain, or any other Mathematician, to find the ratio of side to side by means of Logarithmic Tables, whether Hutton's or Norie's, or any other set of Tables. Let them try!

The following was my reply to Captain Morrison's Letter:—

BARKELEY HOUSE, SEAFORTH,
9th April, 1870.

MY DEAR SIR,

Like other "*recognised Mathematicians*," you attempt to prove me in error by a calculation based on, and worked out by, an application of Logarithmetic Tables of Sines, which you assume to be accurate. I do not object to the use of Sines as you suppose, but I impugn the accuracy of the values of Sines, Cosines, &c., as given in these Tables.

If a regular polygon of 90 sides be inscribed in a circle, and radii be drawn from the centre of the circle to the angles of the polygon, the angles at the centre of the circle will be angles of 4 degrees, and will be subtended by arcs of 4 degrees. But, the angles at the centre of the circle will also be subtended by the sides of the polygon, and I make the sides of the polygon = 3° 50′ 24″. In all regular polygons of more than four sides inscribed in

a circle, the ratio between the sides of the polygons and their subtending arcs is the same. Hence: $3^\circ\ 50'\ 24'' : 4^\circ :: 3 : 3{\cdot}125$.

I have resolved to "*bide my time*" and not enter into further controversy with "*recognised Mathematicians*," for after upwards of ten years' experience, I find it is indeed a hopeless task to attempt to enlighten *them*.

I am about to publish another work on the ratio of diameter to circumference in a circle, of which I herewith send you a few sheets. It will contain the whole of a recent correspondence between me and the Rev. Geo. B. Gibbons. I shall insert your Letter and my reply, giving your name, if you do not raise any objection.

Yours very truly,

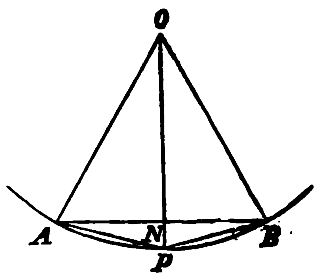
JAMES SMITH.

CAPT. R. J. MORRISON, R.N.

I have this day (April 11th) posted to the address of Captain Morrison, a slip of my Letter which appeared in the *Liverpool Leader* of the 5th February. This number of the *Leader* I had not sent him. To it I appended the following note:—

According to the Imperial Dictionary, to which several of my Correspondents have appealed:—"*The sine of any arc is half the chord of twice that arc*."

Let A O B denote an angle of 4°. Bisect the arc A B in P. It is axiomatic, if not self-evident, that the radius O P bisects the chord A B in N; and it follows, that O N A and O N B are right-angled triangles, and the angles N O A and N O B angles of 2°. N A is the Sine of the angle N O A, and N B the Sine of the angle N O B; but it does not follow, that 2(N A) or 2 (N B) is the Sine of the angle A O B. By Logarithmetic Tables, the natural Sine of an angle of 2° is ·0348995, and according to the "*reasoning and calculation*" of certain "*recognised Mathematicians*," $\cdot 0348995 \times 2 = \cdot 0697990$ should be the natural sine of an



angle of 4°: but the natural sine of an angle of 4° is given in Logarithmetic Tables to 7 decimals as ·0697565, and is less than ·0697990.

Take the natural numbers 2 and 5, and the *powers* of these numbers play an important part in the question at issue between me and "*recognised Mathematicians.*"

Natural number 5 ....Log. 0·6989700
Natural number 2 ....Log. 0·3010300

5 × 2 = natural number 10...............Log. 1·0000000

Hence: 10 × 90 = 900, and if the circumference of a circle be 900 inches, $\frac{900}{2\pi} = \frac{900}{6 \cdot 25} = 144$ inches = radius of the circle. In the Arithmetical rule of proportion, that must be a *false* rule that will not work backwards and forwards. If 1 : 2 : : 2 : 4; conversely, 4 : 2 : : 2 : 1. Is not 900 : 144 : : 3·125 : ·5 ? Is not ·5 the radius of a circle of diameter unity ?

J. S.

In the Appendix E will be found several Letters taken from the *Correspondent,* to which I call the special attention of readers, as proving the untruthfulness of Capt. Morrison's *assertion,* in his Note of the 29th March last. It will be seen that in his communications to the *Correspondent* on the value of π, he never refers to my Letter in No. 40 *(February 3rd)* of that Journal.

13th April, 1870.

This morning's post brought me a very extraordinary communication, of which the following is a copy :—

LIVERPOOL, *12th April,* 1870.

SIR,

The copyright of the Articles in the *Leader* entitled "*Curiosities of Mathematics*" is my property. The series will probably be continued when you have ceased inserting Advertisements under the same title.

But in any case, I purpose making a new use of my Articles, and I cannot permit my rights with respect to them to be infringed. I value the copyright of each of the Articles at twenty guineas, but considering the very limited publicity which they would be likely to obtain from insertion in your forthcoming work, I make you the following offer, (without prejudice to the claim which I should make against you, in case you decline my offer and infringe my rights) namely : that, in consideration of the sum of ten guineas paid to me with respect to each one of the Articles in question, you be at liberty to publish the same in one edition only of your forthcoming work.

As you have published some of my former Letters to you without my permission, I hereby give you notice, that I henceforth reserve to myself all copyright of the communications I may address to you. If you desire it I will communicate to you the terms on which I can allow you to publish extracts from any such communications.

I am, Sir,
Your obedient Servant,
W. ALLEN WHITWORTH.

Mr. Whitworth must have been a little *dazed* when he penned this epistle.

April 15, 1870.

By this morning's post I received the following communication from Capt. Morrison :—

SANDOWN VILLA, RED HILL, SURREY,
14th April, 1870.

MY DEAR SIR,

I am glad to find you speak of withdrawing from the ten years' controversy. It is only wearing you out for nothing. Mathematicians ought to have acted more fairly with you than they have done. The only question should have been to learn *whether*

you were right or wrong; but they have overlain this simple one with attempts to prove *themselves to be right.* Now, this is useless, for if you cannot be shewn to be wrong, their pretensions must fail; at any rate, their pretensions were not so much in question as were yours. You started by asserting and attempting to prove, that a diameter of 1 gives a circumference of 3·125; and, of course, that any other diameter gives the same proportion: thus, if it be 8, it will give 25. *You are wrong beyond dispute,* as the following facts demonstrate:—You say in plain words that a radius of 144 gives a circumference of 900, but I say it gives *more* than 904·59504. Now to prove this. You admit *sines,* you declare: very well, take the sine of half the arc of 4° = sine of 2°. But how to find this? Why, take the sine of 2° as given in all books of trigonometry, and as *worked out in p.* 32, *vol.* 2, *Hutton's Mathematics,* under the head of *Analytical Plane Trigonometry.*

The sine of 2° is there found to be ·0348995; but this is the *natural* sine of 2°, computed to the *radius unity.* Then, as *our radius* is 144, we must multiply it by that number:

Thus, ·0348995 × 144 =	5·025528
To find the *chord* of 4°	× 2
The *chord* of 4° =	10·051056
To find the *measure,* or the *Perephery,* of 90 such chords	× 90
Hence 90 CHORDS =	904·59504

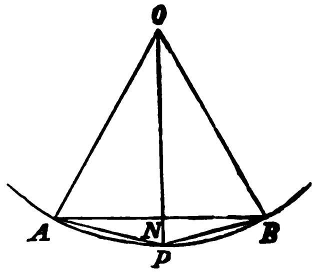
Hence, my assertion is *true,* that the CHORDS of 90 angles of 4° are just 904·59504. But you make the *circumference* to be only 900! Now I am not bound to find out what the circumference really reaches, since you will not, and dare not, deny it *exceeds the perephery of the polygon* of 90 sides *inscribed therein.*

You say that ·0348995 × 2 = ·0697990 should be the *natural sine* of an angle of 4°. Herein, wherever you have got it, you have hold of *error;* for it is the CHORD of 4° with *radius unity,* and NOT THE SINE of 4°. So that, according to the "reasoning and calculation" of certain "*recognised Mathematicians*" you have a great blunder in your bosom; and the sooner you get rid of it the better.

The rule to find the *natural sine* of 2° is very plain, brief, and

simple; nor have I been able to detect any flaw in the rule. Here it is: given the Sin. $1^\circ = \cdot 0174524$ with *radius unity*.*

Then Sin. $2^\circ = 2$ Sin. $1^\circ \sqrt{1 - \text{Sin.}^2\, 1^\circ} = \cdot 0348995$ with *radius unity*. You say truly that, "it does not follow that twice N A or twice N B is the SINE of the angle A O B." Certainly not, but it does *undoubtedly follow* that 2 N A or 2 N B is the CHORD of the angle A O B. We wanted the chord, and not the sine.



With it, by multiplying into 90, we get the *measure*, or perephery, of a *polygon of* 90 *sides*, inscribed in a circle. And as this is *much more* than *your circumference*, it follows that you MUST BE WRONG.

I have no other feeling than respect for you; since I admire your pluck, and I feel that Mathematicians are under obligation to you. The controversy has not been barren; for it has made known several curious points that you have discovered, which were unknown before, or certainly very little known.

Very faithfully yours,

R. J. MORRISON.

On reading over for errata, I discovered an omission on page 15. In giving the construction of the diagram, after saying:—" In the geometrical figure (Fig. 4), let A B C be an equilateral triangle, by construction:" I should have said, with A as centre, and A B or A C as radius, describe a circle.

I also detected a glaring error on page 59. Near the top of that page I have said:—" Produce A B to meet and terminate in the circumference of the circle at a

* Where does Capt. Morrison get the Arithmetical value of the natural Sine of an angle of $1^\circ = \cdot 0174524$, if not from Tables?

point, say E, and join C E. Then: A E C will be an equiangular and equilateral isosceles triangle, and the three angles will be angles of 60°, and together equal to two right angles." I meant, and should have said:— Then: A E C *will be a right-angled isosceles triangle, and the angles at the base will be together equal to the angle at the apex.* How did Mr. Gibbons fail to detect my *egregious blunder?* Did he not read more than the first paragraph of my Letter of the 5th August, 1869? A careful reader of that Letter — if a "*reasoning geometrical investigator*" — could hardly fail to perceive that my "*egregious blunder*" is inconsistent with the context; and any opponent — *if an honest Mathematician and fair controversialist* — would either have made the necessary correction, or directed my attention to the lapsus, and so have given me the opportunity of correcting it. Whether Mr. Gibbons did or did not detect the blunder, he certainly never called my attention to it. So far as my experience of "*recognised Mathematicians*" goes, with reference to the question at issue between us, they struggle for victory, but never battle for TRUTH, notwithstanding the plausibility of their professions.

JAMES SMITH.

APPENDIX.

APPENDIX A.

FROM THE "ATHENÆUM" OF AUGUST 21ST, 1869.

ARTICLE: OUR LIBRARY TABLE.

The Geometry of the Circle, and Mathematics as applied to Geometry by Mathematicians, shown to be a Mockery, Delusion, and a Snare. By James Smith, Esq. (Simpkin & Co.)

This is the old story with some new correspondents. Professor Whitworth and Mr. J. M. Wilson, the Coryphæus of the assailants of Euclid, now fill the foreground. Of course there is a little about an illogical journal called the *Athenæum*, and an "unscrupulous critic and contemptible mathematical twaddler" named De Morgan. But all this is old echo, or rather we have the tunes which were frozen up in the horn: we pass it over. Mr. Whitworth and Mr. Wilson both take to flight, and probably feel they were not wise in making any attempt upon J. S. Mr. Wilson declares he will not have his Letters published, and threatens J. S. with law and a solicitor: to which J. S. replies that he will publish, and accordingly he does publish. We recommend Mr. Wilson to let J. S. publish: the law is slow to remedy such wrongs, where moral character is not involved. There is a much better ground of action, for which law would do nothing. Mr. Wilson, when quite sick of it, writes "I have been under a mistake in corresponding with you, and I should prefer that the correspondence now closed." Mr. J. S. replied by a stroke of biting satire, "When I last addressed you...I had not seen your work. I have since obtained a copy...As a text-book on Geometry it is infinitely superior to Euclid, and I say this without

the slightest hesitation." It is hard upon a real Geometer, be his views of the value of Euclid ever so wrong, to be patted on the back by a pretender, who has printed two different ways of finding *one* mean proportional as a solution of the problem of finding *two* mean proportionals; and who considers it as proof to assume what is wanted, and then to show that the assumption makes nonsense of other assumptions. If any one who is caught for a moment by J. S. would read the "Budget of Paradoxes," he might be saved trouble. But not certainly: Professor Whitworth went on writing after he had detected J. S. in proving that a certain arc is equal to its own chord. But when J. S. charged him with being satisfied in his own mind that $3\frac{1}{8}$ was π, and still maintaining $3{\cdot}14\ldots$, Mr. Whitworth declared off. We are sorry to hear that J. S. has been out of health. We trust he will get round; but we are afraid this can never be until his circle is a little more than $3\frac{1}{8}$ diameters.

APPENDIX B.

FAC-SIMILE OF Mr GLAISTER'S FIRST LETTER TO JAMES SMITH.

Septr 14th 1869.

11. Lyndhurst Road —
Peckham Surrey S.E

Dear Sir

I have just finished the perusal of your splendid work entitled — "The Geometry of the Circle" — I am quite sure that you are at present ~~will always remain~~ in the class of — un- ~~very~~ recognised mathematicians; ~~However, it~~ in fact you are beyond those. However it is not often we get a good jest on so dry a subject as that of the Quadrature of the Circle and wish — many thanks for your (the pleasure which)

P.S. There are several —
burlesque theatres in London.
"The Strand" - "Gaiety" "Globe"
"Charing Cross" - I believe
there is an old play or —
poem ~~called~~ termed 'Loves
of the Triangles' -

RTG

P.P.S. Prof. De Morgan was only
4th wrangler in 1827. a very —
slim year. - He has written a
little work on ~~the Differential~~.
[illegible] so [illegible] impugned by you
Calculus, & I think he acts 'HONESTLY'
~~He may be an 'unscrupulous critic' but~~

very comical production which has afforded me — Remain

yours truly

R J Glaister

[Late Mathematical Scholar of King's College London]

James Smith Esqre

Page viii. of the Introduction — entitled "To the Reader" is — charming. I also much admire your extremely gentlemanlike tone towards a somewhat humble mathematician — named "De Morgan"

RJG

e VIII 4 zone [illegible]
11 & 12 from top

~~[illegible]~~ R J G

APPENDIX B.

FAC-SIMILE OF MR GLAISTER'S LAST LETTER TO JAMES SMITH.

17/12/1869. 11 Lyndhurst Road, Peckham, Surrey. S.E.

~~Dear~~ Sir

If $\frac{355}{113}$ be taken as π, it is merely an approximation true to 6 decimal places the circumference of your required ⊙ will be = $\cancel{3{\cdot}1415}$ $\frac{1420}{113} \times \frac{\sqrt{2}}{\sqrt{3}-1}$ or in a more computable form $\frac{710}{113} \times (\sqrt{6}+\sqrt{2})$. Say 24·2 &c.

I shall not trouble with an

extended computation —
"Your knowledge" of Common
Arithmetic" will surely —
enable you to " manage the
little matter of multiplication
division & evolution. —

$\frac{355}{113}$ as an approximate &
convenient π was found
by Metius about the end
of the 16th century but
its ~~inventor~~ discoverer
was far too sensible a
man to consider it as an
exact π. — It is true to
6 places of decimals and
a little in excess of
the true value

– Well, after above is my approx. answer to your problem it cannot be exactly answered: So obtain it thus – If r = radius $2r \sin \frac{180}{12}$ = side of dodecagon $= 2r \cdot \sin 15° = 2r\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ ~~$r(\sqrt{3}+1)\sqrt{2}$~~ But you make side = 2

$$\therefore 2 = 2r\,\frac{\sqrt{3}-1}{2\sqrt{2}} \therefore$$

$2r$ or diameter $= 4\sqrt{2} \div \sqrt{3}-1 = \frac{4\sqrt{2}}{\sqrt{3}-1} =$

$$\frac{4(\sqrt{6}+\sqrt{2})}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{4(\sqrt{6}+\sqrt{2})}{2}$$

$$= 2(\sqrt{6}+\sqrt{2})$$

which I add by the process the circumference

There is the answer to your "plain & simple question" —

I revise & send back your proof sheet. Your courtesy is coextensive with your mathematical knowledge

Your
Sir
Your obedient
Servant
R T [illegible]

P.S. This is my very last letter to you R.T.G.

J Smith Esqre

APPENDIX C.

FROM THE "LIVERPOOL LEADER," SEPTEMBER 19, 1868.

EUCLID AT FAULT.

One of the most curious phenomena of our age of science is found in the existence of a very considerable class of men who devote themselves as amateurs to some scientific questions, upon which they fancy they have made some wonderful discoveries: while their conclusions are such as the merest tyro in mathematics could at once disprove.

If these men would attend an ordinary night school for six months, or go through a single course of mathematical lectures in a mechanics' institute, they would learn to see the fallacy of their principles and the absurdity of their conclusions. But although such opportunities abound, they prefer to remain in their ignorance, and deluge every reputed man of science with their ridiculous pamphlets.

A gentleman to whom it once occurred "that nothing short of proving the 47th proposition of the first book inconsistent with some other theorem of Euclid would ever convince a recognised mathematician that the arithmetical value of π is a finite and determinate quantity" (pages 6, 7), has just brought out a pamphlet professing to demonstrate Euclid to be "at fault in the theorem, prop. 8, book 6; and theorems 12 and 13, book 2."

Of the modesty of an author who styles himself, on his title page, "James Smith, Esq.," and who repeatedly (pages 4, 7, and 8, &c.) speaks of his mare's nest as an "important discovery;" while he patronises (page 7, note) with an air of condescension such a mathematician as Professor de Morgan, we shall say nothing. Nor will we characterise the logical mind of a man who mistakes for argument a

series of essentially independent propositions arbitrarily strung together, and connected (or shall we say separated) by a profuse sprinkling of such words as "therefore" and "consequently." These are points on which an unkind reviewer might enlarge: but we would rather employ the little space at our disposal somewhat more profitably, by pointing out the fundamental error of principle which leads astray so many dabblers in geometry, and exhibiting the particular fallacy of Mr. Smith's so-called proof.

We observe that nearly all the impugners of geometric truths have fallen into their difficulties by confusing and mixing up two methods of proof which may be distinguished as the *graphic* and the *abstract*. Suppose it be required to prove that the greater side of every triangle subtends the greater angle. A mechanician will adopt the graphic method and will satisfy himself of the truth of the proposition by actually drawing a considerable number of triangles and ascertaining by actual measurement that whenever two sides are unequal the opposite angles are unequal also, the greater angle being opposite to the greater side. It is plain that when a proposition is to be verified by such a method as this the greatest accuracy is necessary in the drawing, the whole force of the verification depending on the correctness of the diagram. But a mathematician adopts a totally different method, the abstract method, not merely verifying, but absolutely proving the proposition. He argues from the definition and nature of the triangle: he probably makes a rough sketch to illustrate his argument and to suggest to the eye the ideal construction which he describes in words; but he cares not whether his diagram is drawn in strict correctness or not, as it is used as a mere illustration to enable the mind to follow more readily the successive steps of his argument, and is not itself part of his proof. He will say, "Let A B C be a right angle," leaving you to draw any right angle as you please, or (if you prefer it) merely to picture an ideal right angle to your mind. If you draw your picture he cares not whether it be a true right angle or not—to him it is merely a sketch of one—yet he proceeds to argue about his ideal right angle A B C as a true right angle, and arrives at conclusions which may or may not be true concerning the sketch, but are necessarily true of the perfect figure which has been described, not graphically, but logically.

But the host of scribblers, who love to attack truths which are established by a logic which is beyond them, arrive at their marvellous results by mixing up these two methods. They begin with the mathematical method, and draw an elaborate figure in which they prove a few properties by Euclidian reasoning from their hypotheses. The properties thus arrived at are, of course absolutely true of the ideal figure which they have described; but only true of the picture they have drawn, if that picture be (as it never is) absolutely correct. Then they suddenly migrate into the graphic method. They take a pair of compasses and a graduated scale, and apply them to their diagram: and thus arrive at a second set of properties, which are absolutely true (or, at least, as true as their measurements are accurate) concerning the diagram they have drawn, but quite untrue concerning the ideal figure they have described. Naturally these two series of properties—the one series true of the ideal or perfect figure, the other series true of the actual and inaccurate figure—are found to be far from identical; and then the wondrous cry is raised, Euclid is at fault and all the mathematicians of the day are deluded.

"James Smith, Esq.," talks of proving things by a figure. He tells us: "The morning of the 2nd May was very wet at Windermere, and it occurred to me—as I could not leave the hotel—that I could not better pass the time than by writing a letter to Mr. S——, enclosing a diagram, represented by the geometrical figure in the margin, in which the angle A and the sides O B and O L in the triangles O A B and O B L are bisected by the line A N. This I intended as introductory to a succession of diagrams, explanatory and demonstrative of the important discovery, that, the eighth proposition of the sixth book of Euclid is inconsistent with the forty-seventh proposition of the first book; and that it is the former, not the latter, that is at fault."

Now, we absolutely deny that any general propositions can be "demonstrated" by a "series of diagrams." But it is quite sufficient for Mr. Smith's purpose (he says) to take one of his diagrams. Let us see what he will do with it. Turning to the description of the diagram on page 8, we read: "On A B describe the equilateral triangle O A B." Now, Mr. Smith, what do you mean? Are we to

think of O A B henceforth as an equilateral triangle, or is it to be such a triangle as you have drawn in your lithograph, in which the sides O A and O B are considerably longer than A B? We don't mind which. We will take your lithograph as roughly representing to us an equilateral triangle, while we argue about that triangle as truly equilateral; or we will talk about your figure and its properties, measuring it as carefully as you like with compasses and rule. But we object to making two contradictory concessions at once. Talk of the true equilateral triangle if you will, or talk of the triangle you have drawn if you will; but don't assume them to be identical, and then, because your diagram is not true, complain that Euclid is at fault!

The fallacy of the whole argument may be briefly pointed out. It is not even necessary to dwell on the unwarranted assumption of page 9, that a circle with B as centre, and B F as interval, will pass through T. Perhaps, it will; but how does Mr. Smith prove it? We leave this to Mr. Smith himself. We only care about three lines on pages 10:—

"Let K B, the diameter of circle, = 8.

Then: By construction:

$$K H = \tfrac{4}{5} (K B) = 6{\cdot}4."$$

Who can tell why $K H = \tfrac{4}{5} K B$? Probably Mr. Smith measured the lines in his incorrect figure, and found them to have this proportion. Very well; but if he had drawn his figure correctly, by making O A B a true equilateral triangle, he would have found a different proportion, and, instead of $K H = 6{\cdot}4$, we should have had $K H = 6{\cdot}324$. By this inaccuracy all the deductions of the following pages are vitiated, and all the inconsistencies which follow are originated. The contradiction does not originate in Euclid, but it is the simple development of Mr. Smith's self-contradiction, worked out into a few figures.

Mr. Smith seems to think that his pamphlet will have the effect of making all orthodox mathematicians ashamed of themselves. It may be so; but their shame will certainly be this, that their efforts for the advancement of science have had so little effect that it is possible for such men as Mr. Smith to remain in their ignorance. Certainly Mr. Smith provides us with one of the strongest arguments

in favour of compulsory education. Compel him to take three lessons from any qualified mathematician, and he will immediately withdraw from circulation his opiniated pamphlets.

But we must not omit to notice the challenge with which Mr. Smith closes his work. He gives us a problem which he says "involves the most serious consequences to mathematical science," and he questions "if there be a living mathematician competent to solve it." The problem is thus stated:—

"Construct a geometrical figure, in which there shall be two dissimilar and unequal right-angled triangles, of which the sides subtending the right angle shall be equal."

Perhaps there is some misprint here; for, of course, as the question stands any fifth form boy would construct impromptu any number of triangles satisfying the required condition. For instance, if P Q R S, &c., be points on a circle, of which A B is a diameter, the triangles A B P, A B Q, A B R, A B S, &c., will be all right-angled triangles, of which the sides subtending the right angle are equal (each being A B), and not more than two of the triangles will be similar or equal. Or, if Mr. Smith would like such triangles to be constructed as would have their sides represented numerically by integers, we would suggest that the four triangles whose sides are

65, 63, 16; — 65, 60, 25; —
65, 52, 39; — 65, 56, 33; —

will all of them be dissimilar and unequal right-angled triangles, of which the sides subtending the right angle are all equal to 65. We anxiously look for the "serious consequences to mathematical science" which are to follow from our solution of the problem!

Note by Mr. Smith. The Rev. W. Allen Whitworth, *in the first Letter he addressed to Mr. Smith, quoted from this article, as if he were not the Author of it! Will, Mr. Whitworth declare that he was not the writer of the article, and so remove all doubt on this point? What are the values of the acute angles in Mr. Whitworth's right-angled triangles A B P, A B Q, A B R, A B S, &c, expressed in degrees? Let A B be the diameter of a circle. From the extre-*

mities of the diameter, that is, from the points A and B, draw straight lines to a point C in the circumference of the circle, so that the sides of the right-angled triangle so constructed, may be denoted by the arithmetical symbols 3, 4, *and* 5. *Such a triangle can be geometrically constructed, whatever Mr. Whitworth may insinuate to the contrary. The acute angles in this triangle will be angles of* $53° 8'$ *and* $36° 52'$. *If Mr. Smith is wrong, Mr. Whitworth can surely prove it. Let him try.*

FROM THE "LIVERPOOL LEADER," AUGUST 21ST, 1869.

CURIOSITIES OF MATHEMATICS.

No. 1.

CIRCLE SQUARING.

EVERYONE has heard of the famous problem of squaring the circle; but not everyone knows precisely what the problem is. Strictly, the problem is this: Given a circle, it is required *by the aid of ruler and compasses alone* to describe a square that shall be equal to the circle in area of surface. The problem cannot be solved; that is, it is impossible to obtain from the circle, by the aid of ruler and compasses alone, the accurate dimensions of the square. But this statement must not be misunderstood as if it implied that no square can exist equal to the given circle, or that the dimensions of such a square cannot be ascertained. The dimensions of the square can be found mathematically, and can be expressed arithmetically to any degree of accuracy. The solution is in no sense indeterminate; it is only *geometrically* unresolvable by ruler and compasses. It is, in this respect, analogous to the problem of trisecting a given angle; the construction of the trisecting lines cannot be made geometrically by ruler and compasses except in some few particular cases (*e.g.*, when the angle is a right angle, or half a right angle); but of course when the whole angle is known arithmetically in degrees, &c., we may at once assign the arithmetical magnitude of the parts, and may construct the parts by means of a scale.

The circle then can be squared arithmetically, but not geometrically. For the sake of the uninitiated, we will explain the principles of the arithmetical squaring of the circle, and shew the point at which the numerous so-called circle squarers dispute the infallible results of mathematical reasoning. The following facts are undisputed :—

1.—The ratio of the circumference of the circle to its diameter is always the same. The circumference is more than three times the diameter, and less than four times the diameter. The actual multiplier is much nearer three than four ; it has a certain definite and determinate value, of which we will say more presently. We only wish to insist here that its value is the same for all circles, and cannot, under any circumstances, change. It is common to denote this multiplier by the Greek letter π. With this notation, the circumference of any circle is π times the diameter, or if r denote the radius, the diameter is $2r$ and the circumference is $2\pi r$.

2.—The area of any circle is π times the area of the square on its radius. If r be the radius, the area is πr^2. Of course, it will be understood that π has here the same meaning as in the previous paragraph, and retains the value there spoken of.

3.—As soon as π is known, the arithmetical squaring of the circle immediately follows. For the area of the circle is π times the square on the radius, and is, therefore, equal to the square described on a line whose length is $\sqrt{\pi}$ (the square root of π) times the radius. Thus we know the dimensions of the square equal in area to any circle whose dimensions are known.

It will be seen from what precedes that all questions concerning the squaring of the circle may be made to depend on the single question, what is the value of π, or by what multiplier must the diameter of a circle be multiplied, that the product may be equal to the circumference ?

If any one will take a tape and measure round a circular table, and again measure it across through the centre, he will find that the one measure is *about* $3\frac{1}{7}$ times the other ;—for instance, if the table be seven feet across (in diameter) it will be *about* twenty-two feet round (in circumference). Hence *appoximately, but not accurately*, $3\frac{1}{7}$ or $\frac{22}{7}$ is the value of π. Archimedes gave this approximation in his book, "De Dimensione Circuli." In the year 1579 a closer ap-

proximation was found by Vieta, who proved that π was intermediate in value between 3·1415926535
and 3·1415926537;
And about the same year Ludolph Van Ceulen showed that it was intermediate between the much closer limits—

3·14159265358979323846264338327950288
and 3·14159265358979323846264338327950289.

But though these results are true as far as they go, they do not determine the value of π absolutely. Even when the decimals are calculated (as they have been) to 600 figures, the actual value of π is still undetermined, as there is nothing to show what the 601st, 602nd, &c., figures are to be. In order to make π *determinate*, we must have some rule for obtaining not merely 600 or even 600,000 figures, but a rule which will never fail to obtain *as many figures as we please*. Such a rule or formula absolutely determining and defining the value of π may be obtained mathematically in a variety of forms. One of the simplest formula, though by no means the most convenient to use in calculation, is the following, viz.: π is equal to eight times the sum of the following series, involving the successive odd numbers in regular order—

$$\frac{1}{1 \times 3} + \frac{1}{5 \times 7} + \frac{1}{9 \times 11} + \frac{1}{13 \times 15} + \&c.,^*$$

continued to an infinite number of terms. The terms in this series are continually decreasing, and presently become very small. (*Granted*). Wherever we stop, in adding up the values of the terms, we stop short of the value of π; but as we may continue to take as many terms as we please, we may get *as near as we please* to the actual value. (*If this were true, the arithmetical value of the symbol π would differ from the value assigned to it by Van Ceulen*). Such a series as this defines π, and the existence of such a series is

* This series calculated to 25 terms makes the value of π less than 3·125, and carried to 50 terms makes it less than 3·133. Is not Mr. Whitworth's assertion, that by this series—if continued to an infinite number of terms—"we may get as near as we please to the actual value" a mere random statement, without any show of proof? To whatever number of terms this series may be carried, 8 times the sum of the terms can never be made to reach 3·14!!

sufficient to show that (even though it be impossible to express the value of π in finite terms in the decimal scale of notation) the value is neither infinite nor indefinite. (*Nonsense*).

The host of circle squarers generally fix on some simple fraction not differing very much from the true value of π, and attempt to verify their assumption either by some process involving mechanical measurements in which small inaccuracies are inappreciable, or else by showing that their assumption is consistent with itself in some elaborate chain of reasoning in which the premises and the conclusion are practically equivalent. We have said that Archimedes gave $3\frac{1}{7}$ as an approximate value of π. Much closer approximations are $\frac{355}{113}$ and $\frac{5702}{1815}$. But the circle squarers very rarely select such close approximations as these; and a well-known Liverpool man, who has just published a large volume on the subject, is nearer the truth than many of his brethern, when he adopts the seriously deficient value $3\frac{1}{8}$ or $3{\cdot}125$. The great difficulty in reasoning with these men is found in the fact that they cannot appreciate the more refined process of mathematics by which the true value of π is proved. The mathematician generally finds it hopeless to bring down a positive proof to the standard of their comprehension. He is obliged to be content simply to point out the fallacies of the arguments which the circle squarers propound; but as these arguments are generally unlimited in number, and a new one is set up as soon as an old one is overturned, the task, is indeed, endless. The circle squarer is like the village schoolmaster of whom Goldsmith wrote—

> "In arguing too, the parson own'd his skill,
> For, even though vanquished, he could argue still."

Without the higher and more subtle process of mathematics, it is impossible to prove that π has the actual value determined by the series we have given. But it has been proved by the most elementary methods that π is less than $3{\cdot}2$. And next week we shall publish a proof by pure geometry and arithmetic that π is greater than $3{\cdot}126$. Consequently it cannot be equal to $3{\cdot}125$. Our proof being entirely independent of trigonometry will be intelligible to the merest tyro, and ought to suffice to dispel for ever the delusions even of the circle squarers of Liverpool.

FROM THE "LIVERPOOL LEADER," AUGUST 28TH, 1869.

CURIOSITIES OF MATHEMATICS.

No. II.

CIRCLE SQUARING.

(Continued.)

It is very surprising that the circle squarers who uphold erroneous values for the ratio of the circumference to the diameter in any circle have not discovered their errors by actual measurement. Actual measurement cannot, indeed, prove the true value of π, because measurement can never be depended upon as strictly accurate. For instance, the most careful measurer would never assert that his measurements were not wrong by the thousandth part of a hair's breadth. He may say his result is practically accurate, or sufficiently acurate for practical purposes; but this is not mathematical accuracy: Mathematical accuracy must be absolutely perfect. But though no actual measurement can prove the true value of π, actual measurement may disprove false values, by exhibiting discrepancies, greater than can be attributed to the error of measurement. For example, in measuring a length of 20 or 30 feet, the measurement may be so carefully made that we may be satisfied of the correctness of the result within an inch, or perhaps half an inch or a quarter of an inch. And if measurements may be depended upon even so far, it is sufficient to exhibit practically the incorrectness of such values as $3\frac{1}{81}$, $3\frac{1}{8}$, $3\frac{1}{6}$, &c., which have been assigned to π.

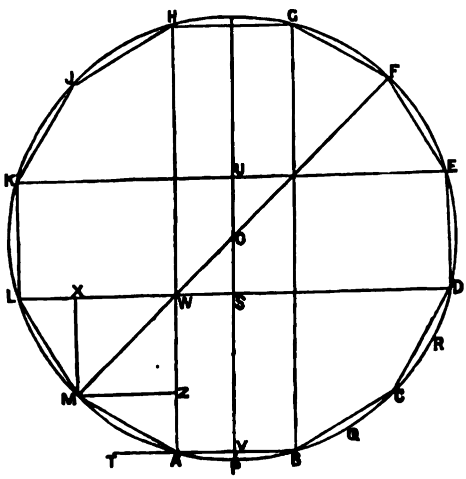
A round table of polished mahogany generally affords a very accurate specimen of a circle, and it is not difficult to meet with such a table of 5 feet, 6 feet, or more in diameter. Of course, the larger the circle the less liable are our results to be vitiated by errors of measurement.

Suppose the circle-squarer take a round table of even 5 feet in breadth, and compare by strict measurement with a tape the circumference and diameter. The circle-squarers, such as Gaetano Rossi (1804) and the Rev. D. Anghera of Malta (1854–1858), who maintained that $\pi = 3\frac{1}{6}$, would find the measure of the circumference of

the table $3\frac{1}{2}$ inches less than their theory would give; while Mr. James Smith, of Liverpool, who maintains that $\pi = 3\frac{1}{8}$, would find the circumference of the table about an inch longer than his theory would lead him to expect. But though many persons have vainly tried to estimate the ratio for themselves by this practical method, and one experimentalist, working in this way, obtained a value which agreed with the truth to three decimal places, yet the enthusiasts who cling to such values as we have instanced seem disinclined to have recourse to a method by which they might so readily satisfy themselves of the falsity of their position.

It is useless to offer to circle-squarers proofs which involve any high mathematics, for, if they had studied high mathematics, they would quickly have delivered themselves from their delusions. And it is very little use to offer them proofs which involve trigonometry, as it is generally the haziness of their notions of trigonometry that has led them most astray. The following proof that π is greater than 3·126 is free from trigonometry, involving only arithmetic and a little pure geometry. It does not go far towards establishing the true value of π, but it is sufficient to show that, whoever be right, those persons are wrong who maintain $\pi = 3\frac{1}{8}$ or $\pi = 3\frac{1}{81}$, or who assign any values so small as either of these.

Let A B C D E F G H J K L M be a regular dodecagon, of which every side = 2.



Let a circle circumscribe the xii.-gon, and let the arcs A B, B C, C D, &c., be bisected respectively in P, Q, R, &c.

And join A P, P B, B Q, Q C, C R, R D, &c., so that the figure A P B Q C R D......is a regular polygon of xxiv. sides inscribed in the circle.

Join A H and B G, evidently parallel to D E and L K.

Join D L and E K, evidently parallel to A B and H G, and at right angles to A H and B G, from consideration of symmetry.

Let D L intersect A H in W, which must (from symmetry) lie on the diameter F M.

Find O the centre of the circle.

Join O P, cutting A B at right angles in V, and bisecting A B.

Let O P cut L D and E K in S and U respectively. *(A lapsus. O P does not cut E K in U.)*

Then,— $OS = OU = \frac{1}{2} SU = \frac{1}{2} LK$

But $LK = 2$

Therefore, $OS = 1$

$OU = 1$

Similarly, $SW = AV = \frac{1}{2} AB$

$SW = 1$

Therefore, $OW = \sqrt{2}$ (Eucl. I., 47).

Draw M Z parallel to A B or L D to meet A H in Z.

Draw M X parallel to L K or A H to meet D L in X. Produce B A to T.

Then, since all the exterior angles of a rectilinear figure are together equal to four right angles, each exterior angle of a regular xii.-gon must be $\frac{1}{12}$th of 4 right angles, or $\frac{1}{3}$rd of a right angle.

Therefore, T A M = $\frac{1}{3}$ of a right angle ;

Therefore, A M Z = $\frac{1}{3}$ of a right angle ;

And, M A Z = $\frac{2}{3}$ of a right angle ;

And the triangle M A Z is, therefore, half of an equilateral triangle.

Hence, since $AM = 2$

it follows, that $AZ = 1$

and $MZ = \sqrt{3}$ (Eucl. I., 47).

In the same way $LX = 1$

and $MX = \sqrt{3}$

xvii

But, $M X = Z W$

Therefore, $Z W = \sqrt{3}$

Hence, in the right-angled triangle M Z W, the two sides M Z and Z W each = $\sqrt{3}$, and, therefore, Hypothenuse M W = $\sqrt{6}$.

But we have shown that O W = $\sqrt{2}$.

Therefore, by addition,

The radius of the circle, or O M = $\sqrt{6} + \sqrt{2}$.

And, therefore, $(O\,M)^2 = 8 + 4\sqrt{3}$.

Again,
$$\begin{aligned} O V &= O S + S V \\ &= O S + W A \\ &= O S + W Z + A Z \end{aligned}$$

And,
$$\begin{aligned} O S &= 1 \\ W Z &= \sqrt{3} \\ A Z &= 1 \end{aligned}$$

Therefore, by addition,

$$O V = 2 + \sqrt{3}$$

But, Radius $O P = \sqrt{6} + \sqrt{2}$

Therefore, by subtraction,

$$P V = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$$

And, therefore, $(P V)^2 = 15 - 10\sqrt{2} + 8\sqrt{3} - 6\sqrt{6}$.

But A V = 1, and, therefore, $(A V)^2 = 1$.

But $(P V)^2 + (A V)^2 = (A P)^2$ (Eucl. I., 47). Therefore, by addition, $A P^2 = 16 - 10\sqrt{2} + 8\sqrt{3} - 6\sqrt{6}$.

Now, the circumference of the circle must be greater than the perimeter of the xxiv.-gon. Therefore,

$$2\pi \times O M > 24 \times A P,$$

dividing by 2, and squaring it, it follows, that

$$\pi^2 \times (O M)^2 > 144 \times (A P)^2.$$

That is—

$$\pi^2 \times (8 \times 4\sqrt{3}) > 144 \times (16 - 10\sqrt{2} \times 8\sqrt{3} - 6\sqrt{6})$$

Divide by 4, and we get—

$$(24 + \sqrt{3}) \times \pi^2 > 36 \times (16 - 10\sqrt{2} + 8\sqrt{3} - 6\sqrt{6})$$

And, therefore,

$$\pi^2 > 36 \times \{8 - 2\sqrt{2} - 2\sqrt{6}\}$$

But $8 - 2\sqrt{2} - 2(\sqrt{6})$ is itself greater than ·2715.

Therefore, *a fortiori*, $\pi^2 > 36 \times (·2715)$

And, therefore, $\pi > 3·126$.

FROM THE "LIVERPOOL LEADER," SEPTEMBER 11th, 1869.

CURIOSITIES OF MATHEMATICS.

No. III.

THE FIRST PRINCIPLES OF ARITHMETIC.

CURIOUS it is to observe the strange conceptions formed by men (often, in other respects, even wise men) concerning the nature of some of the simplest processes in arithmetic, and the meaning of some of the simplest statements of arithmetic. We find persons who distinguish between "truth" and "arithmetical truth," who assert that that which is really untrue may be arithmetically true, in which assertion of theirs the adverb "arithmetically" seems to throw a mystic shade over the adjective which it qualifies—a cloud too dense for any shaft of argument to penetrate.

The statement has recently been published that a circle whose diameter is 4 has its circumference equal to its area. The circumference and the area equal! The line which surrounds the circle equal to the whole surface included by the circle! Why, even a donkey tethered by a string to a post in the middle of a field would discover a considerable difference in the grass available for his dinner, if he were compelled by force of whip to keep the string tight and describe only the circumference of a circle, instead of being allowed to graze over the whole surface within the limits determined by the length of the string. But we suppose this equality is only "arithmetically true," a pleasant euphenism * for "not true at all." It might as well be said that the tenth part of an hour is equal to the sixth part of a yard, because each may be represented in convenient units by the number 6. But, worse than this, people have no hesitation in adding together a length and an area, or an area and a volume. They think that 5 miles and 7 acres must make something; or that it must be possible to say what a bushel and a yard will come to when added together. A clerk who has been some years in an office will sit down and attempt, in perfect good

* Query: euphonism?

faith, to substract 7 yards 2 feet 5 inches, from 9 square yards 1 square foot 3 square inches, and will expect to get an answer expressed in some species of yards, feet, and inches.

But if the notions of the people on the subject of addition and subtraction are vague, their ideas of multiplication and division are infinitely vaguer. Money gets multiplied by money to any extent, and tons get multiplied by pounds, and no one ever stops to think what is this multiplication after all. Many people, for example, will maintain the correctness of the following statements:—

2 crowns	× 2 crowns	=	4 crowns	= £1.
5 florins	× 5 florins	=	25 florins	= £2 10s.
10 shillings	× 10 shillings	=	100 shillings	= £5.
120 pence	× 120 pence	=	14400 pence	= £60.

And when we point out that the sums of money multiplied being the same, the results £1, £2 10., £5, and £60 must be consequently the same, they will stare in confused amazement, and declare that "figures will prove anything."

Verily, figures *will prove anything* when they are manipulated by persons to whom figures *may mean anything*. When people attach their own ideas to arithmetical symbols, or use them without any ideas at all, and work with them by processes which they accept on the faith of arithmetical treatises misinterpreted by themselves, and submit them to rules which they do not understand—rules which were established for very different cases—then, indeed, figures may seem to prove anything. But when figures are fairly used, when their meaning is strictly defined, and when the meaning, once defined, is adhered to, when the work is not done in the dark, the intermediate steps unexplained, and the result simply interpreted by a juggle, then there is no reasoning so certain as that of arithmetic, and no results so conclusive as those which figures establish.

And now, reader, let us suppose you are an amateur mathematician, or, it may be, an amateur arithmetician. We will give you two rules which, if you will only faithfully observe them, will preserve you safely from the pitfalls in which your companions are so often destroyed:—

RULE I.—You may use any term in any sense you please, if you will only define that sense and *stick to it.*

For instance, if you like to define the five letters b-l-a-c-k as expressing the colour of snow, we have no objection to the convention so long as you adhere to it consistently throughout. Your arguments will be in no degree vitiated ; your conclusions will be perfectly correct. Only be consistent. If we agree that b-l-a-c-k shall mean white, you must not, in the course of your argument, turn round and say "No, but it spells *black*," and argue thence that black is white. All we ask for is consistency. Any term, any symbol, any sign shall mean anything you like, only say what you mean and adhere to your definition. You may make — mean *plus* and + mean *minus;* you may make ÷ the sign of multiplication, and × the sign of division, and as long as you preserve the convention your arguments will remain sound ; but you must not take up an algebra, and quote a formula in which these signs have their more usual meanings, and still expect the formula to hold good when you interpret it according to your own convention.

RULE II.—BEWARE OF RULES.

There is no rule in arithmetic but the rule of common sense, and if there be any rule given which you cannot by common sense prove to be true, you have no right to use that rule, or to expect others to accept it. If you are unable to justify the rule, you are unable to recognise under what conditions it is applicable :

"Trust her not,
She is fooling thee !"

Here we pause for the present. We only add two questions for our readers' consideration, and shall be happy to receive their solutions of them for next week's publication :—

QUESTION 1.—What is the meaning of "two," "three," &c. ?

QUESTION 2.—What is meant by "Multiplication ?"*

(*To be continued.*)

* In the three foregoing Articles from the *Leader*, contrary to "*the rule of common sense*," and by a misapplication of Euclid : I. : 47, Mr. Whitworth (without knowing it) attempts to measure *directly* a curvilinear figure by means of a rectilinear figure.

J. S.

FROM THE "LIVERPOOL LEADER," FEBRUARY 5TH, 1870.

CURIOSITIES OF MATHEMATICS.

TO THE EDITOR OF THE LIVERPOOL LEADER.

SIR,—Under the above title, there appeared in your valuable journal of August 21st, 28th, and September 11th last, certain articles, in one of which my name is introduced. The author of these articles, is obviously the Rev. Professor Whitworth, formerly Professor of Mathematics in Queen's College, Liverpool. I say this advisedly, for, in August last, he sent me an elaborate paper, entitled—"A proof of π without trigonometry, for the benefit of those who maintain that π is equal to 3·125.' This paper was evidently prepared for publication, and the Professor requested me to return it, which I did, only retaining it long enough to read it over three or four times with great càre. In this paper he introduced the diagram, which appears in the article in the *Leader* of the 28th August, and by means of which the learned Professor fancies he can prove that π is greater than 3·126.

I will now give your readers a proof without trigonometry, that π cannot be either greater or less than 3·125, if you do me the honour to insert this communication.

Let c denote the circumference of a circle; let d denote the diameter; let r denote the radius; let sr denote the semi-radius; let a denote the area; and let x denote the area of a regular dodecagon inscribed in the circle.

Let the sides of the dodecagon be represented by the arithmetical symbol 2, as the Rev. Professor Whitworth puts it (in the Article in the *Leader* of the 28th August) in his attempt to prove that π is greater than 3·126. Then: $12\ (2) = 24 =$ the perimeter of the dodecagon. Add $\frac{1}{24}$th part $= \frac{24}{24} = 1$. Then: $(24 + 1) = 25 = c$: $\frac{c}{3·125} = 8 = d$: $\frac{8}{2} = 4 = r$: and $3\frac{1}{8}\ (r^2) = 3·125\ (4^2) = (3·125 \times 16)$

$= 50 = a$; therefore, $\frac{24}{25}(a) = 6(r \times sr)$; and this equation $= 48 = x$, and is equal to three-fourth parts of the area of a circumscribing square to the circle; and it follows, that x is equal to the area of a square on the side of an equilateral triangle inscribed in the circle, when all the angles of the equilateral triangle touch the circumference of the circle; it also follows, that π—which denotes the circumference of a circle of diameter unity—cannot be either greater or less than $\frac{25}{8} = 3{\cdot}125$.

THEOREM.

From a given value of c, find the value of x.

Let $c = 100$. Then: $\frac{24}{25}(c) = \frac{24 \times 100}{25} = \frac{2400}{25} = 96 : \frac{96}{3} = 32 = d : \frac{d}{2} = \frac{32}{2} = 16 = r : \frac{d}{4} = \frac{32}{4} = 8 = sr$; therefore, $3\frac{1}{8}(r^2) = (c \times sr)$, and this equation $= 800 = a$: and it follows, that $\frac{24}{25}(a) = 6(r \times sr)$, and this equation $= 768 = x$.

Can the Rev. Professor Whitworth find the value of x from a given value of c? Certainly not, unless he "*plough*" with my heifer. How will the reverend gentleman explain this?

Let x denote the area of a regular dodecagon; let y denote the area of a circle circumscribed about the dodecagon; let z denote the area of an inscribed square to the circle; and let r denote the radius, and sr the semi-radius of the circle.

Then: $6(r \times sr) = x$, and $\frac{25}{24}(6 \times r \times sr) = y$: and if r be represented by a whole number, or by the square root of a whole number, $\frac{6(r \times sr)}{24}$ is a *finite* and *determinate* arithmetical quantity. For example: Let $r = 20$. Then: $\frac{20}{2} = 10 = sr$: $\frac{6(20 \times 10)}{24} = \frac{(6 \times 200)}{24} = \frac{1200}{24} = 50$; therefore, $(1200 + 50) = 3\frac{1}{8}(r^2)$, and this equation $= 1250 = y$. Again: Let $r = \sqrt{7}$. Then: $\frac{1}{2}(\sqrt{7}) = \frac{\sqrt{7}}{2} = \sqrt{\frac{7}{2^2}} = \sqrt{\frac{7}{4}} = \sqrt{1{\cdot}75} = sr$: $6(\sqrt{7} \times \sqrt{1{\cdot}75}) = 6(\sqrt{7 \times 1{\cdot}75}) = 6(\sqrt{12{\cdot}25}$:) therefore, $\frac{6(\sqrt{12{\cdot}25})}{24} = \frac{\sqrt{6^2 \times 12{\cdot}25}}{24} =$

$$\frac{\sqrt{36 \times 12{\cdot}25}}{24} = \frac{\sqrt{441}}{24} = \frac{7}{24}\left(\frac{6}{2}\right) = \frac{7 \times 3}{24} = \frac{21}{24} = {\cdot}875 \text{ exactly.}$$

Well, then, it cannot be controverted that $\frac{6(r \times s r)}{24}$ is a *finite* and *determinate* arithmetical quantity, whether r be represented by a whole number, or by the square root of a whole number. How happens this? It happens because $\frac{24}{8} = 3 =$ the perimeter of a regular inscribed hexagon to a circle of diameter unity; and $\frac{24}{6} =$ the *mystic* number 4; and it follows, that $4(r \times s r) =$ area of an inscribed square to every circle; and $8(r \times s r) =$ area of a circumscribed square to every circle. Hence: $\left\{(z + \frac{1}{4} z) + \frac{1}{4}(z + \frac{1}{4} z)\right\} = y$: and conversely, $\left\{(y - \frac{1}{5} y) - \frac{1}{5}(y - \frac{1}{5} y)\right\} = z$: and it also follows, that $\frac{24}{25}(y) = x$.

THEOREM.

From a given value of x, find the value of y.

Unless x be represented by an arithmetical quantity that is divisible by 3, or some multiple of 3, without a remainder, it is impossible to solve this theorem with arithmetical exactness.

For example: Let $x = 100$. Then $\frac{25}{24}(100) = \frac{25 \times 100}{24} = \frac{2500}{24} = 104{\cdot}1666$ with 6 to infinity, an *indeterminate* arithmetical quantity. But, let $x = \frac{24}{25}(100) = \frac{24 \times 100}{25} = \frac{2400}{25} = 96$, which is divisible by 3, and 6, without a remainder. Then: $\frac{25}{24}(x) = \frac{25 \times 96}{24} = \frac{2400}{24} = 100 = y$ exactly.

Euclid treated geometry as an exact science, but nowhere treats of the areas of geometrical figures, nor could he, without travelling out of the domain of pure geometry. All the demonstrations in his first four books are by lines, angles, and geometrical constructions, and he could go no further, without the aid of "*that indispensable instrument of science*, Arithmetic."

According to "*recognised Mathematicians*," "if the circumference of a circle be finite, away goes the diameter into decimals without end;" and "if the diameter of a circle be finite, away goes the circumference into decimals without end;" and if either circum-

ference or diameter be *finite* and *determinate*—according to recognised Mathematicians—away goes the area into decimals without end.

So recently as the 18th January, 1870, a paper was read at an extraordinary meeting of the Liverpool Literary and Philosophical Society, by the Rev. Dr. Jones, Principal of King William's College, Isle of Man, and formerly of Queen's College, Liverpool, on "*The Unsuitableness of Euclid as a Text-book of Geometry.*" An abstract of this paper appeared in the *Liverpool Courier* of January 20, *(See Appendix D)*, from which I learn that the Rev. Dr. Jones said—"His (Euclid's) system was narrowed by the exclusion of arithmetical considerations, without which no practical geometrical inquiry was possible." I also learn that he said—"His (Euclid's) doctrine of proportion was immaterial, and a violation of common sense."

The day will come when "*recognised Mathematicians*" will think it worth their while to make a study of the relations that exist between the square roots of numbers and commensurable quantities, and will then discover, that one of Euclid's greatest faults was his attempting to make his fifth book, on proportion, alike applicable to commensurables and incommensurables.

The Articles which have appeared in the *Leader* on "*Curiosities of Mathematics*" were to be continued. How happens it that, after a lapse of more than four months, Article 4 has not yet made its appearance ?—I am, Sir, yours obediently, JAMES SMITH.

P.S.—Public attention is now being called to Euclid's faults, "*as a text-book for teaching geometry to beginners,*" by some of the first "*mathematical authorities*" in England ; and it appears to me that the time is opportune for directing the attention of your numerous readers to this interesting and important subject. J. S.

FROM THE "LIVERPOOL LEADER," FEBRUARY 12TH, 1870.

CURIOSITIES OF MATHEMATICS.

TO THE EDITOR OF THE LIVERPOOL LEADER.

SIR,

Mr. James Smith has amused mathematicians for some years by his vagaries concerning the value of π, which he asserts to be $3\frac{1}{8}$. I merely wish to point out to your readers what he means, and any one of them who has a bicycle and a practising ground wherein to exercise it can test this idea for himself. Let us hope that some one will do so, in the interests of science.

Mr. Smith asserts that in 8 turns of a wheel one traverses a distance equal to 25 times the diameter of the wheel, and, consequently, that in 64 turns of the same, one traverses a distance equal to 200 times the diameter of the wheel. "Recognised mathematicians," as he delights to call them, say that in 64 turns of a wheel one traverses a distance equal to at least 201 times the said diameter.

Now, this is a practical question enough. All the experimentalist has to do is to measure the diameter of his *fore*-wheel carefully, and then count 64 turns of it. He can then measure whether the distance is 200 or 201 times the measured diameter. A white spot painted on the nave would assist a bystander to help the performer to count. Who will try the experiment half a dozen times, in the interest of science?

WALTER W. SKEAT.

FROM THE "LIVERPOOL LEADER," FEBRUARY 19TH, 1870.

CURIOSITIES OF MATHEMATICS.

TO THE EDITOR OF THE LIVERPOOL LEADER.

BARKELEY HOUSE, SEAFORTH,
12th February, 1870.

SIR,

You have done me the honour to insert the letter I addressed to you, under the above title, in your publication of the 5th inst., and I posted a copy of the paper to the Rev. W. Allen Whitworth. (I have since received a note from him, in which he says—"*As I resigned my professorship last June, I am not entitled to the prefix with which you adorn my name.*") In return, he sent me a report of the "Speech-day of Derby Grammar School," taken from the *Derby and Chesterfield Reporter*. Mr. Whitworth was selected as the examiner of the boys of the school in Mathematics, and I quote the following from his report:—"I am glad to see the study of Geometry taken up by such young boys in this school. I am sorry that the persistence of the Universities in recognising Euclid, and Euclid only, as a text-book, forces the same book upon schools. Still I think it would be found very beneficial to give the lower boys a course of elementary geometry less technical than Euclid's,—a course of practical geometry and mensuration combined,—and leave them to take up Euclid when they rise to the fourth or fifth form." Hence, on the unsuitableness of Euclid as a school-book for teaching geometry to beginners, Mr. Whitworth, the Rev. Dr. Jones, and I are agreed. I also sent a copy of the *Leader* of the 5th inst. to an old correspondent, and have since received a note from him, of which the following is a copy:—

Werrington, Launceston, 9th February, 1870.

My Dear Sir,—I suppose it was by your suggestion that I received the *Liverpool Leader* of 5th February. I have therefore

answered what seemed a sort of *invitation* to me ; but so little do I wish to prolong the old controversy, that I leave it entirely to you to send, or not, my letter to the *Liverpool Leader*.

Yours very truly,
G. B. GIBBONS,
Now Vicar of Werrington.

The following is a copy of the letter referred to, and I send you the original herewith :—

TO THE EDITOR OF THE LIVERPOOL LEADER.

SIR,

I have had a long and not unfriendly controversy with Mr. Smith about the value of π, and I therefore presume it was at his suggestion that I have received your paper of 5th February.

If the perimeter of the dodecagon is 24, Mr. Smith assumes, by simply adding 1-24th, that the circumference of the inscribing circle will be 25 ; but he only states this, without any attempt at proving it.

He also states that if the side be 2, the radius will be 4, again without any demonstration.

No doubt, if the circumference of the circle is 25, *while the radius is* 4, π must be 3·125 ; but to assert this is quietly to assume the whole question in dispute.

As the central angle of this dodecagon is 30°, whose sine and cosine are known without reference to tables, it is easy enough to find the radius if the side is given, or *vice-versâ ;* and the calculation does not depend on, or in any way involve, the value of π.

c being the central angle, r the radius, s the side, it is known from elementary trigonometry that $sr = 2r^2 - 2r^2 \cos. c$.

Now, $\sin. 30^\circ = \frac{1}{2}$: $\text{Cos. } 30^\circ = \frac{\sqrt{3}}{2}$

Hence: $$\begin{aligned} s^2 &= r^2(2 - \sqrt{3}) \\ &= r^2(2 - 1\cdot73205080\ldots) \\ &= r^2(\cdot26794920\ldots) \end{aligned}$$

The root of 3 runs off into endless decimals, entailing the same misfortune on π.

By assigning any value to s or r, we get the corresponding value of the other. Extracting the square root

$$s = (\cdot 5176)\, r.$$

Let $r = 4 : s = 2\cdot 0704$, not 2 exactly.

Any one conversant with the elements of trigonometry and able to extract a square root, can follow out and test this.—I am, Sir, your obedient servant, G. B. GIBBONS, Vicar of Werrington.

Werrington, Launceston, 9th February, 1870.

I admit that the central angle is an angle of 30°; and I also admit that the Sine of this angle is $\frac{1}{2} = \cdot 5$, and its Cosine $\sqrt{\cdot 75}$, or, if Mr. Gibbons likes the expression better, $\frac{\sqrt{3}}{2}$. But, if unity be adopted as the radius of a circle, our unit of length being 1, the sides of a regular inscribed dodecagon are arithmetically *indeterminate*. Now, Mr. Whitworth, in his fancied proof that π is greater than 3·126, adopts, as a datum or premiss, a side of a regular dodecagon, representing its value by the *finite* and *determinate* arithmetical symbol 2. On this assumption he would make similar angles unequal. Can a central angle of 30° be subtended by a chord that is *indeterminate* in one case, and *finite* and *determinate* in another? Certainly not! Mr. Whitworth's argument would make the central angle to be, *not* angle of 30°, but some other angle. Well, then, Mr. Gibbons and Mr. Whitworth differ from each other, and both fail to perceive that, *mathematically*, the central angle is connected *directly* with its subtending arc, and only *indirectly* with its subtending chord. Let these Reverend gentlemen settle their differences. It is enough for me to show that they do differ; to go further would open too wide a field for a newspaper communication.

In a very recent correspondence with a Mathematician, I extracted from him the following admissions:—First: $\frac{1}{4\pi}$ denotes the area of a circle of circumference unity. Second: If $\pi = 3\cdot 125$, the area of a circle of circumference unity $= \cdot 08$. No further admissions are necessary from any Mathematician to enable me to demonstrate that the circumference of a circle of diameter unity (which is denoted by the symbol π) is 3·125, and can be nothing else.

Because $\frac{\text{circumference}}{2\,\pi}$ = radius in every circle, it follows, that $\frac{\text{circumference}}{4\,\pi}$ = semi-radius in every circle, whatever be the value of π.

Now, it is axiomatic, if not self-evident, that, to find the area of a circle from a given circumference, we must first find either the diameter, radius, or semi-radius of the circle, and, to find one or other of these, the arithmetical value of the symbol π is involved; and it is self-evident that neither diameter, radius, nor semi-radius can be found accurately with a false value of π.

Let c denote the circumference of a circle, and be represented by unity; let d denote the diameter of the circle; and let sr denote the semi-radius of the circle.

Then: Our unit of length being 1, $\frac{1}{4(3{\cdot}125)} = \frac{1}{12{\cdot}5} = {\cdot}08 = sr$: $4\,(sr) = (4 \times {\cdot}08) = {\cdot}32 = d$: and $3{\cdot}125\,(d) = (3{\cdot}125 \times {\cdot}32) = 1$ = unity = c. But $2\,(s\,r) = (2 \times {\cdot}08) = {\cdot}16$ = radius of a circle of circumference unity; and $3\frac{1}{8}\,({\cdot}16^2) = (3{\cdot}125 \times {\cdot}0256) = {\cdot}08$ = area of the circle; and it follows, that the semi-radius and area of a circle of circumference unity are represented by the same arithmetical symbols. This is *unique*, and can be predicated *only* of a circle of circumference unity, and work out with arithmetical exactness. But the property of one circle is the property of all circles, and it follows, that $\frac{c}{12{\cdot}5} = sr$, whatever be the value of c. For example: Let $c = 360$ *linear* yards. Then: $\frac{360}{12{\cdot}5} = 28{\cdot}8$ *linear* yards $= sr$: $2\,(s\,r) = (2 \times 28{\cdot}8) = 57{\cdot}6$ *linear* yards = radius of a circle of which c denotes the circumference; and it follows, that $c \times s\,r = 3{\cdot}125\,(r^2)$; that is, $(360 \times 28{\cdot}8) = (3{\cdot}125 \times 3317{\cdot}76)$, and this equation = 10368 *square* yards = area of a circle when the circumference is 360 *linear* yards; and it cannot be controverted that $c \times s\,r$ = area in every circle.

When Mr. Gibbons, Mr. Whitworth, and other "*recognised Mathematicians*" can find a value of π greater than 3·125, and make the area of a circle, of which the circumference is 360 *linear* yards, either equal to, or greater than, 10,368 *square* yards, they will be

able to prove that I am a *Pseudomath*. This word is the coinage of Professor De Morgan's brain, and the meaning he gives to it is, "*a person who handles mathematics as the monkey handled the razor.*"

I shall notice the letter of Mr. Walter W. Skeat, which appears in this day's number of the *Leader*, in my next communication. There are many good points in the Rev. W. Allen Whitworth's letter on National Education.

I am, Sir,

Yours obediently.

JAMES SMITH.

FROM THE "LIVERPOOL LEADER," FEBRUARY 26TH, 1870.

CURIOSITIES OF MATHEMATICS.

TO THE EDITOR OF THE LIVERPOOL LEADER.

SIR,

Your correspondent, Mr. James Smith, certainly *is* a "Pseudomath," with all respects to him notwithstanding. In his Letter to you in the *Liverpool Leader* of the 19th, he says he can demonstrate that π is 3·125 from the following two data, and from them alone:—

"First: $\frac{1}{4\pi}$ denotes the area of a circle of circumference unity. Second: If $\pi = 3{\cdot}125$, the area of a circle of circumference unity = ·08." But, in fact, he proves nothing of the sort. All he does is merely to show that *if* $\pi = 3{\cdot}125$ (which is the point to be proved), then "10,368 square yards = area of a circle, of which the circumference is 360 linear yards." He concludes by challenging any one to "find a value of π greater than 3·125, and make the area of a circle, of which the circumference is 360 linear yards, either equal to or greater than 10,368 square yards." Of course, it cannot be done; but what then? 10,368 square yards is the area on the supposition that $\pi = 3{\cdot}125$. A greater value of π gives a less area. A less value of π gives a greater area. Mr. Smith has proved nothing.

I am, Sir,

Yours obediently,

T. S.B.

FROM THE "LIVERPOOL LEADER," MARCH 5TH, 1870.

CURIOSITIES OF MATHEMATICS.

TO THE EDITOR OF THE LIVERPOOL LEADER.

SIR,

In your number of to-day there is a letter on the above subject, signed "T. S. B." Not only from the signature, but from the letter itself, I discover that the writer is an old correspondent of mine. Will "T. S. B." deny that he has made the following admissions to me? First: $\frac{1}{4\pi}$ denotes the area of a circle of circumference unity. Second: If $\pi = 3{\cdot}125$, the area of a circle of circumference unity $= {\cdot}08$. And, in the last communication I received from him (dated 19th October last), he says:—"There is nothing ab surd in π being indeterminate. It is no more absurd that π should be indeterminate than that the ratio between the diagonal and a side of a square should be so." His first admission is quite sufficient to enable me to demonstrate the absurdity of this assertion.

π denotes the circumference of a circle of diameter unity. To π add x, and from π substract x; x denoting the same arithmetical quantity in both cases. Then: $\frac{(\pi + x) + (\pi - x)}{2} = \pi$, whatever be the value of π. No Mathematician can controvert this formula. Now, π may *hypothetically* be anything greater than 3, and less than 4. By hypothesis, let $\pi = 3{\cdot}14159$, &c. Then: $\frac{(3{\cdot}14159, \&c. + x) + (3{\cdot}14159, \&c. - x)}{2} = \pi$. Let "T. S. B." or "W. A. W." find the value of π on this *supposition*, and demonstrate that π may be an indeterminate arithmetical quantity.

"T. S. B." admits that when the circumference of a circle is 360 *linear* yards, the area is 10,368 *square* yards, *on the supposition that* $\pi = 3{\cdot}125$. He also admits that *a greater value of* π *gives a less*

area, and *a less value of π gives a greater area*, and arrives at the conclusion that Mr. Smith *has proved nothing*. If Mr. Smith has proved nothing, "T. S. B." has proved much. He has proved his inability to perceive the fallacy of his own arguments, and he has also proved his incapacity to detect the absurdity of his own conclusions.

For the future I shall decline to notice any letters that may appear in the *Leader* on this subject, unless the writers have the moral courage to give their names and addresses publicly. I can assure you, Sir, that you would not get rid of this subject for years if I were to notice every anonymous writer; for I know from past experience that anonymous scribblers will persist in making *assertions* in direct opposition to common sense, and expect these assertions to be treated as if they were sound and legitimate arguments.

Since I last addressed you on this subject, I have received a communication from one of your correspondents,—a well-known "*recognised Mathematician*,"*—in which he adopts the following argument:—"*Let the side of a square* = 4. *Then: Perimeter of square* = 16, *and the area of square* = 16; *but we cannot say perimeter = area, for they are magnitudes of different kinds—the one denotes* 16 *units of area, and the other* 16 *units of length*." Now, about a circle, circumscribe a square. Let the diameter of the circle = 4. Then: There are as many units of area in the circle as there are units of length in the circumference, and I presume, it would be considered by this gentleman as an outrage on common sense to say, that the circumference and area of the circle are represented arithmetically by the same symbols.

In my letter, which appeared in your valuable journal of the 5th inst., I gave the following theorem:—From a given value of c find x, c denoting the circumference of a circle, and x denoting the area of its inscribed regular dodecagon. I solved this theorem, and proved that $x = 768$, when $c = 100$.

Mr. Gibbons, in his letter which appeared in your number of the 19th inst., makes the following assertions:—First: "*If the perimeter of the dodecagon is* 24, *Mr. Smith assumes, by simply adding* 1-24*th, that the circumference of the inscribing circle* (this should be circumscribing circle) *will be* 25: *but he only states this, without any*

* Revd. W. Allen Whitworth.

attempt at proving it." Second: "*He also states that if the side be* 2 *the radius will be* 4, *again without any demonstration.*" To prove these assertions, Mr. Gibbons should have demonstrated that I am wrong in making $x = 768$, when $c = 100$. If Mathematics will not enable Mr. Gibbons to do this, of what value can Mathematics be? If, on the other hand, Mathematics will enable Mr. Gibbons to prove me wrong, and he will not prove it, can he be an honest mathematican and a fair controversialist? Nay, is he not doing the science of Mathematics a positive injustice? Mr. Gibbons, in his correspondence with me, when foiled in argument, has invariably wriggled out of the difficulty, either by passing by the argument or perverting it, and taking care never to admit anything.

Now, it is known to, and admitted by, all Mathematicians, that 6 (radius × semi-radius) = area of a regular inscribed dodecagon in every circle, whatever be the circumference of the circle. This enables me to bring this interesting and important subject under the notice of your numerous readers in another form.

Let c denote the circumference of a circle; let p denote the perimeter of a regular hexagon or six-sided polygon inscribed in the circle; let d denote the diameter, r denote the radius, and sr denote the semi-radius of the circle; let a denote the area of the circle; and let x denote the area of a regular dodecagon inscribed in the circle.

THEOREM.

From a given value of c find x, and prove that $x = 6\,(r \times sr)$.

Let $c = 360$ *linear* yards. Then: $\frac{24}{25}\,(360) = \frac{24 \times 360}{25} = \frac{8640}{25}$ $= 345{\cdot}6$ *linear* yards $= p$: $\frac{p}{3} = \frac{345{\cdot}6}{3} = 115{\cdot}2$ *linear* yards $= d$: $\frac{p}{6} = \frac{345{\cdot}6}{6} = 57{\cdot}6$ *linear* yards $= r$: $\frac{p}{12} = \frac{345{\cdot}6}{12} = 28{\cdot}8$ *linear* yards $= sr$: therefore, $6\,(r \times sr) = 6\,(57{\cdot}6 \times 28{\cdot}8) = (6 \times 1658{\cdot}88) = 9953{\cdot}28 = x$: and it follows, that $\frac{25}{24}\,(x) = \frac{25 \times 9953{\cdot}28}{24} = \frac{248832}{24}$ $= 10368 = a$.

Hence: $\frac{24}{25}\,(a) = x$: and conversely, $\frac{25}{24}\,(x) = a$: and, by analogy or proportion, $p : c :: x : a$: that is, $345{\cdot}6 : 360 :: 9953{\cdot}28 : 10368$. These facts harmonise with each other, and dovetail into each other,

and so, as Bacon says, "*like an arch or dome, mutually sustain each other, and form a coherent whole;*" making $\frac{25}{8} = 3{\cdot}125$ the arithmetical value of π, and 8 circumferences = 25 diameters in every circle.

If Mr. Gibbons and Mr. Whitworth can "*upset*" this conclusion, and decline to do it, will they not be traitors to their profession? If they can't upset it, and decline to admit it, what will they be?

The day will come when my conclusion will be admitted by the universal consent of Geometers and Mathematicians in every part of the world, and I can afford to "*bide my time.*"

With reference to Mr. Skeat's Letter, which appeared in the *Leader* of the 12th instant, I may observe:—Hitherto "*recognised Mathematicians*" have looked upon mechanical circle-squarers as little better than maniacs, and yet, *they* can suggest a mechanical test with great plausibility when it suits their purpose. No mechanical operation can ever produce a geometrical and mathematical demonstration. The force necessary to give rotation to a carriage wheel must be either a *push* or a *pull.* In either case, in addition to the rotation of the wheel, there is a slightly sliding motion along the ground, so that in any number of revolutions of the wheel, the ground passed over will give a longer line than is indicated by the circumference of the wheel; and I venture to say that no half-dozen men would, from 64 revolutions of a wheel, arrive exactly at the same result. From such an experiment I could never get the same result twice over. Until Mr. Skeat can give rotation to a wheel without the application of any force, he had better abandon his mathematico-mechanical demonstrations.

I am, Sir,
Yours obediently,
JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
February 26th, 1870.

TO THE EDITOR OF THE LIVERPOOL LEADER.

SIR,

In your paper of 19th February you printed my Letter to Mr. Smith, on the values of the side of a dodecagon in a circle. I gave, as the approximate value to 4 decimals, $s = {\cdot}5176\,r$. A calcula-

tor who followed it out further to 6 decimals would get $s = \cdot 517638\, r$. But the difference is unimportant to the question in hand.

I notice a small misprint of $s\,r$ instead of s^2 in the equation, $s^2 = 2\,r^2 - 2\,r^2$ cos. c, but this is corrected in the subsequent lines.

My object in writing you now is to correct the idea, suggested by Mr. Smith, that I differ from the Rev. W. A. Whitworth. If Mr. W. asserted that the radius was exactly 4, while the side was exactly 2, I should, indeed, differ from him, and Mr. Smith might bid us, as "*recognised Mathematicians*," to settle our differences; but Mr. W. never makes $r = 4$ exactly, when the side is 2. Since the square root of 3 cannot be expressed in finite decimals, any exact value assigned to the radius will give an inexact value to the side, and *vice versâ*. Mr. Smith simply *assumes* $\pi = 3\cdot125$, and is quite correct in saying that no other value would give 360 linear yards of circumference to correspond with 10,638 square yards of area. But this does not prove that his fundamental value is correct. Indeed, all through his numerous publications, which he has been so kind as to send me, he never treats π as an unknown quantity whose value is to be investigated, but at once sets it down at $3\cdot125$, and then shows correctly the results that would follow such a value.

Yours obediently,

G. B. GIBBONS,

Vicar of Werrington.

LAUNCESTON, CORNWALL,
1st March, 1870.

[ALL FURTHER LETTERS ON THIS SUBJECT MUST BE PAID FOR AS ADVERTISEMENTS.—*Ed. Leader.*]

FROM THE "LIVERPOOL LEADER," MARCH 12TH, 1870.

(ADVT.)

CURIOSITIES OF MATHEMATICS.

TO THE EDITOR OF THE LIVERPOOL LEADER.

SIR,—I am delighted to find that in my native town we have a Journal devoted to science among other subjects, the Editor of which is prepared to let both sides have a *fair hearing;* and there are two

sides to every question, whether it be a political, literary, or scientific question. Such a Journal has long been a great desideratum in this country. "*Hear both sides*" is a good old proverb, but a sadly abused one in scientific journalism. The *Athenæum* is called the leading scientific Journal; but unless a writer's scientific views happen to run in the orthodox groove, there is no possibility of his getting a *fair hearing* through that channel, even if he be willing to pay for it.

When Mr. J. M. Wilson (mathematical Master of Rugby School, and senior wrangler at Cambridge in 1859) brought out the first part of his treatise on "Elementary Geometry," it was unfavourably reviewed in the *Athenæum* of July 18, 1868. This led me to write to Mr. Wilson, not with a view to controversy, but, as I thought, to suggest to him certain facts that he would find useful in bringing out his second part of "Elementary Geometry," *which is to embrace the geometry of the circle and the application of proportion to plane geometry.* "I soon found that, although Mr. Wilson thought Euclid's Elements *unsuitable as a text-book for teaching Geometry to beginners*, the views of the Editor of the *Athenæum* (Mr. W. Hepworth Dixon) did not more securely run in the orthodox groove, than, in certain respects, did the views of Mr. Wilson himself. My correspondence with Mr. Wilson was a very short one. With this exception—can it be called an exception?—I have never sought controversy on the vexed question of the ratio between the diameter and circumference of a circle; but I have never shrunk from controversy when it has been forced upon me.

My controversy with your correspondents—Mr. Whitworth and Mr. Gibbons—was of their seeking, not mine. They both attacked me. Mr. Whitworth charged me with making a false accusation against "*recognised Mathematicians*," and gave as his proof the following *assertion*:—"I *know*, and *always teach*, that the value of π is a finite and determinate quantity." Mr. Gibbons charged me with *assuming* the thing to be proved. In his last communication to me (dated so recently as February 25), he says:—"All you do, is, not to treat π as an unknown quantity whose value is to be discovered, but to *assume* a value for it, and then show correctly what would be the results of such a value." Hence, with both these

gentlemen, I was forced to act on the defensive. My plan of defence was to construct Problems, and found Theorems upon them. In this way I have examined the properties of more than 100 geometrical figures, connecting circles with triangles, polygons, squares, and unequal-sided rectangles, and find that they all harmonise with and dovetail into each other, and so, "*like an arch or dome, mutually sustain each other, and form a coherent whole.*" and further, they all combine to demonstrate that $\frac{25}{8} = 3{\cdot}125$ is the arithmetical value of the circumference of a circle of diameter unity, making 8 circumferences = 25 diameters in every circle.

I am no longer on my defence, and shall now adopt a new mode of argument, and prove, that if the Mathematicians of even twenty years ago could have got out of the orthodox groove, the symbol π would not have appeared in any modern treatise on plane Trigonometry.

The circular measure of an angle is of the *utmost importance* in the *theory* of Mathematics, and an angle at the centre of a circle, subtended by an arc equal to radius, is of the *utmost importance* in the *theory* of plane Trigonometry.

Todhunter, Fellow and principal Mathematical Lecturer of St. John's College, Cambridge, a living "*recognised Mathematician*" (according to "Men of the Time," senior wrangler at Cambridge in 1848), and the most recent author of a treatise on plane Trigonometry that I know of, in his second edition, page 8, says:—"*The symbol π is invariably used to denote the ratio of the circumference of a circle to its diameter; hence, if r denote the radius of a circle, its circumference is $2\ \pi\ r$;*" *but immediately adds,* "*where π =* $3{\cdot}14159$..." It is hardly conceivable that a gentleman of Todhunter's known ability could have fallen into so gross a blunder as this. Is it not self-evident, that $2\ \pi\ r$ = circumference of a circle when r denotes the radius, whatever be the value of π?

Let the radius of a circle be represented by unity = 1, our unit of length being 1. It is admitted by all Geometers and Mathematicians, that 6 times radius = the perimeter of a regular inscribed hexagon, or six-sided polygon, in every circle. Now, it is self-evident, that the circumference of a circle is greater than the perimeter of its inscribed regular hexagon; and, because the sides of the hexagon

are equal to the radius of the circle, it follows, that an angle at the centre of a circle may be subtended by an arc equal to radius. But it is axiomatic, if not self-evident, that there may be six angles at the centre of the same circle, subtended by arcs equal to radius; and it is also axiomatic, if not self-evident, that the six arcs subtending these angles do not exhaust the circumference of the circle. If radius $= 1$, the sum of the six arcs $= 6$: $\frac{1}{24}(6) = \frac{1 \times 6}{24} = \frac{6}{24} =$ $\cdot 25$; therefore, $(6 + \cdot 25) = 6 \cdot 25$. This is simply a fact. But the following facts must not be either overlooked or lost sight of in the consideration of what this simple fact leads to. $2 \pi r =$ circumference of a circle of radius unity; semi-circumference of a circle of radius unity $=$ circumference of a circle of diameter unity; and the circumference of a circle of diameter unity is the thing we are in search of. Is it not self-evident, that the circular measure of an angle at the centre of a circle subtended by an arc equal to radius must be arithmetically the same, whether we *assume* the circumference of a circle to be 2π or $360°$?

It would carry me beyond the reasonable limits of a Letter to prove that from these facts we are enabled to find the exact arithmetical value of the circumference of a circle of diameter unity, and I shall reserve my proofs for another communication.

I am, Sir,

Yours obediently,

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
5th March, 1870.

P.S.—This Letter was in manuscript before your Journal of to-day came into my hands. Mr. Gibbons, in his Letter to you, which appears in this day's number of the *Leader*, says: "My object in writing you now is to correct the idea, suggested by Mr. Smith, that I differ from the Rev. W. A. Whitworth." Mr. Gibbons, however, *coufirms* the idea that he and Mr. Whitworth differ from each other. Can the chord subtending the same angle at the centre of a circle be *arithmetically indeterminate* and also *arithmetically finite*? Again, I say, let these reverend gentlemen, (both "*recognised Mathematicians*") settle their differences. J. S.

FROM THE "LIVERPOOL LEADER," MARCH 19TH, 1870.

(ADVT.)

CURIOSITIES OF MATHEMATICS.

TO THE EDITOR OF THE LIVERPOOL LEADER.

SIR,

Todhunter *admits* that $2\pi r$ denotes the circumference of a circle when r denotes the radius; and it follows, that when r denotes the radius of a circle, and = unity, the semi-circumference = the circumference of a circle of diameter unity, whatever be the value of π. The circumference of a circle of diameter unity is supposed to be unknown, and Mathematicians have adopted the symbol π to denote it. Now, the arithmetical value of the symbol π is the thing we are in search of, and it is self-evident, that the circular measure of an angle at the centre of a circle subtended by an arc equal to radius, must be arithmetically the same, and be denoted by the arithmetical symbol 1, whether we *assume* 2π or $360°$ to denote the circumference of the circle.

The circumference of a circle is divided into 360 degrees of 60 minutes each for astronomical and nautical purposes; indeed, I may say, for trigonometrical and all other practical purposes.

The mathematical formula for finding the circular measure of an angle is $\frac{\text{Angle} \times \pi}{180°}$ where π denotes the arithmetical value of the circumference of a circle of diameter unity. At this point, "*that indispensable instrument of science, Arithmetic*," steps in, and asserts its authority. Is it not self-evident, that to find the circular measure of an angle, we must either first find the arithmetical value of the symbol π, or *assume* an arithmetical value of it? Now, we can *assume* an arithmetical value of the symbol π, for if not, what means Mr. Gibbons' oft-reiterated charge that I *assume* the thing to be proved? In his Letter in your number of the 5th instant, he says:—"*He* (Mr. Smith) *never treats* π *as an unknown quantity whose value is to be investigated, but at once sets it down at* 3·125, *and then shows correctly the results that would follow such a value.*" These facts furnish me with "two strings to my bow." The symbol

π *may* denote 3·1416, or it *may* denote 3·14159—the former rather greater, and the latter rather less, than the arithmetical value assigned to the symbol π by orthodox Mathematicians; but, whether we adopt one or other of these values of the symbol π, we discover that $\frac{\text{a right angle} \times \pi}{180^\circ \times 60'} = \frac{5400' \times \pi}{10800'} = \frac{\pi}{2}$ and is equal to a quadrant of the circumference of a circle of radius unity, or the semi-circumference of a circle of diameter unity.

For example: By hypothesis, let $\pi = 3{\cdot}1416$. Then: $\frac{\text{A right angle} \times \pi}{180^\circ} = \frac{90^\circ \times \pi}{180^\circ \times 60'} = \frac{5400' \times 3{\cdot}1416}{10800'} = \frac{16964{\cdot}64}{10800} = \frac{3{\cdot}1416}{2} = 1{\cdot}5708 = \frac{\pi}{2}$. Again: By hypothesis, let $\pi = 3{\cdot}14159$. Then: $\frac{\text{A right angle} \times \pi}{180^\circ} = \frac{90^\circ \times \pi}{180^\circ \times 60'} = \frac{5400' \times 3{\cdot}14159}{10800'} = \frac{16964{\cdot}586}{10800} = \frac{3{\cdot}14159}{2} = 1{\cdot}570795 = \frac{\pi}{2}$. On both showings, $\frac{\pi}{2}$ is equal to a quadrant of the circumference of a circle of radius unity, or the semi-circumference of a circle of diameter unity. This result is constant, whatever hypothetical arithmetical value of π, intermediate between 3 and 4, we may adopt, so that it be *finite* and *determinate*. Mark what follows: $\frac{\pi}{2} \div \frac{8\pi}{100}$ is constant $=$ 6·25; and because the circumferences of circles are to each other as their radii, it follows, that $\frac{6{\cdot}25}{2} = 3{\cdot}125$, is the arithmetical value of the circumference of a circle of diameter unity; or, in other words, the arithmetical value of the symbol π.

But further: The perimeter of a regular inscribed hexagon to a circle of radius unity $= 6$. Hence: Having found the true arithmetical value of the circumference of a circle of diameter unity,—that is, of the symbol π—we discover that $\frac{6}{2\pi} \div \frac{8\pi}{100} = \frac{6}{6{\cdot}25} \div \frac{25}{100} = \frac{{\cdot}96}{{\cdot}25} = 3{\cdot}84 =$ the perimeter of a regular inscribed hexagon to a circle of circumference $= 4$; for, by analogy or proportion, $3{\cdot}84 : 4 :: 3 : 31{\cdot}25$. Is not the perimeter of a regular inscribed hexagon to a circle of diameter unity $= 3$? Is not $\frac{25}{24}$ (3) 3·125? From these

facts, it follows, that $\frac{24}{25}$ (circumference) = the perimeter of a regular inscribed hexagon in every circle.

Now, it is not only axiomatic, but stands to common sense, that the circular measure of an angle at the centre of a circle subtended by an arc equal to radius must be arithmetically the same, whether we adopt *unity* as the radius of the circle, or 360° as the circumference. But, to find the circular measure of any angle, the symbol π—which denotes the circumference of a circle of diameter unity—is involved, and it is self-evident, that we can never find the true circular measure of an angle with a *false* value of π; or, in other words, with a *false* value of the circumference of a circle of diameter unity.

THEOREM.

Let an angle at the centre of a circle be subtended by an arc of 57° 36′. What is the circular measure of the angle?

Answer: $\frac{57^\circ\ 36' \times \pi}{180^\circ} = \frac{3456' \times \pi}{180^\circ \times 60'} = \frac{3456' \times 3{\cdot}125}{10800} = \frac{10800}{10800} = 1$, and the arithmetical symbol 1 is the circular measure of an angle of 57° 36′.

Proof: 6 (57° 36′) = 345° 36′ = the perimeter of a regular inscribed hexagon to a circle of circumference = 360°; and, by analogy or proportion, 345° 36′ : 360° : : 3 : 3·125; and because every other arithmetical value of the circumference of a circle of diameter unity but 3·125, would make the circular measure of an angle of 57° 36′ either greater or less than 1, it follows, not as an *assumption*, but as a logical *deduction*, that $\frac{25}{8}$ = 3·125 is the true arithmetical value of the circumference of a circle of diameter unity, making 8 circumferences = 25 diameters in every circle.

All the foregoing facts are inseparably connected. They all harmonise with each other, and dovetail into each other; and so, according to Baconian philosophy, *are like an arch or dome, mutually sustaining each other, and forming a coherent whole.*

Could Todhunter have got out of the orthodox groove, and divested himself of the prejudices engendered by false teaching, we should not have had him making the following fallacious and absurd *assertions; absurd,* because opposed to common sense. First:

"*The numerical value of the ratio of the circumference of a circle to its diameter cannot be stated exactly;*" Second: "*The value is approximately equal to* $\frac{22}{7}$, *and still more nearly equal to* $\frac{355}{113}$, *the value correct to eight places of decimals is* 3·141 59265..."

Your correspondents, T. S. B. and Mr. Gibbons agree, that if the symbol π = 3·125, the area of a circle, of which the length of the circumference is 360 *linear* yards, is 10,368 square yards (in Mr. Gibbons' Letter in your number of the 5th instant there is a slight misprint—10,638 square yards should be 10,368 square yards); but T. S. B. goes a step further, and admits that "*a greater value of* π *gives a less area, and a less value of* π *a greater area.*" Mr. Gibbons knew better than make this admission, for *he* could see that it would be fatal to all his data, reasonings, and conclusions. Mr. Gibbons wishes it to be inferred that he does not differ from the Rev. W. A. Whitworth; but he has stated, and I have admitted, that a central angle of 30° is subtended by a side of a regular dodecagon; and he proves, and I admit his proof, that the chord subtending a central angle of 30° is arithmetically *indeterminate*, whatever be the circumference of the circle. Now, Mr. Whitworth adopts, as a datum or premiss, a side of a regular dodecagon, denoting its value by the *finite* and *determinate* arithmetical symbol 2, to prove that the symbol π, which denotes the circumference of a circle of diameter unity, is greater than 3·126. I put the following question to every reflective and *honest* Mathematician:—Can a geometrical something, known to be arithmetically *indeterminate*, be rationally and legitimately adopted as a datum or premiss in any geometrical inquiry involving Mathematics?

Now, it cannot be controverted—and will be admitted by every honest Mathematician—that a line in the form of the circumference of a circle, will enclose a larger surface or area than in any other form whatever; and it is self-evident, that the longer the circumference of a circle the greater will be the surface or area enclosed by it. Bearing this in mind, mark the absurdity of T. S. B.'s conclusion! He admits that a circle, of which the length of the circumference is 360 *linear* yards, encloses a larger surface or area "*on the supposition*" that π = 3·125, than "*on the supposition*" that π is greater than 3·125; and it follows, that a circle of which the circumference

is 360 *linear* yards in length, encloses a larger surface or area "*on the supposition*" that $\pi = 3{\cdot}125$, than "*on the supposition*" that $\pi = 3{\cdot}126$; it also follows, that a circle of 360 *linear* yards circumference encloses a smaller surface or area "*on the supposition*" that $\pi = 3{\cdot}125$ than "*on the supposition*" that $\pi = 3{\cdot}124$; and it is self-evident, that π is involved in finding the area of a circle, when the circumference of the circle is the given quantity. What, then, in the name of common sense, can the arithmetical value of the symbol π be, but the arithmetical mean between 3·126 and 3·124 $= \frac{3{\cdot}126 + 3{\cdot}124}{2} = \frac{6{\cdot}25}{2} = 3{\cdot}125$?

I might rest the question in dispute between Mathematicians and me, on the following very simple issue:

If Mathematicians can find a greater arithmetical value of the symbol π than 3·125, by which they can make a circle of which the length of the circumference is 360 *linear* yards, enclose a larger surface or area than 10,368 *square* yards, they will be able to prove that I am a Pseudomath. T. S. B. adopts De Morgan's *assertion* that I am a Pseudomath. Let him prove it!!

I am, Sir,

Yours obediently,

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
12*th March*, 1870.

FROM THE "LIVERPOOL LEADER," MARCH 26TH, 1870.

(ADVT.)

CURIOSITIES OF MATHEMATICS.

TO THE EDITOR OF THE LIVERPOOL LEADER.

SIR,

It is not only admitted by Todhunter, but by all Geometers and Mathematicians, that $2\pi r$ = the circumference of a circle, when r denotes the radius. Todhunter, in his Treatise on Plane Trigonometry (second edition, page 8), says—"*The symbol π is invariably used to denote the ratio of the circumference of a circle*

to its diameter." This being admitted, is it not self-evident that if the diameter of a circle be represented by unity = 1, the symbol π denotes the circumference of the circle? Now, π is supposed to be unknown, and its arithmetical value is the thing we are in search of. With reference to this, Todhunter makes the following assertion—"*The numerical value of the ratio of the circumference of a circle to its diameter cannot be stated exactly*." "*Recognised Mathematicians*" agree with him; even the Rev. W. Allen Whitworth agrees with Todhunter, although he has made the following inconsistent assertion—"*I know*, and *always teach*, that the value of π is a *finite* and *determinate* quantity." I flatly deny the truth of Todhunter's assertion. Hence, the first question at issue between me and "*recognised Mathematicians*" is—"Can the value of the symbol π be stated exactly, or can it *not* be stated exactly, in arithmetical notation?

Let p denote the perimeter of a regular hexagon or six-sided polygon inscribed in a circle. Then: Because 6 times radius = the perimeter of a regular inscribed hexagon in every circle, it follows, that $6(r) = 6(\frac{1}{2}) = 6 \times {\cdot}5 = 3$ = the perimeter of a regular inscribed hexagon to a circle of diameter unity. Hence, the value of p is a known arithmetical quantity, and is denoted by the arithmetical symbol 3, when p denotes the perimeter of a regular inscribed hexagon to a circle of diameter unity.

Now, "*on the supposition*" that $\pi = 3{\cdot}125$, $(\pi - \frac{1}{25}\pi) = \left(3{\cdot}125 - \frac{1 \times 3{\cdot}125}{25}\right) = 3{\cdot}125 - {\cdot}125 = 3$, = the *known* arithmetical value of p. "*On the supposition*" that $\pi = 3{\cdot}126$ $(\pi - \frac{1}{25}\pi) = (3{\cdot}126 - {\cdot}12504) = 3{\cdot}00096$, and is greater than the known arithmetical value of p. Again: "*On the supposition*" that $\pi = 3{\cdot}124$, $(\pi - \frac{1}{25}\pi) = 3{\cdot}124 - {\cdot}12496 = 2{\cdot}99904$, and is less than the *known* arithmetical value of p.

Again I put the question to every reflective and honest Mathematician:—What, then, in the name of common sense, can the arithmetical value of the symbol π be, but the arithmetical mean between $3{\cdot}126$ and $3{\cdot}124 = \frac{3{\cdot}126 + 3{\cdot}124}{2} = \frac{6{\cdot}25}{2} = 3{\cdot}125$?

When "*recognised Mathematicians*" can find the arithmetical value of the circumference of a circle of diameter unity, from which they can deduct an aliquot part and make the remainder the *known*

arithmetical value of the perimeter of an inscribed regular hexagon to a circle of diameter unity, they will be able to prove that "James Smith, Esq., of Liverpool, *is nailed by himself to the barn-door as the delegate of miscalculated and disorganised failure.*" (See *Athenæum*, July 25, 1868: Article, "Our Library Table.") Will Professor de Morgan declare that he was *not* the writer of that article?

I am, Sir,

Yours obediently,

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
17th March, 1870.

P.S.—Since this Letter was in manuscript I have received a communication from your correspondent, Mr. Gibbons, in which he says: "Cos. 30° = $\frac{\sqrt{3}}{2}$, and the diagonal of a square (side = 1) = $\sqrt{2}$, must be the *results* of reasoning and calculation; but the mere fact of a something being known to be arithmetically *indeterminate* does not hinder its legitimate employment in the way you name, otherwise how could you ever use $\sqrt{2}$ or $\sqrt{3}$ which are *indeterminate?*" This refers to the adoption of a geometrical something known to be arithmetically *indeterminate*, as a datum or premiss in a geometrical inquiry. It is no doubt true that, by "*reasoning and calculation*," we find the arithmetical expression $\frac{\sqrt{3}}{2}$ to be the trigonometrical cosine of an angle of 30°; but, because the cosine of an angle is the sine of the complement of that angle, it follows, that $\frac{\sqrt{3}}{2}$ is also the trigonometrical sine of an angle of 60°. These truths harmonise with the following fact, known to, and admitted by, all Geometers and Mathematicians, namely—"*The sine of any arc is half the chord of twice that arc.*" In the course of my very long correspondence with Mr. Gibbons, I proved, in a variety of ways, that in a right-angled triangle of which the sides that include the right angle are in the ratio of 7 to 1, the lesser of the acute angles is an angle of 8° 8′ exactly, and "*by reasoning and calculation*" I also proved, that the trigonometrical sine of this angle is $\frac{\sqrt{2}}{10} = \sqrt{\cdot 02} =$

·1414213 to seven decimals, without the aid of tables. Mr. Gibbons has never attempted to *grapple* with my proofs of this fact. He dealt with it, by assuming the infallibility of Logarithmetic tables of sines, cosines, &c., giving as a reason that Logarithmetic tables have been calculated by experts in every country of Europe, and all agree, barring clerical errors; and then, by the following process of "*reasoning and calculation,*" makes the angle an angle of $8^\circ\ 7' + x$.

$$\begin{array}{lrl} \text{By Tables:} & \text{Sin. of } 8^\circ\ 8' = & \cdot 1414772 \\ & \text{Sin. of } 8^\circ\ 7' = & \cdot 1411892 \\ \hline & & \cdot 0002880 = \text{difference}; \end{array}$$

therefore, ·1414213 is not the sine of an angle of $8^\circ\ 8'$, but the sine of an angle of $8^\circ\ 7' + x$. Mr. Gibbons has never attempted to *prove* the value of $+ x$ in seconds of a degree. Let him try, and where will he be? The fact is, according to the "*reasoning and calculation*" of Mr. Gibbons, an angle of a given number of degrees and minutes would be a trigonometrical impossibility, and so, Mr. Gibbons would make the science of Trigonometry itself "a mockery, a delusion, and a snare."

J. S.

FROM THE "LIVERPOOL LEADER," APRIL 2ND, 1870.

(ADVT.)

CURIOSITIES OF MATHEMATICS.

TO THE EDITOR OF THE LIVERPOOL LEADER.

SIR,

None of your correspondents have attempted to controvert or disprove the demonstrations I have given in my letters which appeared in your valuable journal of the 12th and 19th inst.,—that $\frac{25}{8} = 3{\cdot}125$, is the true arithmetical value of the circumference of a circle of diameter unity. What has become of the Rev. W. Allen Whitworth?

The following facts are known to, and admitted by, all Geometers and Mathematicians, and can be demonstrated from a given value of the radius of a circle, by means of any hypothetical arithmetical value of the symbol π intermediate between 3 and 4, so that it be *finite* and *determinate* :—

First fact: $\pi\ (r^2) = (c \times s\,r)$, where c denotes the circumference of a circle, r denotes the radius, $s\,r$ denotes the semi-radius, and π denotes the circumference of a circle of diameter unity.

Second fact: $x = 6\ (r \times s\,r)$, where x denotes the area of a regular dodecagon, r denotes the radius, and $s\,r$ denotes the semi-radius of a circumscribing circle to the dodecagon.

Third fact: $\frac{\pi}{3}$ expresses the ratio between the circumference of every circle and the perimeter of its inscribed regular hexagon, whatever be the value of π.

Fourth fact: $\frac{\pi}{3}$ expresses the ratio between the area of every circle and the area of its regular inscribed dodecagon, whatever be the value of π.

Now, if the data of "*Recognised Mathematicians*" be sound, their reasonings logical, and their conclusions irrefragable, it follows, and is self-evident, that there cannot be an arithmetical value of the symbol π, by which they can get the equations $\pi\ (r^2) = (c \times s\,r)$ and $x = 6\ (r \times s\,r)$. Will your correspondents—Mr. Whitworth, Mr. Gibbons, Mr. Skeat, and "T. S. B."—deny this fact? Will the "*Recognised Mathematician*," Todhunter, attempt to controvert or disprove it?

It is conceivable, that some of your mathematical correspondents may have the temerity to make the following assertions :—It is not a fact, that $\pi\ (r^2) = (c \times s\,r)$. It is not a fact, that $x = 6\ (r \times s\,r)$. Now, *suppose* Mr. Whitworth, Mr. Gibbons, Mr. Skeat, or "T. S. B." to make these *assertions*. Would they not *assert* what is untrue, and what they would *know* to be untrue? Have I not proved that we *can* get both these equations from a given arithmetical value of the circumference of a circle, "*on the supposition*" that $\pi = \frac{25}{8} = 3{\cdot}125$? Can these gentlemen say they have not seen the letters on "Curiosities of Mathematics" which have recently appeared in the

Liverpool Leader? Having seen those letters, can they plead ignorance of my proofs? If they think my demonstrations defective, surely, their knowledge of Mathematics should enable them to prove it. Let them try!

Your correspondent Mr. Gibbons has for once made an admission. In his Letter in the *Leader* of the 5th inst., he admits that, "*on the supposition*" that $\pi = 3{\cdot}125$, "Mr. Smith *is quite correct in saying that no other value would give* 360 *linear yards of circumference to correspond with* 10,368 *square yards of area.*" Your correspondent "T. S. B.," in his letter in the *Leader* of the 26th February, makes the same admission, but goes a step further, and admits, that "*a greater value of* π *gives a less area, and a less value of* π *gives a greater area.*" Mr. Gibbons knew better than make this admission. Had he not a motive for declining to admit it? Do men speak or write, or decline to speak or write, without some motive? Have not all rational men some motive for every action of life?

Now, on the supposition that $\pi = 3{\cdot}125$, the fact is incontrovertible, that 360 *linear* yards of circumference correspond with 10,368 *square* yards of area, as Mr. Gibbons puts it. But, on the *same* supposition, $\pi\,(r^2) = (c \times s\,r)$, and $x = 6\,(r \times s\,r)$, and if c denotes the circumference of a circle, p denotes the perimeter of its inscribed regular hexagon, a denotes the area of the circle, and x denotes the area of a regular dodecagon inscribed in the circle: then $p : c :: x : a$, whatever be the value of the symbol π. Are not all these facts inseparably connected? Can Mr. Gibbons or "T. S. B." *upset* or disprove their inseparable connection? Impossible! Well, then it follows, that when "*recognised Mathematicians*" can find an arithmetical value of the symbol π either greater or less than $\frac{25}{8} = 3{\cdot}125$ by which they can get the equations $\pi\,(r^2) = (c \times s\,r)$ and $x = 6\,(r \times s\,r)$, from a given arithmetical value of the circumference of a circle, they will be able to prove that I am a Pseudomath. Will Mr. Whitworth, Mr. Gibbons, Mr. Skeat, "T. S. B.," or Todhunter, attempt to furnish the proof?

No matter how many demonstrations I might give to "*recognised Mathematicians*" of the true arithmetical value of the circumference of a circle of diameter unity; or, in other words, of the true ratio

between the circumference and diameter in a circle, all rationally and legitimately derived from geometrical constructions, and all harmonising with each other, and logically dovetailing into each other, and so, like an arch or dome, mutually sustaining each other, and forming a coherent whole; judging from my past experience of that confraternity, they would either pass over my demonstrations in silent comtempt, attempt to burke them by abuse and ridicule, or meet them with assertions directly opposed to reason and common sense. How can "*recognised Mathematicians*" reconcile their theories with Baconian philosophy? Impossible! Do not all men of intellect and education in the present enlightened age—"*recognised Mathematicians*" excepted—admit the truth of the philosophy of Bacon?

Let c denote the circumference of a circle, and let p denote the perimeter of an inscribed regular hexagon or six-sided polygon.

THEOREM I.

From a given arithmetical value of c find the arithmetical value of p.

THEOREM II.

From a given arithmetical value of p find the arithmetical value of c.

It is self-evident that Theorem 2 is the converse of Theorem 1; and it is axiomatic that the symbol π, which is invariably adopted to denote the circumference of a circle of diameter unity, is involved in the solution of these theorems.

SOLUTION OF THEOREM I.

Let $c = 360$. Then: $\frac{c}{2\pi} = \frac{360}{6{\cdot}25} = 57{\cdot}6$ = radius: and 6 (radius) $= (6 \times 57{\cdot}6) = 345{\cdot}6 = p$.

SOLUTION OF THEOREM II.

Let $p = 345{\cdot}6$. Then: $\frac{p}{6} = \frac{345{\cdot}6}{6} = 57{\cdot}6$ = radius, and 2π (radius) $= 6{\cdot}25 \times 57{\cdot}6 = 360 = c$.

Now, according to common sense and the rules of common arithmetic (and mathematics can never be made to over-ride common arithmetic, which is the foundation of all mathematics), the rule that will not work both ways—that is, backwards and forwards—must be a false rule. J. Radförd Young,—a living "*recognised Mathematician*,"—in his Treatise on Arithmetic, says, with reference to the rule of proportion :—"The only condition which four quantities must fulfil, in order that they may form a proportion, is this, namely—

$$\frac{\text{First term}}{\text{Second term}} = \frac{\text{Third term}}{\text{Fourth term.}}\text{"}$$

Apply this to the proportion given in a previous part of this Letter, namely $p : c :: x : a$, where c denotes the circumference of a circle, a denotes the area of the circle, p denotes the perimeter of a regular hexagon inscribed in the circle, and x denotes the area of a regular dodecagon inscribed in the circle. Then: $\frac{\text{First term}}{\text{Second term}} = \frac{\text{Third term}}{\text{Fourth term}}$. Hence: $\frac{p}{c}$ and $\frac{x}{a}$ are equivalent ratios, and both express the ratio between the perimeter of every regular hexagon and the circumference of its circumscribing circle.

These facts narrow the differences between me and opponents as to the arithmetical value of the symbol π, to a very simple issue.

If "*recognised Mathematicians*" can find some other arithmetical value of the symbol π than $\frac{25}{8} = 3 \cdot 125$—within admissible limits—that is, intermediate between 3 and $3\frac{1}{7}$, by which they can solve the two foregoing theorems—they will be able to prove that "Mr. Smith *has convicted himself of ignorance and folly*, *with an honesty and candour worthy of a better value of* π." (See *Athenæum*, June 24th, 1865 ; Article—"Budget of Paradoxes," by A. de Morgan.)

I quote the following from a Letter I recently received from a non-professional, but, nevertheless, a high-class Mathematician :—"As to the ratio $\frac{25}{8}$ I have no doubt at all of its truth, and I think *a priori* we might be most certain that if the radius or diameter be definite, determinate, so certainly must the circumference and the area of the circle itself be. For, to have a *defined* and true *thing* to be the cause of another with which it is inseparably connected, and to suppose that this other is *undefined* and *indeterminate*, we should say is contrary to every notion of our being."

In conclusion :—This very day I have received a Letter from a gentleman (not recognised as a Mathematician by the British Association, but whose mathematical attainments are known to be unsurpassed by any Mathematician in Europe), from which I quote the following paragraph :—" I see from the *Liverpool Leader*, which, I presume, is forwarded by your orders, that you are still at war to the knife and scissors with the modern school of Mathematicians. I wish you every success in your arduous task, though it is rather an unequal fight, as you have all in your favour, whilst they, blinded by their own conceit and obstinacy, have not a leg to stand on."

I am, Sir,
Yours obediently,
JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
22nd March, 1870.

P.S.—Since this Letter was in manuscript I have received another communication from Mr. Gibbons, from which I quote the following :—" You may safely challenge all the Mathematicians of Europe in the terms you adopt, for they will, one and all, confess that *for a given circumference a larger* value for π would give a *smaller* value for the area. If circumference = 360 and π were 3·125, area would be 10368 ; but increasing π while keeping the same circumference would diminish the area of the circle." In making these admissions, Mr. Gibbons practically proves (apparently without knowing it) the truth of the *theory* that 8 circumferences = 25 diameters in every circle, making $\frac{25}{8}$ = 3·125, the true arithmetical value of the circumference of a circle of diameter unity.

APPENDIX D.

FROM "THE LIVERPOOL COURIER" OF JANUARY 20, 1870.

EUCLID AS A SCHOOL BOOK.

AN extraordinary meeting of the Literary and Philosophical Society was held at the Royal Institution on Tuesday evening last, Dr. Nevins, the President, occupying the chair. A paper was read by the Rev. Dr. Jones, principal of King William's College, Isle of Man, on "The Unsuitableness of Euclid as a Text-book of Geometry," of which the following is an abstract :—

The question was one of the greatest interest, and, although not a new one, was beginning to excite attention among ourselves. Long latent dissatisfaction was gathering strength, and the struggle was fairly commencing. The matter was first brought into prominence by the report of the Schools Inquiry Commission ; while the existence of a wide-spread conviction as to the unsuitableness of Euclid as a Text-book of Geometry rendered inquiry desirable, the defenders of Euclid should be called upon to justify their position. It had long been abandoned on the Continent. Were we alone right, and all the rest of the educated world wrong ? Evidence in support of such a view should be forthcoming. It was written 2,000 years ago : during that time the science had been progressing ; new principles, methods, and applications had been introduced ; the experience of teachers for twenty centuries had been accumulating. Could a book written in the infancy of the science, when no general view of it was possible, be a suitable Text-book now ? In the time of Euclid, it may fairly be said, a good elementary Text-book could

not have been written. Who in any other branch of knowledge would use for beginners a treatise written even 100 years ago? Again, Euclid's work was never intended as an elementary Text-book, and was never in his time so used. It was written for a different purpose. But still the opponents of Euclid must act on the offensive. It had possession of the field and a long historical tradition in its favour; it was defended by that narrow conservatism which thinks that whatever we have been accustomed to must be best,—in short, by the same weapons which have resisted all progress and reform. The question was decided by many at once without examination; the force of prejudice came in, and Euclid was retained. In such a case active measures must be taken. Rash innovation was to be condemned; but in the stolid opposition which defends a practice on the score of long usage it was hard to acquiesce. The attack was against Euclid as an elementary Text-book, not as a scientific treatise. It might have its merits, but they were not as numerous as was generally supposed, and they were often such as to disqualify it as an introduction to the study of geometry. The faults of Euclid might be classified under the heads of phraseology, method, matter, and particular doctrines. He was verbose: in his definitions he explained simple ideas by circumlocutions, and in the propositions impeded the argument by a needless mass of verbiage. His language was stiff and formal, often obscuring the thought, and rendering the student's grasp of the reasoning difficult in his efforts to bring the argument under a certain form. The nomenclature was antiquated and infelicitous. When he had once used a word in a certain sense he had no idea of modifying or generalising it. His method was impaired by the unnecessary limitation of his first principles. He selected only a few out of many which the common sense of mankind would recognise as true. This was probably done with design, but it prevented the natural development of the subject; and his treatment of it was thereby narrowed and constrained. Another fertile source of evil was the "rejection of hypothetical construction," *i. e.*, refusing to examine the properties of a figure before the construction of it was actually effected. This involved the natural order of ideas, prevented proper classification, and rendered the solution of problems often inelegant and of no practical value. Another defect was the neglect of the principle of

superposition, which would have simplified the proof of many propositions, and was the ultimate test of geometrical equality. His system was also narrowed by the exclusion of arithmetical considerations, without which no practical geometrical inquiry was possible ; and so another geometry had to be learned by practical men. Sciences should not be kept rigidly apart, but be allowed to shed light one upon another. Again, not the slightest reason was given why any particular course was adopted ; and so what he was learning seemed to the student meaningless and tedious. There was no classificatiou worthy of the name in the book ; no salient points ; no indications of leading propositions : all was in confused medley. As regarded matter, Euclid erred in various ways. Of his definitions, some were mere verbal statements ; others did not explain their terms ; others were theorems ; some were superfluous ; some were wanting. The third postulate was needlessly restricted. Some of the axioms were not self-evident ; others were wanting. There was a large number of superfluous propositions. Book III. in particular might be much reduced ; one proposition was mentioned which included six or seven others as corollaries in Mr. Wilson's book. Again, Mr. Wilson nearly covered the ground of Book IV. by two problems and two preceding theorems, and a far greater power of construction was thereby acquired. On the other hand, there were many important omissions ; there were proofs which could be simplified ; others which could be extended ; and others which were imperfect. All the above points were amply illustrated. In his treatment of particular doctrines Euclid was defective. Thus his definition of an angle was inadequate. He never entertained the idea of an angle equal to, or greater than, two right angles ; and so many simplifications were neglected ; and in one case at least the restriction was ignored. His doctrine of parallels was objectionable, because based on a negative definition and an axiom which was not self-evident. The large number of substitutes proposed for these was evidence of the general dissatisfaction with Euclid's treatment. His doctrine of proportion was immaterial, and a violation of common sense. No one ever went through the process of the 5th definition of Book V. He should have used freely arithmetical considerations, which, after all, were involved in his definitions. Book V. was practically abandoned, and

a substitute was necessary. It was suggested that we should use an amended Euclid, with notes and commentaries. But surely it was an unwarrantable waste of time to teach a boy a subject in a faulty form, and then put him to the trouble of learning a commentary to correct the faults. He might remember these and forget the comments. Such a course would not be tolerated in any other subjects; and the mere suggestion of it was a sufficient condemnation of Euclid. Viewing the subject on its practical side, young beginners were discouraged by Euclid, because their grasp of ideas was feeble and their progress slow. This arose from the difficulty, the tedious length of the propositions, and other defects. Difficulty was no argument against a subject if unavoidable, but every subject should be presented to the beginner in the simplest form—difficulties would be found soon enough. To illustrate this point, the proof of Book I, Proposition 5, by the method of superposition in Mr. Wilson's book, was mentioned in contrast with that of Euclid. As a matter of fact, boys did not learn geometry from Euclid; they understood the propositions, and could reduce them accurately, but acquired but a limited stock of geometrical ideas, and had little or no skill in the application of them to original questions. This point was illustrated by a reference to the author's own experience and to the testimony of others. And yet the great test of mathematical knowledge was the ability to apply it to original questions. We should not be content without this in algebra, trigonometry, chemistry, &c. It might be said the fault was in the teacher. But all were not bad teachers; good teachers of analysis should be good teachers of geometry; the book was enough to paralyse the efforts of any teacher. There were no salient points in it; no natural order; it was not suggestive; the mere mastering and "saying" of the propositions occupied all the time that could be allotted to the subject. It was all very well to say that "time was an essential element in education;" but that was no reason for wasting time. Indeed, the number of subjects required to be studied rendered it absolutely necessary to economise time. Just complaints were being made that our pupils were being overburdened. The pressure of public opinion forcing upon us one subject after another, the crushing requirements of the various examinations, obliged us to act against our better judgment. Unless some of the old subjects could be taught on better methods,

our present system of education would break down. In geometry alone, of all branches of education, we were inferior to other nations. Mr. M. Arnold's testimony, from what he had heard abroad, was quoted on this point. The foreign *consensus* against the use of Euclid was striking. This inferiority was due to the use of Euclid. But, it was urged, Euclid supplied an admirable mental discipline. That could be secured as well from modern text-books, as the case of the French, Germans, &c., who were not inferior to us in the power of reasoning and intelligence, clearly proved. Indeed, the greater knowledge of the subject which would be acquired, and the larger amount of original work which would be possible, were modern text-books used, would involve a wider discipline. Acquisition of knowledge and mental training were separable in idea but not in fact. The discipline was in direct proportion to the knowledge gained. At all events, discipline, plus a larger amount of knowledge, was better than discipline with the minimum of attainments. Again, if Euclid were abandoned, more original work would be possible; and this supplied a better mental training than comprehending the reasoning of others, and called out the original faculties. It was said Euclid was a good training ground, because it was well "to be exercised in the field of danger." But some in that field fell to rise no more. Undue difficulties should not be thrown in the way of immature minds; this often crushed the powers which were developed by adequate tasks. Though it might be regarded as heresy to say so, Euclid's type of reasoning was not perfect. No one reasoned so in other sciences, or in common life. It was like dancing in armour or on stilts—very wonderful as a feat, but not useful. Better learn to reason naturally and simply, and not in an artificial or constrained form. Euclid reasoned from the smallest number of first principles; in reasoning on other matters we freely drew our premises and common notions from all quarters. Euclid appealed to the memory rather than to the intelligence. The "saying" of propositions exercised the memory more than the reason, which was often stunted and enfeebled thereby. At all events, the repetition of propositions without original application crushed originality, and made the mind reason in a groove. This was the great defect of cultivated minds; they mastered and reproduced, but could not originate. And so many of the

greatest works were composed by men who had not been subject to the course of culture marked out by our schools and universities. The evil was on the increase, owing to the baneful influence of competitive examinations. There must be no mistake; geometrical studies were the best possible discipline of the reasoning faculties; our quarrel was with Euclid, from the study of which the results in the way of intellectual training were not so satisfactory as they might be. The arguments were cumulative, and enough to sink any book not sustained by prejudice. Against modern text-books it was asserted that none of the existing ones was quite satisfactory. This might be so, as the authors were trained on Euclid; but to ignore these was not the way to get a good book at last. Then it was said they were more faulty than Euclid. Let any one compare the list of faults brought against Mr. Wilson's book by the *Athenæum* reviewer with his own statement as to the faultiness of Euclid, and with the long list of defects collected by Professsor De Morgan in the Companion to the British Almanack for 1849, and he will see the balance to be in favour of the modern treatise. It was said that the modern geometry was a system of "unlimited assumption." But Euclid frequently assumed postulates which he has never stated, and the offence is worse in his case, as he does pretend to lay down definitely all his postulates. After all, the practice is not so objectionable, provided the postulates and axioms be such as commend themselves to the common sense and experience of mankind, and be distinctly stated where required. The requirements of a good text-book were, that all necessary first principles should be freely assumed. Subject to the above conditions, the proof should be in each case as simple and direct as possible; the consequences should flow easily and naturally from the premises; the reasoning need not in every case be drawn out with verbal fulness. The variety of text-books would create no difficulty. It did not do so in other subjects, or on the Continent. The objection had reference to examinations; but, what was forgotten so often in the present day, examinations existed for the sake of students, and not *vice versâ*. Indeed, the variety of text-books would tend to improve teaching, by correcting one another's defects and widening the teacher's view of his subject. If modern text-books were used, the study of geometry could commence earlier, and a larger knowledge of the subject would be acquired; boys

would have time to proceed to advanced geometry, as they do in France. We were obliged to continue the use of Euclid, because the universities and other examining bodies forced it upon our pupils, whose future prospects depended upon their success in examinations. This was one of the cases where the universities, which ought to lead, retarded the educational progress of the country, and suffered improvements to be forced upon them from without which ought to be the results of spontaneous movements from within. It was then suggested that the universities and other leading examining bodies should be petitioned to set geometry, instead of Euclid, as the subject of examination, leaving it to the discretion of the teacher to use what text-book he pleased. This was already done by Oxford. Next it was recommended that mathematicians and schoolmasters should combine to produce in concert a good text-book; this was likely to be done by the committee of the British Association and by the University of Oxford; and that until this was effected we should use one of the existing modern text-books. It was further suggested that those engaged in education should communicate their ideas and the results of their experience to those who had composed, or were engaged in composing such treatises. The practical difficulty in the way of composing a good text-book must be great at first; but assistance might be derived from France and Germany. The question could no longer be trifled with, convictions were too strong, schoolmasters who were in earnest could not allow generation after generation of their pupils to suffer while people were making up their minds. We should all concentrate our efforts to substitute for Euclid a text-book written on modern principles. Though all our prejudices were naturally in favour of Euclid, yet we were forced to the conviction that it must be given up, and that but few, if any, educational reforms would be fraught with such beneficial results.

APPENDIX E.

FROM "THE CORRESPONDENT," JANUARY 27TH, 1866.

A SLICE OF THE SEAFORTH MINCE PI.

SIR,

Mr. James Smith makes frequent use of Euclid's 47th proposition, which, I conclude, therefore, he admits, as well as the 20th. Moreover, his last letter but one informs us (*Correspondent*, p. 23) that no less than three natural sines, those of 30°, of 45°, and of 60° are "correctly given in our tables;" and to leave no doubt about their values, Mr. Smith himself states them as ·5, as ·707107..., and as ·866025...

If, then, on any quadrant, whose radius may be called unity, we mark two points, at 30° and 45° from one extremity, join them, and draw the sine and cosine to each; these latter four lines, he assures us, measure, the longest, ·866025, &c., the two next, ·707107, &c., and the shortest, ·5.

Now the two points being also joined by a straight line, he will allow this, I suppose, to be the chord of 15°, and also the slant side of a certain right-angled triangle, whose other sides are, by his showing,

The difference between	·866025...
and	·707107...
namely	·158918...
and the difference between	·707107...
and	·5
namely	·207107...

By squaring these two, then, ·158918.. and ·207107.., adding their squares together, and taking the root of the sum, we have (if by Mr. Smith's leave, the 47th proposition is not as great a delusion

as the tables) ·261052.. for the chord of 15°. But the *arc* of 15° is a twelfth of the arc of 180°, and may be had by dividing the Smithian π into twelve, thus :—

$$12)3·125000$$

$$·260417 = \text{arc of } 15° \text{ (Smithian)}$$

which take from ·261052 = chord of 15° (popular and Smithian)

leaves ·000635 = excess of chord over arc. *

By ·000635 of a radius, then, does this straight line exceed the curve (both Smithian) between the same two points! Not having read any earlier letter, I know not what are the "difficulties" out of which Mr. Smith calls upon his opponents to help each other, but surely here is one that your readers have a right to see explained before hearing any more about "dogmatic blunderers," &c. Is the non-"mysterious" 3·125 found a *mince* π after all (in the French sense of *mince*), and if so, how much is it to be eked out? Or is the arc of 15° *really* shorter than its chord?

I remain, Sir,

Yours very respectfully,

E. L. GARBETT.

FROM "THE CORRESPONDENT," FEBRUARY 3RD, 1866.

THE QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

I am afraid some of your readers will be heartily tired of circle-squaring, and had not a fresh champion of Orthodoxy sprung up in the person of Mr. E. L. Garbett, I should not have troubled you with another "*slice of the Seaforth mince* π," without pausing for a time. Were I to remain silent, however, it would be taken for granted that his Letter is unanswerable, and I must therefore beg of you to favour me with space in your next publication for a reply.

* I dispute this conclusion. ·261052 is not the chord of 15° (Smithian), and Mr. Garbett's conclusion is fallacious. J. S.

Well, then, I at once admit the correctness of Mr. Garbett's figures, but I dispute the argument he founds upon them. The chord of 15° is a side of a regular polygon of 24 sides inscribed in a circle; and it is obvious that to whatever extent we may double the number of sides of polygons inscribed in a circle, as we increase the perimeters we increase the areas in like proportion, but can never make a polygon equal in perimeter and area to the circumference and area of its circumscribing circle. Now, if the radius of a circle = 1, the perimeter of an inscribed regular hexagon = 6, and the area of the hexagon = 2·598075 approximately. If to this be added the sum of the areas of the 12 right-angled triangles about the hexagon, which make up a dodecagon or 12-sided polygon = ·401925 approximately, we obtain the area of the dodecagon = 6 (radius × semi-radius) = 3. Now, if x denote the area of a circle equal in circumference to the perimeter of the hexagon, y the area of the dodecagon, and z the area of the circumscribing circle, then, $3 : \pi :: x : y$; and $3^2 : \pi^2 :: x : z$, whatever the arithmetical value of π may be. But, $3\left(\frac{6}{6\cdot25}\right) = 3\cdot125$ $\left(\frac{6}{6\cdot25}\right)^2 = 2\cdot88$ = area of a circle of circumference = 6; therefore, area of a circle of circumference = 6 — area of a regular hexagon of perimeter = 6 = 2·88 — 2·598075 = ·401925 = area of the 12 right-angled triangles about the hexagon approximately. Thus area of the circle, plus area of the triangles = 2·88 + ·401925 = 3·281925. This is an arithmetical quantity greatly in excess of the area of a circle of radius 1. But, if to the area of a circle equal in circumference to the perimeter of the dodecagon, or 12-sided polygon, we add the difference between the areas of 12 and 24-sided polygons, we obtain a smaller arithmetical quantity than 3·281925; and by continuing this process we may still further reduce this quantity at every step, but when we have extended the calculations to the exhausting point, we have still a quantity in excess of the area of a circle of radius 1. Hence, the 47th proposition of the first book of Euclid is inapplicable (directly) to the measurement of a circle.

Within the last ten days I have received three Letters from a Cornish gentlemen, written in the most kindly spirit, in one of which the writer gently reproves both my opponents and myself for using hard words, which he truly says are "*useless and irritating.*" He

observes :—" *When two disputants find they cannot agree in axioms or fundamentals, it is best to leave off disputing, which, if pursued, generally becomes mere reviling.*" I shall be glad if for the future all parties to the controversy on the ratio of diameter to circumference in a circle, will bear these facts in mind. The gentleman in question differs from me and adopts the same line of argument as Mr. Garbett, but puts it in a somewhat different form. He treats the subject, however, so distinctly and intelligibly as to leave no difficulty in dealing with it.

Let C denote a circle, and O A B represent one of 25 equal isosceles triangles inscribed in the circle. Let O denote the angle at the centre of the circle, A B its subtending chord, A C B its subtending arc, and O D a straight line bisecting the angle O and its subtending chord A B. A diagram of this figure may readily be constructed by any Geometrician.

The following is an illustration of my correspondent's argument and conclusion :—

$$\text{The angle O} = \frac{360^\circ}{25} = 14^\circ\ 24'$$

$$\text{The angle A O D} = \frac{14^\circ\ 24'}{2} = 7^\circ\ 12'$$

Therefore, A D = half the chord A B = A O sine $7^\circ\ 12'$ = sine $7^\circ\ 12'$, for A O radius = 1. The natural sine of $7^\circ\ 12'$ as per tables is ·125333 ; therefore, 2 (·125333) = ·250666 is the value of the chord A B, and is greater than $\frac{3{\cdot}125}{12{\cdot}5}$ = ·25 ; therefore, my correspondent draws the conclusion that this would make the chord A B greater than its subtending arc A C B.

Now, this argument is, no doubt, plausible, but I meet it in the following way :—

The circular measure of the angle O = arc A C B = $\frac{14^\circ\ 24' \times \pi}{180} = \frac{\pi}{12{\cdot}5}$, whatever the arithmetical value of π may be. Hence, 12·5 (arc A C B) = 12·5 × $14^\circ\ 24'$ = 180°, = semi-circumference of the circle ; 4 (arc A C B) = 4 × $14^\circ\ 24'$ = $57^\circ\ 36'$ = radius ; $\frac{12{\cdot}5}{4}$ = 3·125 = π ; and 2 π (radius) = 6·25 × $57^\circ\ 36'$ = 360° = circumference. Again : The perimeter of a regular in-

scribed hexagon to a circle of diameter unity = 6 times radius = 3.; therefore, since the property of one circle is the property of all circles, and as π denotes the circumference of a circle of diameter unity, it follows of necessity, that $\frac{3}{\pi}$ expresses the ratio between the perimeter of every regular hexagon and the circumference of its circumscribing circle. Thus, the angle O is to an angle of 15° as the perimeter of a regular hexagon to the circumference of its circumscribing circle, that is to say, 14° 24′ : 15° : : 3 : 3·125 ; therefore, 6 times radius = 6 × 57° 36′ = 345° 36′ = perimeter of a regular inscribed hexagon to a circle of circumference 360° ; therefore, $\frac{\pi}{3}(345° \; 36′) = \frac{3·125 \times 345° \; 36′}{3} = 360°$ = circumference. But further: The angles at the centre of a circle are to each other as their subtending arcs ; therefore, the arc A C B is to an arc of 15° in the ratio of 3 to 3·125.

Now, the circular measure of an angle of 14° 24′ to a circle of radius 1 = $\frac{864′ \times 3·125}{180° \times 60′}$ = ·25 = arc A C B ; therefore, 12·5 (arc A C B) = 12·5 × ·25 = 3·125 = semi-circumference of a circle of radius 1 ; 4 times ·25 = (4 × ·25) = 1 = — radius ; and 2 π (radius) = 6·25 = circumference. We thus arrive at a similar conclusion, whether we take a circle of circumference 360° or a circle of radius 1. But further: The circular measure of the arc A C B = semi-radius of a circle of diameter unity ; therefore, π times arc A C B = π times the square of the radius in a circle of diameter unity, = $\frac{\pi}{4}$ = 12·5 times $\frac{\pi}{50}$ = 12·5 times $s\,r^2$ = area of a circle of diameter unity ; and since the property of one circle is the property of all circles, it follows of necessity, that 12·5 times $s\,r^2$ = area in every circle. My opponents may readily convince themselves that 12·5 times $\frac{\pi}{50}$ = area of a circle of diameter unity, by means of any hypothetical value of π. These facts I challenge Mr. Garbett to controvert, and I ask him, as an honest controversialist, to make the mysterious "*mince* π" 3·14159265, &c., harmonise with them. If he find this impossible, let him admit that he is checkmated.

Mr. Alex. Edw. Miller seems to have been sadly afraid of my seizing upon a typographical error and making a handle of it. I

am not the man to play any such game. I had observed the error, but understood perfectly what he meant. If I had had the correction of the printer's mistakes, I might probably have employed the expression $\frac{1}{2}\sqrt{2}$ instead of $\frac{1}{\sqrt{2}}$, the former being more readily worked out arithmetically.

In conclusion, I may observe :—The question of whether the problem of the quadrature of the circle can or can not be solved, depends upon the possibility of ascertaining the true value of π. Notwithstanding the many letters of mine which have appeared in the *Correspondent* on this subject, Mr. Miller would appear to have been quite oblivious as to my object, for he observes :—"*As to discussing with him* (Mr. Smith), *or any one else, the question whether π does or does not = $3\frac{1}{8}$, it never entered into my head.*" This looks very like an attempt to wriggle out of a difficulty.*

I am, Sir,

Yours very respectfully,

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
January 29th, 1866.

FROM "THE CORRESPONDENT," FEBRUARY 3RD, 1866.

THE DISTANCE OF THE SUN.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

I think the argument of "Terra" as to the distance of the sun, and its consequent apparent semi-diameter, as entirely opposed to the facts of observation, which my figures agree with

*This letter completly refutes the conclusions arrived at by Mr. E. L. Garbett and Captain Morrison. Neither of those gentlemen have attempted to grapple with my demonstrations, indeed, they have never referred to this Letter in any of their communications to the *Correspondent*.

J. S.

to one second. This argument he ignores, but I must beg to bring his nose to the grindstone, however unwillingly. Till he answers this I have no more to say on the subject. As to the matter of agreeing with Mr. Smith, what right has "Terra" to call upon me to avow my opinion, unless, indeed, he be fighting under false colours, and that "Terra" and Mr. James Smith are all one person. If this be so, then I will avow that I have defended Mr. James Smith, not because I think he is right, but because he has made many valuable discoveries, and I think him deserving of a cool, deliberate, and incontrovertible reply and refutation at the hands of the Mathematicians, which he has not yet received. True, Mr. E. L. Garbett has gone nearer than any other person to fix Mr. Smith, and we shall see how he will get over this refutation.

Another of your correspondents, Mr. J. Reddie, has attacked my "Astronomy in a Nutshell,"—a work printed only to teach how to work the leading problems of the solar system. He says that he has never seen my larger work, "The Solar System as it Is," &c., which does not fix the rate of the sun's motion "precisely," as Mr. Reddie heedlessly asserts. However, it seems to me that when a man undertakes to criticise and carp at the writings of another person, he should feel bound, not only to see, but to carefully read, all that other person has written on the subject. Not to do so is not only very absurd, but quite unworthy of any man who claims to teach us how to "recast" Astronomy. He—Mr. J. Reddie—clearly does not understand that in naming "the West" as that point in space towards which the sun ever moves, no allusion to this earth, or any of its belongings, was ever intended. If we look at the Pole star, we face (almost exactly) the true North point; the opposite is the true South; and the point which is at right angles thereto, on our right hand, is the East, the opposite to which is "the West." Let Mr. Reddie ascend in a balloon, and he will find that the North will remain the North to him, even though the earth deserted him and vanished away entirely. For assuredly the great concave of Heaven would be there, and the sun would move, as move he does to "the West," although the earth, and all that it inhabits, were to melt away and "leave not a wreck behind."

Now, as to the "glaring error" Mr. Reddie speaks of in my smaller book, I plainly tell him that there is no error at all, but

what his heedless, overhanded criticism has invented. The question was to prove that a body falling in a square under the influence of "*two equal forces*," acting at right angles, would form the ***diagonal*** of the square. This I did, and I defy Mr. Reddie to show any, the least, error in my argument. He next proceeds to attacking the theory of the motion of the sun in space towards the "West," which I have above disposed of, as I think, satisfactorily. Mr. Reddie speaks with very comfortable complacency of having "forced Professor Airy to give up the notion of solar motion in space altogether," and treats it as "exploded." But he has merely pointed out an omission in the mode of observing this phenomenon which by no means affects the fact, but merely acts upon the degree of the motion. Professor Airy says it is "at present in doubt and abeyance;" but Mr. Reddie has no "doubt" about it, not a jot. He rushes to the conclusion that all the great astronomers who have observed the phenomenon are in error, dark as Erebus. Does he not know that Professor Airy teaches in lectures that the moon's path is nearly a circle round the earth? And does not Mr. Reddie himself dispute this absurdity? Why, then, does he take the Professor's "doubts" as conclusive against a well established fact? Is this wild enmity against the idea of "the motion of the sun in space" indicative of a desire to prove that the earth is the fixed body? This was Mr. Prescott's hobby, and it is one that has hitherto thrown all its riders. I hope it will not bring Mr. Reddie to grief. I should be sorry if it did, as I greatly respect his talents, and only regret that he is in too great hast to settle most points in astronomy. *

Yours,

R. J. MORRISON.

* R. J. Morrison regrets that Mr. Reddie "*is in too great haste to settle most points in astronomy.*" This is amusing, coming as it does from one who has since reduced his estimate of the Sun's distance, from upwards of ninety millions of miles to about 360,000 miles. I suspect this "***hobby***" will throw its "*rider.*" What then? J. S.

FROM "THE CORRESPONDENT," FEBRUARY 3RD, 1866.

"NATURAL-BORN NOBLES."*

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

There is truly both instruction and amusement to be got from the rather swollen "Physics and Science" department of the *Correspondent*, apart even from the letters of "X," and Professor De Morgan (if these be not, as I suspect, identical), and many readers of the *Athenæum* will probably thank the latter gentleman for being informed of your "most panoramic" periodical's existence. In the new year's gift from "Terra," p. 8, that bounteous and venerable Dame quotes a dictum of her son—it now appears no less renowned a son than Zadkiel—that the All-Wise having made the whole universe by weight and measure, "this consideration enables us (who of course, can see how All-Wisdom must have apportioned things), from the observed distance and diameter of the moon, to determine those of the sun," which they accordingly have each determined in his or her own way. Now, as the "whole universe," made by measure, comprises some things even nearer than the moon, I would beg to suggest that there are great advantages every way in a nearer problem, on which I propose they should first try their hands, and then proceed to the sun. It is this—All the distances and dimensions of Ireland and of the Isle of Wight are even more accurately settled now than those of the earth and moon. My proposal

* This idea of a heading for a letter originated in a review of one of my works in Zadkiel's Almanac for 1866 (see Appendix F). In a letter to the Editor of the *Athenæum* dated 12th June, 1865, I communicated the proof introduced by Zadkiel into this review, and about the same time also gave it to Captain Morrison in a private letter; but it does not appear in the work the gallant Captain reviews, and was not made public until nearly the end of the year 1866, when I brought it out in my work entitled:—*The British Association in Jeopardy, &c.*

J. S.

is : " From the observed" measures of Ireland and the Isle of Wight, "to determine those of" Great Britain, which are not yet known with any approach to the same accuracy. This would check, if not save, much labour of our ordnance surveyors, would be a far more satisfactory test of the ability that Zadkiel and "Terra" claim, and has, it seems to me, the advantage in all conceivable points. For, First, the three islands are nearer of a size, and the ratio far less extravagant from even the *least* of the given ones to that sought than from the *greatest* dimension in the case of the earth and moon to the least of those sought from *them*. Next, the three islands, though not forming the whole of a group, are far less mixed up with others than in the case of the three bodies. The second of these is one of eight and one of the smallest of its order; but in the isles, Ireland forms the whole secondary order in itself. Lastly, this group make a far less significant part of *their* universe of all-known lands, than the three bodies make, I will not say of the known or seen starry "universe," but even of that corner thereof within non-telescopic eye-shot (and you *can* have no more proof that the "whole" universe consists chiefly of suns and planets than chiefly of Isles of Wight, or than a mite has that it consists chiefly of cheese). On all grounds, therefore, I submit that the chances must be in favour of my problem, rather than Zadkiel's present one, if he and "Terra" would condescend to attempt it.

But to return : "Terra" supposes herself in need of "the moon's horizontal parallax," for ascertaining which (p. 8) she assures us "two elements are essential—namely, the moon's distance and the earth's semi-diameter ; and if we know not the true ratio or proportion between them, it is certain we can never ascertain the moon's mean horizontal parallax correctly." (!) This is equivalent to saying : For ascertaining the length of a railway two elements are essential—the average breadth of the works, and the whole number of acres they occupy ; or, for ascertaining the area of the Holborn viaduct two elements are essential—the total cost when finished and the average cost per square yard ; and if we know not the true ratio between these it is certain we can never ascertain the vaiduct's area correctly! Zadkiel, who, as a gallant navigator, retains probably some impression from school-days that astronomers (and even astrologers) deduced a body's distance from its parallax, and therefore not the

parallax *also* from the distance, ventures to doubt, in your next number, p. 23, whether "Terra" goes the right way to ascertain what is sought. "Terra" (p. 31) "respectfully requests him to find a better;" and to this no reply appears ready from the world's astrological instructor, though another of your contributors *may* be Reddie. He evidently, like the gallant Captain, sticks at admitting quite, *both* "Terra's" essentials. There is some hazy perception that to require distance (which can only be known from parallax) before you get the parallax, would be a somewhat circular kind of requirement, but he still thinks the other necessary. In his own words, p. 43, as "Terra" well observes, "without a *true* value of the earth's semi-diameter, we could not ascertain the parallax either of sun or moon." He is sure that to ascertain the railway's or vaiduct's length, we must have the average cost per mile or per yard; without a *true* value of *that* element, we could tell the length of neither. But the necessity of also knowing the total cost he leaves to be insisted on by "Terra." Will she never learn that you may ascertain "the ratio between" two things without the slightest knowledge of the absolute measure of *either*, and that this "ratio between" the earth's semi-diameter and a body's distance *is* the body's "parallax," and has been, and must be, measured *independently of all knowledge of either;* and when it has been so settled, *then* the perfectly independent measure of one of them, be it 5,000 miles or 50,oco, will give the other?

This brings me to the question of Mr. Reddie's, which I would answer. After displaying the uncertainties about solar parallax and motion, he asks: "What are the differences between $\pi = 3 \cdot 125$ and $\pi = 3 \cdot 14159\ldots$, in importance, to *these* differences?" I reply, considerably more than as the loss of the "London" to the loss of a pin, or perhaps in about the ratio of the importance of that event to the loss of a "natural-born noble's" wits. As Mr. Reddie seems to fancy, there is some sort of "importance" that is measured by the number of millions, or of cyphers, in the figures involved, let me request his attention a moment to a fact or two about his "differences," as certain as that twice two are four.

First, he will not find, nor can any astronomer, among all the statements he quotes, *any two* whose interchange will make a difference of *one second* of space, or fraction of a second of time, in any

single line of the "Nautical Almanac" for any year yet published; or *one figure* of any other almanac or any ship's reckoning in the world. Ten million miles in the sun's distance will not do so—though altering, we admit, in the same ratio as itself, "Professor Kelland's base line" and every dimension in the universe beyond the moon. And as for solar motion, if Mr. Reddie will enquire, he will find that a star's "annual parallax" has never meant, with any professor, its change of apparent place in a particular six months, but between the apparent place to-day and the *mean* of the places six months *ago* and six months *hence;* and will see (if he is not a natural-born noble) that in that case *no* solar motion, or any number of millions of what he will, in whatever direction, make absolutely no difference whatever.

But, secondly, if $\pi = 3{\cdot}125$, there is not one figure throughout the Nautical and all other almanacs correct enough (unless by pure hazard and miraculous coincidence) to be not mere mischievous delusion; and every ocean-crossing vessel that gets to port is a miracle. For there is not one figure in their reckonings which did not depend on tables every figure of which was calculated on $\pi = 3{\cdot}1459265$. Let Mr. Reddie open a book of Navigation Tables or any almanac, and observe, I repeat, that *every single figure* in them involves and depends on the value of π, and would be altered by taking $\pi = 3{\cdot}125$. Moreover—and this is the point I am especially to observe, Sir—Mr. James Smith knows as well as the Astronomer Royal that this is so! He glories in it. His last letter challenges the world (p. 39) to produce any tables that do not give the sines of certain arcs greater than the arcs themselves (as valued by the Smithian π). Of course the world cannot produce one. It contains *no* table which has not made the sine of 9°, and of every arc below 9°, greater than the Smithian calculation of the arc. I cannot stop to find the exact arc which he equals to its sine. It is above 9°, and all below it are the "certain" arcs of which he speaks.

It is true that he was inadvertent enough to speak (p. 23) of "one of the very few sines given correctly in our tables," and to admit three (or rather *four*, those of 90°, 60°, 45°, and 30°,) to be so; but this was shipping four seas, any single one of which, unless pumped out, will sink him! He will find that even one tabular sine, and any one, except that of 0° = 0, is more than he can take in

without drowning. For if the tabular sine of any one angle is right so is its tabular chord, and hence the sine of its half, and so of its quarter, its eighth, and its sixteenth. But before he has bisected any arc down to its sixteenth, he will have got below $9°$, and obtained a sine longer, as he well knows, that he allows the arc to be. Not one tabular figure can be right with $\pi = 3{\cdot}125$, because in that case every degree used in our calculations is an arc, 358 of which (and a very small fraction) will reach round the circle. As all astronomers then calculate on 360 of them being required, it is plain that every heavenly body must make its revolution, or any part thereof, in $\frac{1}{180}$ less time than they calculate. Every transit, or aspect, or lunar distance, if happening to correspond with the almanac statement to-day, will in future occur sooner than predicted —every one 18 hours hence, 6 minutes sooner—everything predicted for six months hence, a day sooner, and everything in the almanac now published for 1870, ten days sooner, and the days from the equinox of next March to that of March 1867, will be three hundred and sixty-*three* and a quarter. Now a man, we are told, *knows* and can demonstrate $\pi = 3{\cdot}125$, which he knows as well as I do must involve this total error of all the tables, and has *said* so, and what is he about? Publishing correct tables? Not one figure! Can he not at least *calculate* an instalment, half-a-dozen correct sines or tangents per week, and send them to the *Correspondent*? We shall be some time accumulating the 144,000 results that go to make the poorest trigonometrical table, but ever so little is better than nothing, and the deceitful tables multiplying shipwrecks every hour! Have we not heard enough of "assuming π to $= 3{\cdot}125$," to draw now from that assumption this consequence in Quadrature of Morals? Instead of disputing whether π *does* $= 3{\cdot}125$, you see I have taken for granted that it does, and the most important result I can deduce is this, Sir, that any creature in the livery of human shape who knows and can prove it to be so, and fiddles while the ships are sinking, and instead of moving a little finger towards the production of the true tables, eats and drinks, and calls astronomers names, must be such a "natural-born noble" as Heaven preserve us natural-born blunderers, Sir, from ever becoming.

Yours very truly,

30th January, 1866. E. L. GARBETT.

FROM "THE CORRESPONDENT," FEBRUARY 10TH, 1866.

THE DISTANCE OF THE SUN.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

Suppose me to put the following question to an intelligent boy of 8 or 10 years of age, who had never been taught anything as to the science of astronomy—Which of the two do you *think* the larger, my boy, the sun or the moon? Would not his probable answer be—*I think*, Sir, they are about the same size? The boy would be right, and this would be the answer of every adult, however intelligent, who had not received some instruction, and made a certain advance in the science of astronomy. Neither the boy nor the adult would have any idea that Captain Morrison's figures, or any other figures, could be made to prove the "distance of the sun and its consequent apparent semi-diameter." So much for "*I think*," and mere observation, and upon the gallant Captain's own showing it is a question of figures whether the inhabitants of our sublunary sphere are ever to arrive at the perfect knowledge of either the one or the other. I do not "ignore" the Captain's argument, nor does he bring my "nose to the grindstone," but I do dispute his *assertion* that his "figures agree to one second."

The gallant Captain puts the question:—"As to the matter of agreeing with Mr. Smith, what right has 'Terra' to call upon me to avow my opinion?" "Terra" replies:—He did not ask Captain Morrison for any opinion, but simply took the publicly-avowed opinion of the gallant Captain upon Mr. Smith's last published work, and drew the inference that upon the main argument of that work he and Mr. Smith were agreed. On this point "Terra" would appear to have been mistaken, and so far begs to apologise to the gallant Captain, but, in doing so, cannot help expressing his conviction that ninety-nine out of every hundred readers of the Captain's review of the work in question would have fallen into the same error.

I hardly think Mr. Smith will thank Captain Morrison for his left-handed compliments. He defends that gentleman, "*not because he thinks he is right*," but for other reasons. If Mr. Smith is not a true quadrator, what is he? To the readers of the *Correspondent* he is known as a circle-squarer with a disputed title, and as nothing more. Will the gallant Captain, then, be good enough to inform us what are the "*many valuable discoveries*" for which he gives Mr. Smith credit?

Captain Morrison concludes the "*Smithian*" paragraph of his Letter as follows:—"*True, Mr. E. L. Garbett has gone nearer than any other person to fix Mr. Smith, and we shall see how he will get over this refutation.*" Is it not the true inference from this, that in the opinion of the gallant Captain, Mr. E. L. Garbett has got Mr. Smith in a "*fix?*" If not, what meaning does he attach to the word *refutation?* Is not the *wish* "father to the thought," and generator of the hope, that Mr. Smith may fail to get out of the "*fix?*" If, after all, he should contrive to get out of it, what will the gallant Captain do then? This would necessarily involve the remodelling of his values of the dimensions and distances of the sun, moon, and earth. Will he entertain such an idea? If not, will he abandon his theory? Time will bring about a solution of these enigmas.*

It appears to "Terra" that public writers should make it one branch of their study to be consistent with themselves.

TERRA.

5th February, 1866.

* If the distance of the Sun from the Earth were exactly 360,000 miles, it would make the Earth's circumference to be only 78¼ miles; which is contradicted, not only by actual measurement, but by the experience of every circumnavigator of the globe. Captain Morrison, in a recent production entitled "*The New Principia*," professes to demonstrate that, "THE EARTH IS PROVED TO BE THE STATIONARY CENTRE OF THE SOLAR SYSTEM, AND THE SUN SHEWN TO BE ONLY 365,006 MILES FROM THE EARTH." What next? J. S.

FROM "THE CORRESPONDENT," FEBRUARY 17TH, 1866.

IMPERTINENCE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

When Squire Western (in "Tom Jones") got rather out of his depth, and wished to give the conversation a pleasant turn, he proposed talking "on politics, or the state of the nation, or something which every one can understand." That might be very well for Squire Western, but, unhappily, just at present "Politics and the State of the nation" is one of those things which no one does exactly understand. Many of us are in the position of the sailor who was accidentally blown up at a fair, and who, taking it for part of the exhibition, when he got on his legs, exclaimed, "I wonder what the devil the fellow will be up to next." True, we have not exactly been blown up, but we have got a new Parliament. Lord Palmerston has most unexpectedly been taken from the helm, like Palmurus of old who fell asleep at the rudder, and no one has a very precise idea "what the devil the fellows are going to do next," or whether they are going to do anything.

Therefore, to select "politics and the state of the nation" as a topic which every one can understand, would be a simple piece of absurdity. Perhaps we cannot do better than select a subject in which, unfortunately, all classes of men are more or less proficient, from the urchin in the street up to . . . .

Well, wait till Parliament has grown a little older, and then we shall be better able to complete the sentence.

I will not occupy the time of your readers by giving a definition of the word *impertinent*, but I will assume that they all possess a copy of the "Imperial Lexicon," "Richardson's Dictionary," and half-a-dozen other authorities; and when they have consulted them all, I will still back a certain dealer in canary-birds against the best of your readers for a ready and impertinent reply to an impertinent question. "Well," said a friend of mine to the bird-fancier, "your

canaries are very yellow; of course you *paint* them?"—"No, Sir; dips 'em in the mustard-pot every morning."

There is no "royal road" to impertinence any more than to knowledge, and it were to be desired that there were no road *at all*, but that impertinent people could find a board stuck up everywhere, with "No Thoroughfare." But some impertinence is so purely the result of ignorance or accident, that the strictest precautions would not put a stop to it, and it would lead to endless litigation.

For example: An extremely worthy old clergyman, whose asthma was so increased by a bad cold as to render him almost inaudible, met one of his parishioners, and remarked to him pathetically, "Well, I am sadly afraid I must leave you. I am sure you could not have understood a syllable of my last sermon." "Lord, Sir," replied the worthy man, "don't you think of going for to leave us. Lord, Sir, we never minds anything *you* says. You just stay where you are."

Of course that was extremely consolatory, and *might* have been extremely impertinent!

But before proceeding any further with this letter, it has just occurred to me under what head (if deemed worthy of insertion) will it be placed.

Perhaps under the head of "Social Science," on the *lucus a non lucendo* principle, because *impertinence* is a most *un*-social "science." The question of classification puts me in mind of a preacher who had been descanting on the virtues of some saintly personage, till, warming with his subject, he passionately exclaimed, "Where shall we place him? Shall we place him among the Patriarchs? He is greater than a Patriarch! Among the Prophets? He is greater than a Prophet! Among the Apostles? He is greater than an Apostle! Shall we place him——" "O, d— it!" said a sailor, most irreverently rising from his seat, "he may have *my* place, and welcome, for I am going." Now, sooner than you should lose all your readers whilst you are making up your mind where to place *impertinence*, let me recommend you, Mr. Editor, to place it anywhere, under the head of "Political Economy," "Trade and Commerce," "Arts and Music"—it is a matter of perfect indifference, for, wherever a society of men are gathered together, *impertinence* is sure to be found in the midst of them.

As for newspaper correspondents,——— ! !

But the impertinence of newspaper correspondents is nothing to that of relations and "dear friends." No one knows—that is, every one knows—how much impertinence often lurks under that terrible "My dear!"

"Sir," once said the Dean of a College to a student who had pleaded cold as the cause of his absence from College Chapel, "I should have thought your religion would have kept you warm." "Sir," replied the youth, "I never made a cloak of my religion;" but I fear that *impertinence* often "makes a cloak" of "my dear," but not to keep friendship warm.

Of what use are friends if they are not to point out each other's faults? "Could not his *friends* tell him of such and such faults?" Of course they *can*, and of course they *do*. Quite forgetting Cicero's golden rule that "amicitia iisdem artibus retinetur quibus ab initio partitur," friendship is retained by the same arts by which it is in the first instance acquired. The celebrated author of the "Fasti Hellenici et Græci" records it as a remarkable fact in one page of his journal, that a relation whose name he mentions had given him hearty praises, and he remarks that he was the only relation who had ever done him justice, or something to that effect. And Sir Fretful Plagiary, in the *Tragedy Rehearsed*, assures Sneer that if the newspapers ever do say an ill-natured thing of a man's works, "he's sure to hear it from some d—d good natured *friend* or another."

And what says the old proverb? "The Lord deliver me from *friends*."

As to our spiritual teachers, if they never are guilty of impertinence, idle people are sure to saddle impertinent stories on them, as, for instance, the one told some time back of the Bishop of Oxford and the late Lord Palmerston. The latter had offered (so goes the story) the Bishop a seat in his carriage, but the Bishop preferred to walk on. At last he was overtaken by Lord Palmerston and by a shower of rain. Lord Palmerston nodded to him out of his carriage, and said—

> "How blessed is he who ne'er consents,
> By ill advice to *walk*."

To which the Bishop replied—

> "Nor stands in sinner's ways nor sits
> Where men profanely talk."

How very absurd these stories must appear to those on whom they are saddled. But as incongruity is the soul of humour, so a Bishop is supposed to give great zest to a story involving a slight touch of profanity.

"What's in a *name?*" Why, *everything!* A clown at Cooke's Circus, some years back, exemplified this important truth. "Do you know," said he, turning to the master of the Circus, the difference between 'elevated,' 'inebriated,' 'intoxicated,' and 'drunk'"? Of course the master of the Circus, who had only heard it several times every day, had forgotten all about it, so the clown had to tell him again.

"If his Royal Highness Prince Albert should take a little too much, people say his Royal Highness is slightly elevated; if Lord John Russell takes too much, Lord John is inebriated; if a respectable tradesman takes too much, he is said to be intoxicated; but if you and I were to take a drop too much, people would say of us, 'Look at those two, they are as drunk as two pigs.'"

Now, no one cares what a pig does, but a statesman or a bishop is quite another affair. And it was told as a capital joke of a very High Church bishop, that when some one, reading before him, substituted "condemnation" for "damnation," the Bishop rose with great wrath, and exclaimed in a loud voice, "Damnation, sir!"

But to return to impertinence. It is sometimes so thoroughly disinterested that one is disposed to excuse it in consequence of the honesty displayed. A hypochondriac who fancied he was very ill, was assured, over and over again, by his medical attendant, that he had really nothing the matter with him, and no longer required his attendance. "Well," said the patient, "I shall be obliged if you will tell me how I must live—what I may eat, and what not?" "My directions," replied the physician, "are few and simple: you must not eat the poker, shovel, or tongs, for they are hard of digestion; nor the bellows, because they are windy; but anything else you like."

Such a patient now-a-days would set down his doctor as an ignorant, impertinent allopathist, and would have immediate recourse to a homœopathic quack, who would fleece him to the tune of perhaps £200 a year.

But it is not always safe even to make a complimentary speech. A man at a party had proposed as a toast—"Woman—without her, man is a brute." But the reporter had it printed, "Woman—without her man, is a brute." A piece of "wrong-hyphenism" which might probably in some families lead to awkward consequences.

Some mere blunders have a very impertinent sound. The following story I met with lately in a newspaper. Lord Shaftesbury, when examining a girl's school, asked one of them (a rather queer question, by the way), "Well, my good girl, who made your body?" "Please, my lord, Betsy Jones made my body, but I made the skirt myself." And another charity girl, under examination in the Psalms, was asked, "What is the pestilence that walketh by darkness?" "Please Sir, bugs."

This puts one in mind of the girl who was asked, what was meant by a backbiter? "Why that's a flay, ma'am." But the following I heard myself in a catechism at a Roman Catholic Chapel. Priest: "Well, my boy, can you tell me where the Virgin Mary has gone?" The boy horrified all present by saying, "To hell."

A quick lad was present with his mother at the examination of a girls' school. The girls were asked what was the first thing God created? But as they hesitated, the lady turned to her son, and said—"Come, *you* can tell them *that*." "O yes," said the boy; "Light, to be sure, and God made light first, in order to be able to see how to make all the other things." The idea was not at all a bad one, but perhaps the story does not throw much light on our subject of impertinence.

Aggressive impertinence admits of no excuse; but defensive impertinence is sometimes no impertinence at all. A lady at an American hotel, who gave herself great airs, was horrified at having to sit down at the same table with a "Yankee" gentleman. No sooner had dinner commenced than she commenced her attack. Seeing the gentleman use his own knife (a clean one) to carve some joint, she turned to the waiter, and said, "Remove that dish, that gentleman has had his knife in it." Presently afterwards the lady helped herself with her fingers, to a piece of dry chip lying in a dish before her. "Remove that dish," said the gentleman, "that lady has had her fingers in it." So far was a good thrust and parry,

but the lady was determined to "conquer or to die." "A fig for *you*, Sir," she said, handing the enemy a fig. "Thank you ; and a fig-leaf for *you*, ma'am," he replied, with a polite bow, handing her a fig-leaf which lay in the dish ; after which the gentleman remained master of the field.

Talking of figs, some of your readers may not have heard of Dr. Jowett and his celebrated fig. The doctor flourished many years ago in the University of Cambridge, and in his garden he had a fig-tree, on which, once in a time, there was so promising a fig, that the Doctor took a card and labelled it "Dr. Jowett's fig." But a profane under-graduate ate the fig, and wrote on the card, "A fig for Dr. Jowett."

But a better specimen of impertinence was given in *Punch* some years ago. A gentleman asked a man to hold his horse for him. "Do a bite?" asked the man. "No," was the reply. "Do a kick?" "No." "Well," said the man, "do a want *two* to hold him?" "No." "Then," said the man, "hold him yourself."

A prosperous man on horseback met a countryman—"Fellow, don't touch your hat to me, you will frighten my horse." "Warnt a goen," was the reply. "A fine day," observed a gentleman to a navvy. "Well, did I say it warnt—do you want to argufy, you beggar?" Perhaps the man thought the gentleman impertinent, to address him without a proper introduction. Sometimes people take a civil question for a piece of impertinence. Thus a stout farmer, in the old coaching days, an inside passenger, sat down to dinner, at a roadside inn, with a gentleman who was travelling outside. When dinner was nearly over, the gentleman asked the farmer, "Full inside, sir?" The farmer, who had eaten an immense dinner, laid down his knife and fork, and turning fiercely on the querist, said—"I'll tell you what, sir, I don't know what you mean by asking me if I'm full inside, but I will only observe, sir, that if, when a gentleman has paid for his dinner, he may not be allowed to eat it without impertinent remarks, things are come to a pretty pass, sir."

Campbell, in his life of Thurlow, tells a good story of Wilkes, who had his keen eye fixed on Thurlow whilst the latter was indulging in a fine rhetorical flourish in the House, which he brought to a close thus :—"And when I forget my King, may my God forget me." Wilkes, who could stand it no longer, growled out—"Forget

thee!—forget thee! He will see you d——d first." But this speech was probably not intended for Thurlow's ears. Thurlow was not very courteous himself, and on one occasion when he parted from the barristers in Circuit without saying a word to them, one of them remarked that "Thurlow might at least have dammed us." It was said of Thurlow that he must be an impostor, as no man could possibly *be* as wise as he *looked*. Thurlow had not a bad opinion of himself, but your readers may have heard of the clergyman who was dilating to a woman of his congregation on his own "exceeding sinfulness," till at last the woman looked him steadily in the face and asked him whether he really meant what he said? He assured her it was only too true, that he was the "greatest of sinners." "Well sir," said the woman, "then my husband was right after all, for he says when you first comed into this parish, says he, mark my words, my dear, *that's a rale bad 'un*." The waiter who, when ordered to prepare dinner for a party of clergymen, rushed back into the room in hot haste to inquire whether they "were High Church or Low Church gents?" assigned as a reason for his enquiry, that "if the gents were High Church" the wines must be good, and if "Low Church, the wittles must be good," existed probably only in the inventive imagination of some clerical wag. But we may safely lay it down as a rule that men of all creeds in an endowed and Established Church were not entirely above "creature comforts," and that the philosophy and religion of most people only enable them to hear with resignation and serenity the privations of others.

As this letter is an *omnium gatherum*, the reader will probably not object to winding up with a quotation from a paper in a late number of the *Saturday Review*, entitled I think "Colloquial Fallacies:"—

"A small class of men of a polemic turn of mind mistake disputation and argument for talk. They do not care for any conversation which does not somehow or other develop an issue, a position which is open to more than one view. A good talk to them is pretty nearly synonymous with a hot and close argumentation. They are like those mythical Americans who go through the world as roaring lions, seeking free fights. People, in their view, only meet for the sharp encounter of native wits. The quiet, easy flow of talk is a tame, dull waste of precious time that ought to have been spent in assertion and replication, in rejoinder and rebutter and surrebutter, in quick

clenching and rapid refutation. A couple of people of this disputatious temper may prove as outragious a nuisance as the most pompous conversational autocrat that ever lived and talked. It is highly proper to be anxious for truth. If you hear anybody say the thing that is not, or that in your opinion is not, and if you have a short and decisive confutation easily within reach, then it is well to lay on, and not to spare. But a sustained duel is a sheer vexation to calm overlookers. Instead of trusting that right may win, they sigh in vain for the descent of some just angel who should inflict upon the disputants the fate of the Kilkenny cats. As De Quincy says, in speaking of Dr. Parr's rudeness in this direction, 'mere good sense is sufficient, without any experience at all of high life, to point out the intolerable absurdity of allowing two angry champions to lock up and sequestrate, as it were, the whole social enjoyment of a large party, and compel them to sit, 'in sad civility,' witnesses of a contest which can interest the majority neither by its final object nor its management.' Now and then, it is true, one meets a fool so hollow and so pretentious that it is impossible to resist the temptation of having a throw with him. But even in such a case as this the execution ought to be swift and certain. If you can impose absolute silence on your fool, it may be worth while to spend a little time and trouble in despatching him. But if he be one of those lively fools who can skip to and fro with the celerity and heartiness of that ignoble but tormenting insect which can leap a hundred times the length of its own body, which is no sooner expelled from one corner than he has entrenched himself in another, then it is much the better plan to leave him to disport at his ease. And though an encounter between a blockhead and a philosopher may, under certain conditions, be amusing and useful, an encounter between two philosophers in society is a distinct absurdity."

The above quotation does not apply to *impertinence* used amidst the many conversational rules which are laid down on all hands for our guidance. We must beware, all of us, that we do not become so timid as to leave the whole of the talk in the hands of the *Saturday Review* and of a few infallible guides.

And now, in conclusion, I will venture to express an opinion, in which I believe the majority of your readers will coincide, namely, that however slight and sketchy this letter may be, it is far more likely to attract and amuse the public than all the lucubrations, however able, of your numerous writers on earth, or moon, or sun, or circle, or square. Lest, however, I should divert from each other and to myself, the heavy cross firing of all these gentlemen, and

thus have to sustain a combined attack from the whole of that ponderous and irresistible phalanx, I will at once concede, in the hope that the admission will meet their views, and spare them all further trouble in the matter,

That I am, Sir,
Yours truly,
IMPERTINENCE.

FROM "THE CORRESPONDENT," FEBRUARY 17TH, 1866.

THE DISTANCE OF THE SUN.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

I must beg permission to take one more turn at the grindstone, before I have done with "Terra." He says that I "assert" that my figures agree to one second. Now I have done more, for at page 34 of the *Correspondent*, I proved that the sun's apparent mean semi-diameter is, by my figures, 16′ 0″; and that by the "Nautical Almanac" it is there shown to be 16′ 1″. I think that "Terra's" nose will be very sharp to overthrow this; and I do not think I shall have to grind any more.

As to Mr. Reddie, he uses the English language so strongly that it is hard to understand him; but he writes like a gentleman; and I have no dislike to the task of correcting his errors. I always thought that "recast" signified "re-adjusted," but it seems that it means, "once more overthrown." Good; let it be so; and then we understand that "astronomy recast" signifies that "astronomy is once more overthrown." This feat being performed by Mr. J. Reddie, no wonder that "Astronomy in a Nutshell" should be overthrown along with it.

Let not Mr. Reddie suppose I am "offended," for I am just now laughing very heartily; but I will not be so rude as to say

what at. However, I do maintain that the sun is ever moving to "the west," and this, *nonobstant clameur de* Mr. J. Reddie. For unless Mr. Reddie deny that there is any such point as "the west"—independent of this earth—I contend that, in astronomy, we have a custom to treat the centre of the earth as the assumed point of observation, and that all computations are made therefrom ; and that, consequently, the north, south, east, and west points, are all equally established. This, too, is quite independent of the mariner's compass ; and these points would, and did, exist without that instrument. The point to which I show that the sun is ever moving is situated in 12 hours of right ascension, or in 180° of celestial longitude. And if Mr. Reddie gain by the admission, he may say that the compass points to it, as "west," only once in the 24 hours, but I am not such a child as to be ignorant of this fact, which does not affect the question. For if we regard the sun, from the centre of the earth, at the time of the summer solstice, we find it in a line with the north exactly ; and then, if we look at it when 90° removed from that position, we shall find it in 180° of longitude, and removed to "the west" of its previous position. I will not fill your page with further argument on this matter, as it must be plain to all, even to Mr. Reddie. But I may quote the words of the editor of the *Athenæum* in reviewing my "Solar System as it is, &c." "If the sun moves, the system moves with it, *in the manner* Mr. Morrison describes." I believe that Mr. Reddie denies that the sun moves at all. Very well ; let him *prove* this, and I will be one of the first to do him honour. But then it must be done more logically than by what he calls an *exposé*, which I fear will only tend to expose himself. Lastly, Sir, I have to enlighten Mr. Reddie as to the question of falling bodies. He seems to think, and I fear he does think, that there is no other force but that of projectiles. For he says that, "when a body which has been projected horizontally, falls, it always moves in a *curve*." Thank ye for nothing, Mr. Reddie. But did you ever hear of the force of traction, which is quite different from that of projection ? The latter is always an *inconstant* force, and when affected by what he calls gravity, it, of course, produces a curve. But the former may be *constant* and equal with that of gravity. This is evident ; for if a railway train travel a mile in a minute, or say 1,800 yards in that time, then a

body let fall from it will have a *force forward* of $\frac{1800}{60}$ = 30 yards in a second, or 90 feet; whereas a body falls only about 15 feet in the first second; so that we may reduce the rate of travelling to bring the ratio of the falling body and that of the body moving by traction to an equality. In such case, the body *will form the diagonal of a square*, which Mr. Reddie ought to know, as it was the very thing mentioned in my work. I am happy to agree with Mr. Reddie in one thing, which is the condemnation he gives of the system followed by astronomers as to the parallax of the fixed stars. But in this, too, I believe that I preceded him, as he will find in a work published by me long before his work, viz., in my "Path of Encke's Comet," published in 1858. I agree also in entirely declining any argument with a person who descends to use such language and such ridiculous arguments as are made use of by Mr. E. L. Garbett.

Yours faithfully,

R. J. MORRISON.

FROM "THE CORRESPONDENT," FEBRUARY 24TH, 1866.

"PERTINENCE."

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

I am a constant reader of what Professor De Morgan terms "the most panoramic of all the periodicals." I am an observer and a thinker, and *think* I can comprehend what I observe; and I venture to trouble you with *some* of my observations and thoughts on *some* of the communications which have appeared in your valuable journal.

In your last Saturday's number, Captain Morrison observes:—"At page 34 of the *Correspondent*, I proved that the sun's apparent mean semi-diameter is, by my figures, 16′ 0″; and that by the

'Nautical Almanac' it is there shown to be 16′ 1″. *I think* that 'Terra's nose will be very sharp to overthrow this; and I do not *think* I shall have to grind any more." This reminds me of the following doggrel lines in the last number of your old series, by the "*round man in the square hole*:"

"When you want to establish a point,
Begin by assuming it true;
For if you are wrong it's no matter,
And if you are right it will do."

(By-the-bye, it strikes me, from certain *Athenæum* recollections, that "*the round man in the square hole*" connects Professor De Morgan with these doggrels; or, if the composer of them be not the learned Professor himself, he is, at my rate, as great a wag.)

So, on this principle, Captain Morrison begins by assuming that the "Nautical Almanac" is true and infallible (which the Captain's opponent, "Terra," disputes), and then jumps to a conclusion founded upon this assumption that *he has made a discovery.* Captain Morrison cannot fail to perceive that he must first prove the infallibility of the "Nautical Almanac" before he can establish an infallible conclusion founded upon its assertions, as to moon's parallax and sun's semi-diameter. If Captain Morrison be a really sincere controversialist and earnest enquirer after scientific truth, why does he not answer the following question, very fairly put by "Terra?" "Will the gallant Captain, then, be good enough to inform us (your readers) what are the 'many valuable discoveries' for which he gives Mr. Smith credit?" This is fairly due from him, in justice to Mr. Smith. It appears to me that until the gallant Captain answers the question, he may grind again and again before he brings "Terra's" nose to the "grindstone."

Mr. E. L. Garbett, by his short letter which appears in your last number, has clearly got Mr. Reddie in a "*fix*," for Mr. Reddie will find himself incompetent to prove that recognised mathematical authorities, in any period of history, have ever done more than come to what they believe to be a closer approximation to the true value of π than that arrived at by Archimedes upwards of two thousand years ago. But Mr. Garbett concludes his letter by observing:—"*There would be more writing than there is time for, if all quacks who rodomontade in newspapers had to be exposed.*" This is no answer to

Mr. Smith, who very fairly puts the question in your last number:—"Has your contributor, Mr. E. L. Garbett, retired from the controversy? If not, how happens it that we have no reply from him to my letter which appeared in your 40th number?" It appears to me, that Mr. Garbett is about as clever as Mr. Alex. Edw. Miller in "*wriggling*" out of a difficulty, and that Mr. Smith, somehow or other, has the happy knack of contriving to get the last word with all his opponents.

Professor De Morgan, by his short letters in your 31st and 34th numbers, made a bold attempt to "*wriggle*" out of a difficulty; but Mr. Smith contrived to get the last word, and has succeeded in clearly establishing his charge of untruthfulness against the learned Professor. Even if Mr. Augustus De Morgan has ceased to be a reader of the "*panoramic periodical*," yet (if a certain contributor in your last number be correct in his notion), he will be sure to hear of this rebuff from some "*good-natured friend or another.*"

A contributor to your last week's number, styling himself "*Impertinence*," ventures to express the following opinion of his own communication:—"*However slight and sketchy this letter may be, it is far more likely to attract and amuse the public than all the lucubrations, however able, of your numerous writers on earth, or moon, or sun, or circle, or square.*" I dare say—indeed, there is not much doubt—that "*Impertinence*" is right. Nevertheless, you have a class of readers, and that a numerous one, who take a deep interest in these subjects, and who would like to see some of our leading authorities take them up, and silence such writers as Mr. Smith, by getting the last word. Until they do so it leaves a lurking suspicion in the minds of many of your readers that there may be, after all, something in Mr. Smith's theory. But "*Impertinence*" calls his letter an "*omnium gatherum*"—and so it is; and towards the close of it he remarks—"The reader will probably not object to winding up with a quotation from a paper in a late number of the *Saturday Review* (No. 532, the same date as the first number of your new series), entitled, *I think*, 'Colloquial Fallacies.'" *I think*, forsooth! Does "*Impertinence*" imagine that he can bamboozle any of your intelligent readers to *think* that he and the writer of "Colloquial Fallacies" are not identical? If so, I suspect he is egregiously mistaken. His communication is not uncomplimentary to the "*pan-*

oramic periodical." It is spicy in its style, and will afford a certain degree of amusement to many readers; but although coming from such a quarter, it will be taken for exactly what it is worth, and no more.

19th February, 1866. PERTINENCE.

FROM "THE CORRESPONDENT," FEBRUARY 24TH, 1866.

THE VALUE OF π.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

In his letter to the *Correspondent* of last Saturday, Mr. Smith remarks:—

"Your contributor 'Neptune' concludes *his* letter with the following observation:—'For assuredly a man might as leave attempt to discover the philosopher's stone as, without their (Dr. Rutherford and William Shanks) aid, to determine the value of π.' That is to say, in the opinion of 'Neptune' it would be folly to make an attempt, without the aid of Dr. Rutherford and William Shanks, to determine the value of π. And yet 'Neptune' calls the writer (who repudiates the aid of both in ascertaining the value of π) 'the High Priest of Quadrature,' and says:—'I am very willing to sit at his feet and be taught.' Would that public writers (as your contributor 'Terra' thinks they should) 'make it one branch of their study to be consistent with themselves!'"

"Your contributor, Mr. J. Renshaw, of Sherwood-rise, near Nottingham, cannot be a geometrical star of the first magnitude. Does Mr. Renshaw know that the edge of his circular iron plate can no more be compared with the edge of the circumference of a geometrical circle (a geometrical line having length but no breadth), than the point of the finest needle with the point of a wasp's sting? Does Mr. Renshaw know that a piece of thin brass wire (no matter how thin) describes a multilateral polygon when wound round the edge of any circular disc, whether of 50 or any other number of inches diameter. If not, he should do. Consequently, if a piece of thin brass wire be wound once round Mr. Renshaw's truly circular

iron plate, it would be longer than the mathematical circumference of the plate. How true it is, as Professor De Morgan says:—'A person of small knowledge is in danger of trying to make his *little* do the work of more.'"

And I think that even Professor De Morgan will be disposed to admit that in the present instance, at least, Mr. Smith has decidedly the best of the argument.

"Neptune" is evidently like one of those ultra-Episcopalians who push their idolatry by Episcopal authority to such extremes that, like vaulting ambition, they o'erlap themselves and fall over on the other side—in plain English, snap their fingers at the whole bench of Bishops whenever they happen to differ in opinion with them. Let "Neptune" leave the quadrature alone, and if he must attack some one, let him attack "Terra."

"Terram quætit Neptunus."

As to Mr. Renshaw, Mr. Smith has decidedly given him his quietus for the present.

The term "thin" is thoroughly vague and unmathematical.

Wire which was considered as extremely "thin" at Liverpool, might be regarded in the light of a "brass rod" at Nottingham, or *vice versâ.*

Who is to decide how much of the length would be lost in an attempt to turn a brass rod round the "circular iron plate?"

The experiment proposed by Mr. Renshaw may be perfectly satisfactory to those who, like Mr. Renshaw and Professor De Morgan, will maintain that $\pi = 3{\cdot}14159$, &c.; but I must confess that, after reading Mr. Smith's letter, I cannot help feeling that it would fail to bring conviction to anyone who was satisfied that $\pi = 3{\cdot}125$.

Mr. Renshaw has given the thickness of his iron plate (50 inches). He should give the *diameter* of the brass wire, and *prove* mathematically to Mr. Smith that only a certain fraction of an inch is lost in length in consequence of its describing (as Mr. Smith says) "a multilateral polygon when wound round the edge of a circular disc." Mr. Renshaw would no doubt say, "Experto crede," but Mr. Smith would reply, "Inexperto non credam."

I am, Sir,

Yours truly,

FAIRPLAY.

FROM 'THE CORRESPONDENT," FEBRUARY 17TH, 1866.

QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

As Mr. Reddie wishes to write "as unlike E. L. Garbett as possible," I will, if you please, give him an opportunity. In his last letter, p. 68, he says, "For years, it is known to those who know the history of mathematics, that $\pi = 3{\cdot}125$ was accepted as correct. . . ."

Now I ask you, Sir, to lay before him, and your readers, three alternatives, one of which he should be required to take:

1. To produce a record that at any period in all history, either "for years," for days, or for one day, "$\pi = 3{\cdot}125$ was accepted" or taught in any one place in the world, civilized or barbarous.
2. Or to retract the above words; or
3. To hold his tongue.

Mr. Reddie will find he is on a dangerous tack. Though I did name him before he named me, he may be assured he would have fared better had he answered instead of railing, or even had he answered as well as railing. There would be more writing than there is time for, if all quacks who rhodomontade in newspapers had to be exposed.

I am, Sir,

Your obedient Servant,

E. L. GARBETT.

Feb. 13, 1866.

FROM "THE CORRESPONDENT," FEBRUARY 24TH, 1866.

THE MEASURE OF TWO MEN.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

I am bound to thank Captain Morrison for tasking himself to correct my errors! As regards the effect of "Victoria Toto Cœlo" upon "Astronomy in a Nutshell," he is right. I can only say, "*Bis vincit qui* se *vincit in* Victoria," and then hold my tongue—in *toto cœlo* quies! As I have set Captain Morrison a "laughing very heartily," while I myself am "in a merry key," surely the public ought to be "amused," if not "attracted," by our Letters. Although not given to paradoxes, I will venture upon one:—If it were *necessary* for me to say more to your readers about the sun's motion in space "*to the west*," it would be *unnecessary!* To the Captain *or* me, in this matter, the proverb must apply, *per multum risum, poteris cognoscere stultum!* And so, as regards a *falling* body "moving by *traction!*" I would as soon think of replying to the wonderful discovery that when the sun is 90° from the meridian of Greenwich it is in 180° of longtitude!" But there is one thing I must reply to. Captain Morrison says he is "happy to agree with Mr. Reddie in one thing, which is the condemnation he gives of the system followed by astronomers as to the parallax of the fixed stars." I shall be sorry to render Captain Morrison *un*happy, but even in this "one thing" he does *not* agree with me. I do *not* condemn "*the system* followed by astronomers as to the parallax of the fixed stars." I adopt their "system," and show that they have made a gross blunder in working it. The point at issue between the "astronomers" and me is definite, and lies "in a nutshell," but *not* in Captain Morrison's. Anything whatever that he has discovered or proved, *veluti in speculum*, I am certain I do not even understand; and, in this sense, I haul down to the gallant captain, and hope he will consider that I am, *per fas* aut *nefas*, beaten!

As regards Mr. E. L. Garbett, he appears to have come down a peg; but still the old ignoble leaven lurks in his style, and crops out

in his concluding words. Suppose, ***in the meantime***, "I hold my tongue," his third polite alternative. Then we shall see what ***he*** can tell us of the nearest value of π before Van Ceulen showed it to be "$=3{\cdot}14159$." If he can't tell us he must be a "quack." I have never thought of replying to his "rhodomontade" in No. 41. (See Mr. Garbett's *Letter*, *page* lxvii.)

Yours faithfully,

Hammersmith, 19*th February*, 1866. J. REDDIE.

FROM "THE CORRESPONDENT," FEBRUARY 24TH, 1866.

THE QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

In my letter on Bennett's quadrature there is a misprint of 2^2 for r^2, which has possibly prevented Mr. Smith from seeing its correctness. Briefly, the proof is as follows:—The area of a circle involves or depends upon π. If π be $\frac{25}{8}$, the area of a circle of radius 4 will be 50. But Bennett's square, equal in area to a circle of radius 4, has also an area = 50. Hence the agreement between Bennett and Smith, and the consequent priority of Bennett.

As Mr. Smith does not seem inclined, notwithstanding the great urgency, to calculate the trigonometrical tables, I shall at once proceed to make known the results at which I have arrived by a careful study of Mr. Smith's writings, and proper application of the Rule of Three—

Sin. $30° = {\cdot}5$; sin. $24° = {\cdot}4$; sin. $18° = {\cdot}3$; sin. $12° = {\cdot}2$; sin. $6° = {\cdot}1$.

I have no doubt these will please Mr. Smith, and if any of his enemies should object to them, let them please to remember that there are such things as paradoxes, and in a week or two I shall

favour you with a list of them, as also a new proof of the value of π, after which Mr. Smith's opponents will find it useless to say another word.

Your obedient servant,

D. J. C.

FROM "THE CORRESPONDENT," FEB. 24TH, 1866.

THE QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

In reply to Mr. James Smith's strictures on my former letter, contained in your paper of the 17th inst., I beg to say that I never set myself up as a geometrical star of the first magnitude. I readily admit that the experiment I recommended Mr. Smith to try had a certain amount of roughness about it, albeit it was a sufficiently accurate test to satisfy Mr. Smith that his value of π was *incorrect.* From what I have seen of Mr. Smith's arguments and replies to his opponents, it would have been useless for me to attempt to convince him by means that have failed in the hands of far abler men than myself. Finally, as Hospur or Diapheuthis would say, "I recommend Mr. Smith to part with his old screw 3·125, and set up the correct thing marked $\pi = 3·14159$, &c."

I am, Sir,

Yours very respectfully

February 19*th*, 1866. S. A. RENSHAW.

FROM "THE CORRESPONDENT," MARCH 3RD, 1866.

THE QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

Your contributor "D. J. C.," in this day's number of the "*panoramic periodical*," has called the attention of your readers to the absurdity of my criticism on his Letter which appeared in your 41st

number, and I admit that it is a fair question for discussion, whether I or Mr. E. L. Garbett is best qualified for the command of the army of "*natural-born blunderers.*" "D. J. C." might have handled me with as much severity as "X." has handled "E. L. G."; but, instead of doing so, he generously observes :—"In my Letter on Bennett's quadrature there is a misprint of 2^2 for r^2, which has possibly prevented Mr. Smith from seeing its correctness." This is the observation of a "*natural-born noble,*" for I cannot disguise from myself the fact, that the first impression of "D. J. C." would necessarily be that the writer must be a stupid fellow. It now appears to me that my remarks on his letter were very ungracious, and (knowing that he wrote under the impression that he had a "*good opportunity of seconding Mr. Smith*") I cannot help feeling that he must have thought them so, and I now beg frankly and sincerely to tender that gentleman my most humble apology.

If you, Sir, were to ask me how I came to make such an exhibition of myself, I could not explain the reason; all I can say is, it affords a proof of the hobbles into which "*natural-born blunderers*" may get by their rashness and stupidity.

Well, then, I admit that John Bennett discovered one method of geometrically producing a square exactly equivalent in area to a given circle, and I cannot say he may not have discovered more than one, for I never heard of him until your contributor "D.J.C." introduced his name to my notice, and I have not yet seen "The Millwrights' and Engineers' Pocket Director," or his larger work, "Geometrical Illustrations." Now, assuming that John Bennett only discovered this one method of producing a square equivalent to a given circle, let us work out this discovery to its legitimate conclusion.

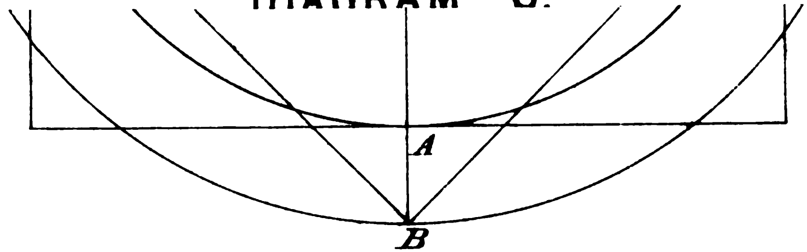
Let X denote a circle. Draw two diameters at right angles to each other. Let O denote the centre of the circle, and the lines O A radii. Produce these radii to points denoted by B, so that the lines O B shall be equal to $\frac{5}{4}$ (O A), and join the extremities of the lines O B, producing the square B B B B. Then, the square B B B B is exactly equal in superficial area to the circle X, and stands upon it. Our recognised mathematical authorities may fumble with their $\pi = 3 \cdot 14159265$, &c., which Professor De Morgan *thinks* "*comes in at every window, and down every chimney*, calling itself circumference *to a unity of diameter*," until (to adopt the language of the same

elegant writer) their "*toe-nails drop off, and their noses bleed blue ink*," but they will never succeed in controverting this geometrical fact. "*Chacun à son gout*," says your contributor "X," and I am a little afraid that this may not be exactly to his taste, or nicely suited to the columns of the *Correspondent*, but I must ask him to pardon us, and plead as my excuse that I am strictly within the limits of the example furnished by *Athenæum* critics.

Now, about the circle X circumscribe a square, making the sides of the square parallel to the diameters of the circle. With O as centre and O B as interval, describe a circle Y. The circumference of the circle Y will cut the sides of the circumscribing square to the circle X at 8 points. Take three of these points, adjacent to each other, say C, D, and E, and join O C, O D, and C E. On C O describe a square C O E F and draw the diagonal F O. The diagonal F O will be equal to the line C E. The square C O E F will be equal to half the square B B B B, and will therefore be equal to half the circle X in superficial area. If on F O a square be described, this square will be exactly equal to John Bennett's geometrical square B B B B, and both squares will be equal to the circle X in superficial area. I wish I could give a diagram of the geometrical figure thus formed, but any of your geometrical readers can readily construct one, and, if they do, will at once perceive that they may construct two squares within the circle Y, having all their angles touching the sides of the circumscribing square to the circle X, and these squares are equal to the square B B B B. We thus obtain three equal squares from a given circle, all of which are inscribed squares to the circle Y, and all equal in superficial area to the generating circle X.

Now, in constructing a diagram of this geometrical figure (*See Diagram* C) we produce two right-angled triangles O A C and O A D, of which the sides that contain the right angle are in the ratio or proportion of 4 to 3; therefore, the sum of the areas of squares on the sides of either of these triangles is equal to the area of the square B B B B, or the area of a square on F O. But, O A a radius of the generating circle X is the longer of the two sides that contain the right angle in the triangles O A C and O A D, and is common to both triangles, and if $OA = 4$, then $3{\cdot}125$ times $OA^2 = OA^2 + AC^2 +$

DIAGRAM C.



OC^2, or, $OA^2 + AD^2 + OD^2 = 50$. But, C E is the hypothenuse of a right-angled triangle C G E, of which C G the longer of the sides that contain the right angle = the sum of O A and A C, or the sum of O A and A D; and the shorter side G E = the difference of O A and A C, or the difference of O A and A D; and when O A = 4, C G = 7, and G E = 1, therefore, $CG^2 + GE^2 = 7^2 + 1^2 = 49 + 1 = 50 = CE^2$, therefore, the area of a square on C E is equal to the sum of the areas of squares on the sides of either of the triangles O A C or O A D: and $CG^2 - GE^2 = 7^2 - 1^2 = 49 - 1 = 48$ = area of an inscribed regular dodecagon or 12-sided polygon to the generating circle X.* But, if A O = 4, then, O B = 5, therefore, 3·125 times $5^2 = 78{\cdot}125$ = area of the circle Y = 100 times $\frac{3{\cdot}125}{4}$ = 100 times the area of a circle of diameter unity. All this will be plain and intelligible to your correspondent "D. J. C.," and every Geometer will see, that we might make the circle Y the generating circle, and by the same process go on *ad infinitum*.†

And now I am about to give "D. J. C." a piece of advice, and I can assure him I do so in a spirit of friendly sincerity. He observes:—"As Mr. Smith does not seem inclined, notwithstanding the great urgency, to calculate the trigonometrical tables, I shall at once proceed to make known the result at which I have arrived, by a careful study of Mr. Smith's writings, and a proper application of the rule of three:—Sin. 30° = ·5 : Sin. 24° = ·4 : Sin. 18° = ·3 : Sin. 12° = ·2 : Sin. 6° = ·1. I have no doubt these will please Mr. Smith; and if any of his enemies should object to them, let them please to remember that there are such things as paradoxes, and in a week or two I shall favour you with a list of them, as also a new

* The difference of the squares of any two numbers is equal to the product of their sum and difference. This fact is of as much importance in practical Geometry, as in Algebra. J. S.

† The square B B B B is an inscribed square to the circle Y, and from the foregoing truths we learn, that if n denote the area of the square B B B B, $\left\{(n + \frac{1}{4}n) + \frac{1}{4}(n + \frac{1}{4}n)\right\}$ = area of the circle Y: and conversely, if n denote the area of the circle Y, $\left\{(n - \frac{1}{5}n) - \frac{1}{5}(n - \frac{1}{5}n)\right\}$ = area of the square B B B B. I drew the attention of Professor de Morgan to these facts in a Letter addressed to him in June, 1864, and he has never attempted to disprove them, but he has never had the candour to admit them. J. S.

proof of the value of π,* after which Mr. Smith's opponents will find it useless to say another word."

Now, I should like "D. J. C." to thoroughly understand me. I do not dispute that Tables of what are called Sines and Cosines of angles, profess to give the arithmetical values of the Logarithmic Sines and Cosines of angles. What I dispute is, that (with certain exceptions†) they give the true arithmetical values of the trigonometrical Sines and Cosines of angles. To make my meaning perfectly intelligible, take ·4, which "D. J. C." thinks is the Sine of an angle of 24°. The Logarithm of ·4 (omitting the index) is ·6020600. By Tables, the nearest Log-sin. corresponding to the Logarithm of the natural number ·4 is 9·6021495, and is given as the Log.sin. of an angle of 23° 35′. The natural sine of an angle of 23° 35′ is given in Tables as ·4000825, but the Logarithm of the natural number ·4000825 is not 9·6020600 but 9·6021495. Now, I do dispute that ·4000825 is the arithmetical value of the trigonometrical sine of an angle of 23° 35′, and maintain that the arithmetical value of the trigonometrical sine of that angle is less than ·4. If "D. J. C." were right, by parity of reasoning, the sine of an angle of 36° would be ·6. If the sine of an angle be ·6, the angle is an angle of 36° 52′. I would advise "D. J. C." to give this matter a careful consideration, and not jump too hastily to the conclusion, that the sines of angles of 24°, 18°, 12°, and 6°, are what he has given as the sines of these angles.††

*The *Correspondent* was not defunct for eight weeks after D. J. C.'s Letter in number 43, but his list of paradoxes and "*new proof of the value of π*" never appeared in that Journal. J. S.

† The sine of any arc is half the chord of twice that arc. Hence: If the radius of a circle be unity, half the diameter is the sine of a right angle = 90°: half the side of an inscribed equilateral triangle to the circle is the sine of 60°: half the side of an inscribed square is the sine of 45°: and half the side of an inscribed regular hexagon or six sided polygon is the sine of 30.° J. S.

†† It is self-evident from a mere inspection of the diagram C, that OEBC is a quadrant of the circle Y, and it follows, that the quadrant O E B C is equal in superficial area to an inscribed circle to the square C O E F. The angle C in the right-angled triangle C G E is an angle of 8° 8′, and its trigonometrical sine is $\sqrt{\cdot 02} = \cdot 1414213\ldots$ Hence: The angle C in the right-angled triangle C G E is equal to half the angle at the apex in the isosceles triangle O D E. J. S.

In my letter of the 12th inst., I remarked "D. J. C." "*need be under no alarm as to a discussion among Mathematicians on the priority of the discovery.*" I said so for the following reason :—"*I do not claim to be the first discoverer of the true ratio of diameter to circumference in a circle.* The discovery was made by a French well-sinker, by name Joseph Lacomme, about the same time that John Bennett published the 'Millwrights' and Engineers' Pocket Director.' All I can say is, I never heard of either Bennett or Lacomme until long after I had made the discovery, and I have probably gone much deeper into the subject than either of them."

The following may not be without interest to some of your readers :—

In the year 1836, at which time Lacomme could neither read nor write, he had constructed a circular reservoir and wished to know the quantity of stone that would be required to pave the bottom, and for this purpose called on a Professor of Mathematics. On putting his question and giving the diameter, he was surprised at getting the following answer from the Professor—"*Qu'il lui etait impossible de le lui dire au juste, attendu que personne n'avait encore pu trouver d'une manière exacte le rapport de la circonférence au diamètere.*" From this he was led to attempt the solution of the problem. His first process was purely mechanical, and he was so far convinced he had made the discovery that he took to educating himself, and became an expert arithmetician, and then found that arithmetical results agreed with his mechanical experiments. He appears to have eked out a bare existence for many years by teaching arithmetic, all the time struggling to get a hearing from some of the learned societies, but without success. In the year 1855 he found his way to Paris, where, as if by an accident, he made the acquaintance of a young gentlemen, son of M. Winter, a commissioner of police, and taught him his peculiar methods of calculation. The young man was so enchanted that he strongly recommmended Lacomme to his father, and subsequently through M. Winter he obtained an introduction to the President of the Society of Arts and Sciences of Paris. A committee of the society was appointed to examine and report upon his discovery, and the society at its *séance* of 17th March, 1856, awarded a silver medal of the first class to M. Joseph Lacomme for his discovery of the true ratio of diameter to

circumference in a circle. He subsequently received three other medals from other societies. While writing this I have his likeness before me with his medals on his breast, which stands as a frontispiece to a short biography of this extraordinary man, for which I am indebted to the gentleman who did me the honour to publish a French translation of the pamphlet I distributed at the meeting of "The British Association for the Advancement of Science" at Oxford in 1860.

I am, Sir,

Yours very respectfully,

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
24th February, 1866.

FROM "THE CORRESPONDENT," MARCH 3RD, 1866.

THE QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

Suppose a man were to write an essay on some complicated question—let us say, for example, on "Population"—what should we think of him if he confessed that he had never read the writings of Malthus, Mill, and other political economists on the subject? It is surely somewhat presumptuous to attempt to handle that which had been treated by a host of eminent writers, without having previously made oneself familiar with their arguments. Otherwise, how is plagiarism to be avoided? How do we know that we are not going over old ground, and wasting time and labour which must issue in a futile result?

Yet this seems to be the position of Mr. Smith, who confesses that he repudiates the aid of Dr. Rutherford and Mr. Shanks in ascertaining the value of π, and admits by implication that he has

never read their works! No wonder that our greatest Mathematicians smile somewhat derisively at his arguments, and at the confidence with which they are propounded!

I cannot plead guilty to the charge of "inconsistency" for having said that I was willing to sit at Mr. Smith's feet and be taught, provided he established his claims to the character of a competent teacher—which he has not yet done. At present he seems to me to resemble (in, I must admit, a very kindly and genial way) the snail in the Hindoo proverb, which "sees only its own shell, and imagines it to be the finest palace in the universe."

I am, Sir,

Yours obediently,

NEPTUNE.

February 28th, 1866.

FROM "THE CORRESPONDENT," MARCH 3, 1866.

"QUACK, QUACK" THE LAST.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

I should never dream of continuing a controversy with the "high priest of quadrature." I sent you a brief note to that effect immediately on his reply to me in No. 40, and had it appeared in No. 41 it would have saved "Pertinence" a serious mistake, injurious both to Mr. James Smith and myself.* The substance of that note was, that if, as *Mr. S. asserted*, my figures ·261... for the chord of 15° were correct, then either his own figures ·260... for the arc of 15° is incorrect, or the curve was shorter than the straight line. I had asked which of these inferences we were to draw,—a humble question surely, and no argument—but Mr. Smith says thereon, "I admit the correctness of Mr. Garbett's figures, but I dispute the argument he founds upon them!" What argument?

* The Editor must have had a reason for not inserting this Note. Was it too scurrilous for insertion? J. S.

Calculations follow that bring him to the conclusion "the 47th proposition of Euclid is inapplicable (directly) to the measurement of a circle." What on earth has that to do with my application of it to the calculation of a *straight line*, which figures, moreover, he has just admitted "the correctness" of? No one has applied it, in the *Correspondent*, directly or indirectly, to measure any *circle*. I give the length of a straight line, he says with "correctness," and merely ask whether a twenty-fourth of a circle between the same points is *longer* or *shorter*; and Mr. Smith has hitherto refused to answer.*

He then gives a whole column, which you print, of answer to the private Letters of a Cornish gentleman, who, I presume, had he wished to be answered in the *Correspondent*, would have written to the *Correspondent*. I cannot see why you printed that column, nor why I should read it,—Smithian "mathematics" with a long numeral sum or two to verify in each line, being far from easy reading, however light; though Mr. Smith be, when not cyphering, a most amusing writer. If the Cornish gentleman "takes the same line of argument as" mine, but "in a slightly different form," pray let the form that your readers have seen be answered, rather than that which we never saw!

But the climax is reached, and Mr. Smith once more "excels himself," when, having "chawed up" the poor Cornishman to his content on p. 56, he turns to tell me I am "checkmated" unless I can "harmonise" the value, $\pi = 3{\cdot}14159265$, &c. (*figures I had never used or mentioned*,) with Mr. Smith's treatment of another man's argument *that I have never seen!* † How can I be checkmated before Mr. Smith has made his move? And above all, why am I to defend either the Cornish Letters or $\pi = 3{\cdot}14159$? What value for π did I give? How can he show I hold it, even now, to be more than $3{\cdot}13$?

Now a word about "wriggling out of difficulties." Mr. A. E. Miller has attempted nothing of the sort. His last words, I find, are:—"If I have written anything of which any one who shows that he understands the point of an *argument* desires an explanation, I shall be ready to give one to the best of my ability, but I decline to enter on a course of *wrangling* with Mr. James Smith." Now,

* This assertion is untrue! † Untrue!

what is the difficulty wriggled out of? If "Pertinence" will state it, Mr. Miller professes himself ready to explain it. Well, in that note which I sent to you for No. 41, I made a rather more liberal offer—it was that if *any one*, I did not limit it to "one who shows that he understands the point of an argument,"—but if any other reader than Mr. Smith himself was puzzled by any part of that gentleman's letters or calculations since January began, and would point out what he wished reconciled with the "orthodox π," I would read that part and do my best to clear it up. If you received that letter and choose to print it along with this I will abide by it, otherwise I would certainly rather not be bound by such a promise. It were perfectly absurd to expect that he, or any one, can afford the time to unravel paradoxes by which we have no reason to suppose anyone deceived but their author, as fast as that author, having nothing else to do, pours them forth! Let "Pertinence" indicate any particular figure-juggle that puzzles *more* heads than Mr. Smith's own, and we are ready to attempt its explanation—Mr. A. E. Miller if it be in last year's *Correspondent*, or myself if in this year's. Where is the difficulty, then, that we "wriggle out" of?

I see no "knack" required, in man, or child, or parrot, to "get the last" of such words as Mr. Smith's "with all his opponents." By the way, if Mr. Renshaw's unlucky wire can only make a polygon, how if we put the polygon *within* the circle? Let some Liverpool owner of an eight-foot fly-wheel, 8 feet within the projecting rim, invite Mr. Smith to line that rim with 25 feet of hoop-iron, or gild its inside with 100 three inch gold leaves, and make it stick everywhere and both ends meet before he goes to dinner. This is not meant to "get the last word," but for fun.

Now "Pertinence" will surely see that my remark on the impossibility of exposing "all quacks" could have no reference to answering Mr. Smith. No one who first imports a strong word into the discussion applies it, like the parrots who so wittily bandy it about in your last number with absolutely no meaning at all; but it happens that in this case I employed it not even so loosely as the dictionaries would warrant. They define it as "a boastful pretender to skill or knowledge which he does not possess." So that, by the way, if I were unable to tell Mr. Reddie what was the "nearest value of π before Van Ceulen showed it" (which he never did) "to

be 3·14159," I should not therefore be a quack, for I never pretended ability to tell him ; while he, according to the dictionary, is one—if for no other reason, for pretending to know, while he did not know, the meaning of the work *quack*. But I have said I do not apply it so widely ; I hold it essential to the quack *proper*, that the skill or knowledge pretended shall be such as he *knows* himself not to possess. Now, doubtless an enemy of Mr. Smith might say, he boasts of both skill and knowledge which he does not possess ; but of none (I say) which he does not believe that he *knows* himself to possess ; and for this reason, friend " Pertinence " will see, I could not use the word with any reference to that quarter, such as he implies in saying my use of it was " no answer to Mr. Smith."

With very few words I shall have done with my other two critics. An apology, perhaps, is necessary, not for interfering where I was " not wanted," but for my singular and irregular motives to do so. The fact is, I greatly resemble Mr. Casely, and was as little wanted by anyone in the *Correspondent* as that gentleman was in any of the three shops in Cornhill ; and as he declares that he did not want the goods he thought proper to take from two of them, so my burglary, effected in No. 37, was exclusively directed against the safe of Professor De Morgan, which, however, being opened and rifled and with the undeniably lawful tools of the Professor's own "*Doctrine of Probabilities*," I could not escape a glimpse, quite unsought, of the amusing stock-in-trade of Messrs. James Smith, Zadkiel, Reddie, &c., which showed me that unless I relieved their safes, too, of a little, I might run a risk, not only, like Mr. Casely, of being laughed at, but worse—of being taken in a manner for one of them. To show any reader of the " *Ecclesiastical*," part No. 37, that I am by no means one of them, was the sole motive of my raid among your men of science, and is now, I trust, accomplished.

I should be sorry were there any such result as " X." may mean by a " bloody nose." The only results I can perceive are broken safes, and their owners unable to mend them, all crying out (with the sole exception of Mr. De Morgan, who knows better than to speak in such circumstances) more and more foolishly. Mr. Reddie, deaf to my friendly warning on p. 84, rushes on and exposes more quackery of somebody's at every step. Not only, it now seems, was the benighted world once content with a " *mince* " value of π, with

one that every child was able to disprove by just putting, like Mr. Renshaw, a tape round a wheel; not only, moreover, was this mince π Mr. Smith's very $3\frac{1}{8}$ till "the Dutchman" improved upon it—but this feat of "the Dutchman" was a *discovery!* And yet "the Dutchman" is not, as we might have supposed last week, that Dutchman who *did* make a real discovery about π—whose $\frac{335}{113}$ is at once one of the most remarkable and most useful things ever discovered in figures or likely to be,—but he is (as the "*tombstone*" already led me to suspect) that wretched Van-Thingamy of the school-books, who thought to preserve his stupid name by wasting years in ascertaining, as any one might have done, who was idle enough, 36 useless figures. (It is to be hoped our Rutherford and Shanks are something better than a couple of Van Ceulen's, though their 500 new figures are of still *less* possible use). Moreover, more quackery yet, the corrector of our astronomers not only imagines these figures were something to discover, but he of the tombstone first "showed" even the six of them, 3·14159; and *these*, too, were something to discover. As if, ever since cyphering without LXV., &c., was introduced, any school-boy, who has advanced to the rule of square root, and Euclid's prop. 47, could not always "*discover*" these and more, for himself in an hour!

"X." I am constrained to admit, has "shown up" my extreme want of penetration. He is most certainly not Mr. De Morgan, by many infallible proofs. If an *amicus curiæ* were wanted, "X." is certainly the remotest from the character it was ever my lot to come across. Passing over what does not concern me—for it cannot matter a millionth of a split hair what such an *Amicus* thinks of any of us "bound" or not bound to do—we are told. "Perhaps Mr. R. might also fairly plead there was an attempt to misrepresent him." If Mr. R. does, we shall certainly not refer the matter to "X." A little above I find:—"He (Mr. R.) said that an accurate base line was essential to an accurate calculation of parallax." Now, I do not choose to ascertain from "page 43, top," whether he said so, or anyone said so. There can be no "misrepresentation," I suppose if I simply represent that *whoever* said or says so, if anyone does—be his name R., X., A., or Z.—is or was both a quack, in the dictionary sense, for pretending to knowledge about "parallax," of which he

knew nothing, not even the exact meaning of the term; and also a "quack proper," in my most restricted sense for pretending to know what is essential to finding parallax, when he not only knew none of the ways it is found, but *knew* that he knew not any. "X." helps us to a very necessary word for qualifying such quackery. It must indeed be "*inveterate*"—such in short as might lead a man to fancy our Astronomers had so applied annual parallax to the estimation of star distances as to have their base line altered by our progress in space—a "gross blunder" indeed.

I remain, Sir,

Your faithful servant,

February 26th, 1866. EWD. L. GARBETT.

FROM "THE CORRESPONDENT," MARCH 3RD, 1866.

THE DISTANCE OF THE SUN.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

I regret to have to occupy your pages with Mr. Smith, who fought for some time under the signature of "Terra," but who now fights under that of "Pertinence." Why these changes? He says that I "assume that the Nautical Almanac is true and infallible." Well, if so, it is because that it agrees with observation. It is to this test we must come at last, and as we know that observation—daily all through the year—proves the Nautical Almanac to be true and infallible as far as regards the apparent semi-diameter of the sun, we know that it condemns "Terra" and all his supporters. There ought to be an end of their fruitless nonsense.

As to Mr. Reddie he still runs his head against the idea of the meridian of Greenwich. I again invite him to drop this silly notion when discussing the true motion of the sun through space. For I

do assure him that the sun would move all the same, even if the earth were not to turn round on its axis. The only way to conceive clearly (which Mr. Reddie really does not) of the sun's motion is to conceive the eye of the observer to be situated in the centre of the earth, and then, when the sun completes 90 degrees of longitude, it will be found that he is to the *north* of the observer. And, after the sun has moved to 180 degrees of longitude, the said motion being to the left hand, he will have moved to "the west." This is true in spite of Mr. Reddie and his extensive knowledge of the Latin language. And he ought to be thankful for being taught the fact, which fact depends partly on the motion of the earth serpentining about the sun, and forming once a-year, from midsummer to midsummer, a cycloid curve, or nearly such. I am quite sure that Mr. Reddie does not "understand" the system that I have taught, not being able to perceive that the turning of the earth on its axis, and the consequent motion of meridians, have nothing to do with the *direction* in which the sun moves. If he were not profoundly ignorant of these simple astronomical truths he would cease to darken counsel without knowledge. Let him remember that this theory of mine has enabled me to give very plain, but very infallible rules for the working of all the great problems of the solar system. Having defeated his argument on the subject of falling bodies, which he mistakenly applied to the laws of projectiles, instead of traction, he ought to be honest enough to acknowledge the fact. But he is clearly too great a man for this.—

Allow me to correct an error of the press in my last letter on this subject. At page 82, for "Path of Tucke's Cornet" read "Course of Encke's Comet." This correction is due to Mr. Reddie, who may otherwise be at a loss to discover the work.

I will, with your permission, here give the passage there alluded to, which will save Mr. Reddie and your readers the trouble of a reference. At page 22 of the above work, published in 1858, are these words, "In short, the discovery that the sun moves through space at the rate of 100,000 miles per hour, and takes the earth with it through 875,696,000 miles in a year, will not only overthrow the darling error of astronomers that the earth and planets, comets, &c., move in ellipses, but *it wholly destroys* the inferences drawn from observations *on the parallax of fixed stars.* These have been made upon the as-

sumption that the earth moves through a lineal measure of space equal to 190 millions of miles in a year, whereas it moves five times as far! Alas! for all the presumed proofs that such and such fixed stars are at such and such distances! They must be enormously farther off. The universe must, indeed, be altogether past our finding out."

I think, Sir, that this passage gives me the right to claim priority of discovery of the very point in astronomy of which Mr. Reddie so complacently assumes to be the originator.

Yours, &c.,

R. J. MORRISON.

FROM "THE CORRESPONDENT," MARCH 10TH, 1866.

THE DISTANCE OF THE SUN.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

In this day's *Correspondent* there is a communication from Captain Morrison, in which he assumes that the Letters of James Smith, Terra, and Pertinence, are written by the same hand. There is also a Letter, in the same number, from the pen of Mr. E. L. Garbett, in which he speaks of "Pertinence" having made a serious mistake, injurious both to James Smith and himself. It is, therefore, obvious that Captain Morrison and Mr. E. L. Garbett differ in opinion. I must leave your readers to draw their own conclusion as to whether either or neither is right.

Now, it so happens that a copy of the pamphlet I distributed at the last Oxford Meeting of "The British Association for the Advancement of Science" fell into the hands of Captain Morrison, and he was the first Mathematician and Astronomer who paid me the compliment of saying, in that work I had demonstrated that $\frac{1}{3 \cdot 125}$

expresses the true ratio of diameter to circumference in a circle. This occurred some five years ago. I have his Letter in my possession, and can produce it if necessary.

Now, Mr. Airy, the Astronomer-Royal of England, makes the equatorial diameter of the earth, in miles, 7925·648, and the polar diameter, in miles, 7899·170. (See "Encyclopedia Metropolitana"—Art., "Figure of the Earth.") In other words, the Astronomer-Royal makes 24·857 miles the earth's mean circumference, which, divided by 3·14159, the Mathematician's and Astronomer's value of π, makes 7912·4 miles to be the earth's diameter approximately. If the Astronomer-Royal be right, I am obviously wrong, for my value of π would make the earth's diameter greater; and it may be admitted that the Astronomer-Royal's conclusion, as to the earth's mean circumference, is derived from actual measurement.

Now, Captain Morrison assumes the Astronomer-Royal to be right as to the earth's mean diameter, and, founded upon this assumption, boldly asserts that *he has made a discovery* of the relations existing between the true dimensions and distances of the sun, moon, and earth. This may be true, but, if so, let the gallant Captain avow his error, withdraw his compliments, and declare to the public, through your columns, that he is no longer of opinion that Mr. James Smith has made any discovery at all. But then, what will become of the following assertion in his Letter, which appears in your 40th number:—"I have defended Mr. James Smith because he has made many valuable discoveries"?

I would not have exposed the inconsistency of the gallant Captain if he had not forced it upon me.

I am, Sir,

Yours very respectfully,

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
3rd March, 1866.

FROM "THE CORRESPONDENT, MARCH 10TH, 1866.

QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

I am the Cornish man referred to by Mr. Smith, and again by Mr. Garbett, in your last number. My suggestion was, that it would facilitate a settlement of the controversy if the following easy problem were thrown out for solution among your numerous readers, and it should (for our present purpose) be solved without using any Tables of sines or their logarithms :—An isosceles triangle has each of its equal sides, unity (1). Its vertical angle is 15°. Find the base. This base will obviously be the chord of an arc of 15°, and $\frac{\pi}{12}$ will be the corresponding arc ; we shall then be able to compare their magnitude. The work should be shewn at length. It may be assumed that sin. 30° = $\frac{1}{2}$, from this, sin. 15° or cos. 15° (and if employed in the calculation, sin. 7° 30′) must be found. Our Tables of sines are correct, but as this has been disputed, we save controversy by not employing them. Any one moderately versed in trigonometry can solve the above problem, and its result on the value of π will be inevitable and conclusive.

I am, Sir,
Your obedient servant,
G. B. G.

LAUNCESTON, CORNWALL,
5th March, 1866.

FROM "THE CORRESPONDENT," MARCH 10TH, 1866.

SOLAR MOTION IN SPACE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

1. Some of your contributors are so long-winded upon very small points, that it ceases to be a joke to reply to them. Between them, Mr. Garbett and Captain Morrison, occupy four of your columns—Mr. Garbett's share being three out of the four! I ask but a small space to say all I can in answer to such Letters; and even these few lines I shall write for such as know least of the points in dispute, lest they should be taken in by "Quack, Quack, the Last" of Mr. Garbett. It is something to have elicited, after all his bounce, that he is unable to tell us what was considered the nearest value of π before $\pi = 3{\cdot}14159\ldots$ was generally received. And as "school-books" are all wrong (*ipse dixit!*), and Dutchmen all "wretched," while Rutherford and Shanks are no better, and nobody but "*Mr.* Casely" is = *Mr.* Garbett, I shall now leave π to his manipulations, any allusion to its value being merely an episode in *my* Letters. It is something new, perhaps, to have a burglar engaged to force "open a fire," and circle-squarers ought now to "begin to sing!"

2. As regards Mr. Casely-Garbett's flat denial that "an accurate base line is essential to an accurate calculation of parallax," I can only refer to my own words and arguments on quoting Professor Kelland, and to the following quotation from Professor Airy's "Six Lectures on Astronomy" (4th ed., p. 89), viz.:—"This is what we mean by *parallax*, that it is the difference of direction of an object as seen in two different places." As regards Mr. Garbett's barefaced contradiction of this,—as I will not condescend to write like him, and yet his effrontery ought to be reproved,—I shall have recourse to a periphrasis, to negative his words. Thus:—*A base line* — *n* = Mr. Garbett's statement.

3. I will now interpolate a remark for the sake of your general readers, bearing upon the subject of this discussion,—which must

not be counted as having reference to Messrs. Garbett and Morrison. I have already done with the former, and have but little to say to the latter.

4. In Professor Airy's interesting and lucid "Six Lectures" (p. 125), he refers to a figure (No. 54) of the earth's circular path round the sun, and explains how, by viewing a fixed star at "opposite sides of the earth's orbit,"—that is, at six months interval, —we find "the 'parallax,' or distance of the star,"—"the *only* way in which we can measure its distance, being one strictly analogous to that used for measuring the distance of the moon, with this difference, that we cannot observe from two points at once."* Hence, Professor Kelland described the length of the base line from which a given star is viewed at six months interval as = 190 million miles,—the earth's distance from the sun, when he wrote, being considered = 95 millions. Now I have pointed out *that that is* ONLY *true if the sun is* AT REST; and all the quacks of "Quacks," or cleverness and brass of literary Casely's, will *not* alter the truth of the matter. Hence, any person who is not a fool can see how the calculations of parallax are affected by "solar motion in space," and it by them. Unless, also, the distances of the fixed stars are known, "their proper motion" or, conversely, "the rate of the sun's motion in space," cannot even be guessed at. Even when supposed to be known, the guessing is very wild. I have noticed some of the various rates given by astronomers, ranging from 18 thousand to 600 million miles an hour! But now I wish to point out a more palpable absurdity that results from this "solar motion," take it at *any* rate, that *any* body likes to assume. By Professor Airy and "orthodox" astronomers generally, the direction of this solar motion—as first supposed by Sir W. Herschel in 1785—was considered to be towards λ *Herculis*—or I may say from the foot of *Orion* to the arms of *Hercules*—that is, nearly at right angles to the two signs of the zodiac *Virgo* and *Pisces*. Now, I ask all rational beings to bethink them, how it could be possible,—if the solar system had been moving merely since the time of Hipparchus, at

* In the case of the moon we *can*; for instance, simultaneously at the Cape and at Greenwich, the distance between these two observatories being our "base line" on which all depends.

the slowest rate of motion assigned to it, namely, at 18,000 miles an hour, or 157 million miles a year, say only for 2,000 years,—that these two constellations should have continued to appear still in their old places in the zodiac?

5. To give a popular illustration. One can easily imagine that a person running in a circle round a fixed point, say in the centre of Hyde Park, would not observe much difference in the relative position of trees and houses and other objects all round about him. If the circle were very small, it might even be regarded "as a point," perhaps, as the earth's orbit has been with reference to the fixed stars and the zodiac. But if, instead of running thus round a *fixed* centre, he had to run round a "Deerfoot" while Deerfoot ran straight for the north, it must be manifest that our runner would soon reach the Regent Park, Primrose Hill, and Highgate, and leave the houses skirting Hyde Park (which here represent the zodiac) far behind.

6. To resume, and bid farewell to Captain Morrison. If he cannot understand that while the north and south astronomically are fixed points, and therefore possible directions towards which a body could move, "the west" is a *series* of circular directions, *ever varying*—I cannot help him. I don't *call* his a "silly notion" merely because it is not only unnecessary, but I intend to continue "to write in a gentlemanly manner, and as unlike E. L. Garbett as possible." It is enough to *prove* a point, without caring to use unmannerly epithets. Captain Morrison's imputation, however, as to my want of honesty in not acknowledging that he is right, in respect of what, to me, is his incomprehensible notion that a "*falling* body" *could* move under the influence of "*traction*," I must resent with the severity it deserves. I do so, by now saying, after refraining long—that to be so accused by the notorious "Zadkiel," I consider will not damage me even with those who know nothing of the point in dispute. What Captain Morrison, *alias* Zadkiel, can mean by claiming a priority of discovery as to a point, of which, *he says*, I assume to be the originator, I do not understand. *He* claims to prove "solar motion in space" at the rate of nearly 100 thousand miles an hour, and precisely to the west. *I* claim to have "discovered" (to use his word), or (as I express it) to have *proved* that *all* "solar motion in space" is an *absurdity*, and that such motion "to the west" is some-

thing indescribably worse, inasmuch as there is no "direction to the west," *as a point in space*, in existence, or even conceivable to a rational being who thinks what words mean. Whatever moves always "to the west" goes *round and round*, as the sun *seems* to do, —and *perhaps* (I am *not* afraid to say) *really does*.

7. It is enough for me, in spite of "Quacks" (dictionary meaning) of every kind, and all their quacks (or quacking), to be able to say that a few months after I put forward my unanswerable objections to solar motion, the Astronomer-Royal, in the Annual Report of the Astronomical Society, admitted the whole question to be "in doubt and abeyance." I know this doesn't alter Captain Morrison's notions. I never thought it would. And while he demeans himself as "Zadkiel," I only hope it won't.

8. Captain Morrison introduces two new points. 1st. His conclusion that, *if* the sun moves in space as he imagines, the fixed stars must be enormously farther off. Rejecting his premises or postulate, I reject also his conclusion. 2nd. That, *if* the sun moves, the earth's path round it is a kind of cycloid curve. Now, *if* there were no laws of motion or of dynamics, I could accept this conclusion, *if* I could accept the premises. But I *reject both*. And this brings me to one of my other and strongest arguments against the *possibility* of "solar motion" in space. To simplify matters, let us take the rate of solar motion as = 65,000 miles an hour (a sort of mean between the 47,000 of "Y." in the *Times* and the nearly 100,000 of Captain M.); and in that case the earth's mean motion in its orbit being also 65,000 miles an hour, the earth's path would be a perfect cycloid. And what follows? Namely, that once a year the earth's orbital velocity would be 65,000 + 65,000 = 130,000 miles an hour, and six months afterwards 65,000 — 65,000 = 0! Every year, in fact, the earth would absolutely stop for an instant, as if to provoke the incredulity and laughter of "Zulu philosophers," when they advance from the study of the ancient Scriptures to that of "modern astronomy!" This result doesn't however, disturb Captain Morrison, because, as I freely admit, his dynamical ideas are both original and new. I happen to be "orthodox" in dynamics, and I don't, therefore, believe it possible that the dull earth *could* hop about in this kangaroo style. Still, the Captain is quite right that, *if* the sun moved, and the earth flew round it while moving, this incredible motion by fits and starts

would be the resultant feat which the earth must necessarily accomplish. Now don't let the Captain suppose I am claiming "priority of discovery" here! The difference between him and me all through is simply this: that some things which he professes to have "discovered" as *true*, and glories in, *I* have pointed to as *impossible*, *absurd*, and done my best to hold up to *well-deserved ridicule!* That he is not the author of all these scientific comicalities, I know. *Fallacia alia aliam trudit;* and it is entirely his own fault that I have ever mentioned his name at all in connection with them.

Your faithful servant,

J. REDDIE.

HAMMERSMITH, *March 2nd*, 1866.

FROM "THE CORRESPONDENT," MARCH 17TH, 1866.

THE QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

In your number of this day there is a Letter from the pen of my Cornish friend, G. B. G., with whom I have been in correspondence for nearly two months on this subject, during which short period I have received from him no fewer than 25 Letters. There can be no doubt that "*any one moderately versed in Trigonometry*" may readily solve his problem after his fashion. What I deny is, that "*its result on the value of π will be inevitable and conclusive.*" In our private correspondence I have been unsuccessful in convincing G. B. G. of this fact, and he has chosen to enter the lists with your contributors, and to your mathematical readers we must now refer the arbitration of our differences.

Let A B C denote a right-angled triangle, of which the sides A C and A B (that is, the hypothenuse and perpendicular) are in the ratio or proportion of 2 to 1, by construction. Produce the side C B to a point D, making C D = C A, and join A D. Then: A C D will be an isosceles triangle, of which the base A D will also be the hypothenuse,—"*or slant side*"—of a right-angled triangle A B D. Let

$CD = CA = 1$. Then: C will be an angle of $30°$, $\frac{AB}{AC} = \frac{1}{2} = \cdot 5$ will be the arithmetical value of the *trigonometrical* sine of the angle C, and in this case, the arithmetical values of the *natural* sine and *trigonometrical* sine are the same. I admit that the arithmetical value of A D—the base of the isosceles triangle A C D—as ascertained by Euclid: Prop. 47: Book 1: is $\cdot 5176380$; but I utterly deny that $\frac{\cdot 5176380}{2} = \cdot 2588190\ldots$ is the arithmetical value of the *trigonometrical* sine of half the angle C. In Logarithmetic Tables to 7 places of decimals the sine of an angle of $15°$ is given as $\cdot 2588190$.*

Again: Let D E F denote a right-angled triangle, of which the sides D F and D E (that is, the hypothenuse and perpendicular) are in the ratio or proportion of 5 to 3, by construction. Produce the

* If an equilateral triangle have one angle at the centre of a circle, this angle is subtended by a chord equal to the radius of the circle: and if a straight line be drawn bisecting the angle and its subtending chord, the equilateral triangle will be divided into two similar and equal right-angled triangles, of which the hypothenuse and shortest side will be in the ratio or proportion of 2 to 1. The acute angles in these triangles will be angles of $30°$ and $60°$, and the lesser of the acute angles is equal to half the greater of the acute angles; and it follows, that if a central angle contained by two radii of a circle, is not greater than $60°$, and the extremities of these radii be joined producing an isosceles triangle, a straight line can be drawn from an angle at the base of the isosceles triangle perpendicular to its opposite side, dividing the isosceles triangle into two right-angled triangles. The lesser of the acute angles in the smaller of these triangles is equal to half the angle at the apex of the isosceles triangle. Hence: If in a right-angled triangle the hypothenuse be to the perpendicular in the ratio of 2 to 1, the acute angles are angles of $60°$ and $30°$: if the hypothenuse be to the perpendicular in the ratio of 4 to 1, the acute angles are angles of $75°$ and $15°$: if the hypothenuse be to the perpendicular in the ratio of 8 to 1, the acûte angles are angles of $82° \, 30'$ and $7° \, 30'$: and if the hypothenuse be to the perpendicular in the ratio of 16 to 1, the acute angles are angles of $86° \, 15'$ and $3° \, 45'$. Let m and n denote the acute angles in a right-angled triangle. Having fixed $90°$ as the measure of a right angle, is not $90° - m = n$: and conversely, is not $90° - n = m$? Well, then, if in a right-angled triangle, one angle be an angle of $7° \, 30'$ the hypothenuse is to the perpendicular in the ratio or proportion of 8 to 1. Hence: $\frac{1}{8} = \cdot 125$ is the *trigonometrical* sine of the angle, and not $\frac{\cdot 261052}{2} = \cdot 130526$ as given in Tables to 6 places of decimals. (See E. L. Garbett's Letter from the *Correspondent*, page lix.)

J. S.

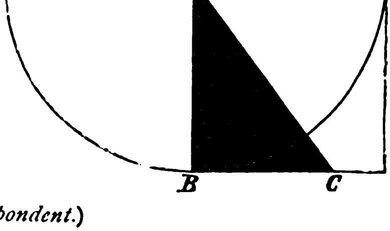
side F E to a point G, making F G = F D, and join D G. Then: D F G will be an isosceles triangle, of which the base D G will also be the hypothenuse—"*or slant side*"—of a right-angled triangle D E G, of which the sides D E and E G which contain the right angle, are in the ratio or proportion of 3 to 1, by construction. Let F G and F D = 5. Then: the angle F at the apex of the isosceles triangle D F G will be an angle of 36° 52′. $\frac{DE}{DF} = \frac{3}{5} = \cdot 6$ will be the arithmetical value of the *trigonometrical* sine of the angle F; and the arithmetical value of D G—the base of the isosceles triangle D F G—as ascertained by Euclid: Prop. 47: Book 1: will be $\sqrt{10}$. But, $\frac{\sqrt{10}}{10} = \frac{3 \cdot 162277}{10} = \cdot 3162277$; and ·3162277 is the arithmetical value of the *trigonometrical* sine of half the angle F. Again: Let F G and F D = 1. Then: D E = ·6, and E G = ·2 and D E G is a right-angled triangle; therefore, $(DE^2 + EG^2) = (\cdot 6^2 + \cdot 2^2) = (\cdot 36 + \cdot 04) = \cdot 40 = DG^2$; therefore, $DG = \sqrt{\cdot 40}$. But, $\frac{1}{2}(DG) = \frac{\sqrt{\cdot 40}}{2} = \sqrt{\cdot 10} = \cdot 3162277$ = the *trigonometrical* sine of half the angle F. Hence: the angle G D E = half the angle D F E = $\frac{36° \ 52'}{2}$ = 18° 26′. In Logarithmic Tables to 7 places of decimals, the sines of angles of 36° 52′ and 18° 26′ are erroneously given as ·5999549 and ·3162010. All my efforts have been unavailing to induce G. B. G. to examine the properties of a right-angled triangle, of which the hypothenuse and perpendicular are in the ratio or proportion of 5 to 3, by construction.*

Now, the area of a regular inscribed dodecagon or twelve-sided polygon to a circle of radius unity = 6 (radius × semi-radius) = 3; and a side of the dodecagon is a chord subtending an angle of 30°. In Hutton's Tables, (and they agree with other Tables to 7 places of decimals, barring clerical errors) the sine of an angle of 15° is given as ·2588190, and the versed sine as ·0340742. Hence: 12 (·2588190 × ·0340742) = ·1058286044376o. If to this be added the semi-perimeter of a regular inscribed hexagon to a circle of radius unity,

* Mr. Gibbons *assumes* the correctness of Tables, and by his "*reasoning and calculation*" makes D F E an angle of 36° 52′ + x, and G D E, an angle of 18° 26′ + y. (See his Letter of the 22nd July, 1869, page 4.) J. S.

which is equal to the perimeter of a regular inscribed hexagon to a circle of diameter unity, we obtain the arithmetical quantity 3·10582860443760, which—according to Tables—is approximately equal to 12 times the sine of an angle of $15°$ = 12 (·2588190) = 3·1058280, and is less than the arithmetical value of π. By Hutton's Tables the sine of an angle of $7° 30'$ is ·1305262. I do not dispute, that as ascertained by Euclid: Prop. 47: Book 1: (assuming the sine of an angle of $30°$ as a starting point,) that ·1305262 is the arithmetical value of half the chord subtending an angle of $15°$. Well, then, it cannot be denied, that 24 times ·1305262 = 3·1325288 is greater than 3·125. It is *asserted* by my opponents that this is fatal to the THEORY that 8 circumferences of a circle are exactly equal to 25 diameters: and this would be true if the champions of Orthodoxy could prove that a polygon of an infinite number of sides can be made to enclose as large an area as its circumscribing circle. This they *assume*, but cannot prove. If the reader be a sincere enquirer after scientific truth, I advise him to give my Letter, which appeared in the 40th number of the Correspondent (See page lx.), a careful perusal in connection with this.*

FIG. 1.



Any geometrical reader may readily construct the following figure. With A as centre and A B a straight line as interval, describe a circle, and on the line A B describe a square. With A B as perpendicular describe a right-angled triangle A B C of which the sides A B and B C shall contain the right angle, and be in the ratio of 4 to 3, by construction. (I could not give the diagram in the *Correspondent*.)

* 6 (Sin. of $30°$), 12 (Sin. of $15°$), 24 (Sin. of $7° 30'$), and 48 (Sin. o $3° 45'$) are equal, and all equal to the perimeter of a regular inscribed hexagon to a circle of diameter unity, or the area of a regular inscribed dodecagon to a circle of radius unity. J. S.

With reference to this geometrical figure, I said in a very recent Letter to "G. B. G." "Now, 3·125 times $AB^2 = (AB^2 + BC^2 + AC^2)$: and π times the area of a square on A B = area of the circle, whatever be the value of π. Now, by some very simple geometrical additions to this figure, we can produce another right-angled triangle D E B similar to the triangle A B C, having all its angles touching the circumference of the circle: and we can produce another circle which we may denote by Y, so that 3·125 times the area of the circle Y shall be equal to the sum of the areas of circles, of which D B, D E, and E B are radii.* I cannot help thinking, that if you give my two or three last Letters a careful perusal, you will have no difficulty in making the additions to the diagram, and that this will be more convincing than if I gave you the proof. You will find that it establishes 3·125 to be the true arithmetical value of π, and that it can be nothing else.

The following is a copy of my last Letter to "G. B. G."

BARKELEY HOUSE, SEAFORTH,
8th March, 1866.

DEAR SIR,

On my return home (from London) late last evening, I found your favours of the 5th and 6th instant awaiting me, and this afternoon I am in receipt of that of the 7th instant, which you commence by observing:—"I have quite exhausted all I can say on the subject of our Correspondence. I see no *connecting* link between a set of areas of squares and a set of circles, that can give the one by means of the other." Be this as it may, you have failed to convince me that there is not a *connection* and exact relations, between circles, rectangles, right-angled triangles, and equal-sided polygons. You again observe:—"I have quite come to the end of my ammunition. I have not another shot to fire." Pardon me, my Dear Sir, and think me not boastful when I say, I have not yet expended all my "*ammunition*," or fired my last "*shot*."

You say in your Letter of the 5th inst.:—"In all your Letters, the point I fail to see, is the equation or connexion between trian-

* This fact may be demonstrated by means of any hypothetical arithmetical value of π intermediate between 3 and 4, so that it be *finite* and *determinate*.

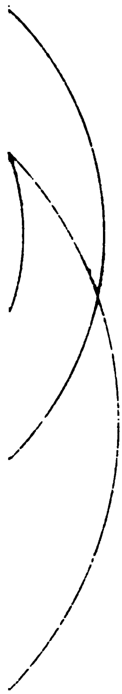
gles and curvilinear figures. I will take your last drawing at the end of your Letter just received (*See Diagram*, *Fig.* 1). If $\frac{AB}{BC} = \frac{4}{3}$, I quite agree with you, that $3{\cdot}125\ (AB^2) = (AB^2 + BC^2 + AC^2)$, as you put it. Also, $\pi\ (AB^2)$ = area of the circle. But how these can be so connected as to give a value of π surpasses my comprehension."

Now, my Dear Sir, I will shew you how they can be so connected as to give the true arithmetical value of π, and for the nonce, we will forget all about the logarithms of numbers, natural sines, and trigonometrical sines.

Well, then, take the drawing you refer to (Fig. 1), and let X′ denote the circle. Produce B A to meet and terminate in the circumference of the circle at the point D. Then, D B is a diameter of the circle X′. From the point D draw a straight line parallel to A C the hypothenuse of the triangle A B C, to meet the circumference of the circle X′ at the point E, and join E B. Then: By Euclid: Prop. 31: Book 3: D E B is a right-angled triangle. Produce D E to meet B C produced at the point F. Then: A B C and D B F are similar right-angled triangles, having the sides that contain the right angle in the ratio or proportion of 4 to 3. (This fact you have admitted in a previous Letter). With D B, D E, and E B as radii, describe the circles X, Y, and Z.* (*See Diagram* D). By Euclid: Prop. 8: Book 6: the triangles on each side of E B are similar to the whole triangle D B F and to each other.†

* It is self-evident, that with D as centre and D B as radius we might describe a circle equal in all respects to the circle X; and it is axiomatic, if not self-evident, that both these circles are equal to four times the circle X′ in superficial area.

† If a rectangle or right-angled parallelogram BD·BF be constructed on DB the diameter of the circle X′, the area of the rectangle $= 6\left(AB \times \frac{AB}{2}\right)$ and is therefore equal to the area of a regular inscribed dodecagon to the circle X′. Again: If B D and F B be produced to meet the circumference of the circle X at points G and H, and if B F be produced to meet the circumference of the circle X at a point L, and G H and G L be joined, then, G H L will be an inscribed equilateral triangle to the circle X, and a circumscribed equilateral triangle to the circle X′. It is *absurd* for any Mathematician either to *assert* or *insinuate*, that there are no definite and exact relations between the areas of rectilinear figures and curvilinear figures; and still more

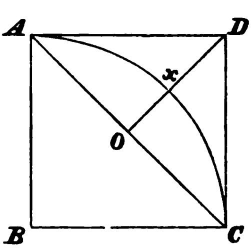


Now, you will admit that the area of a square on any line is equal to 4 times the area of a square on half that line, and it follows of necessity, that the area of a circle of any diameter is equal to four times the area of a circle of half that diameter ; therefore, the area of the circle X is equal to four times the area of the circle X′. Again : A B C and D E B are right-angled triangles, and they are similar triangles, by construction : and since the circumferences and areas of circles are to each other as their radii, it follows from Euclid : Prop. 47 : Book 1 : that the area of the circle X is equal to the sum of the areas of the circles Y and Z. Again: since from a given radius we can ascertain the area of a circle, (and it will be admitted that we can do so approximately even with the ratio of Metius "the Dutchman"), it follows of necessity, that if we work with the true ratio, we may ascertain the radius of a circle from a given circumference or area. Again: Since we know the ratios or proportions, between the sides of the triangles A B C and D E B, it

absurd to *assert* that we can find the circumference or area of a circle from a given radius, with an indeterminate value of π. E. L. Garbett had the temerity to make such an assertion. I quote the following from a Letter of his which appeared in the *Correspondent* of the 17th March, 1866. "It is due to Mr. Smith to tell him that he has been misinformed about Joseph Lacomme. Whatever that worthy man may have received a medal for, it will be found no Society (unless possibly a very ignorant club of mechanics) has given any person anything for discovering a ratio of circumference to diameter. If any Smith have arisen in France he had no Zadkiel, and he died, like the brave men before Agamemnon. And no Mathematical Professor gave Lacomme the answer Mr. S. transcribes, for such is the force of prejudice, that there is no schoolmaster nor surveyor in Europe who would not have professed to give the area to a *square barley corn* if required, were the reservoir as large as Liverpool, the earth, the solar system, or the Milky Way! In the first case they would even maintain they gave the circumference to a barley corn by only using Metius "the Dutchman" his ratio, 113 : 355. I fear the high priest of quadrature will find some stupid hoax has been practised upon him." In this quotation "*temerity*" abounds, but audacious "*Impertinence*" much more abounds! It is observed by Montucla (a well known *mathematical authority*) that if we suppose a circle whose diameter is a thousand million times the distance of the sun from the earth, the *approximative* measure of the circumference as computed by De Lagny, would differ from the *true* measure by a length less than the thousand millionth part of the thickness of a hair. Think of this with an *indeterminate* value of π! What next?

follows of necessity, that if we put an arithmetical value on one side of either triangle, we can find the values of all the sides in both triangles.

Let a, b, and c denote the areas of the circle X, Y, and Z: and let a represent the area of X the largest circle, and b the area of the intermediate circle Y: Then: $\frac{2}{3\cdot125}\,(a) = b$. But, $\frac{3\cdot125 - 2}{2} = \frac{1\cdot125}{2}$; and, $\frac{1\cdot125}{2}\,(b) = c$.*



* On the straight line AB describe the square ABCD. With B as centre and BA or BC as radius, describe the arc A x C, and draw AC a diagonal of the square A B C D. Bisect A C in O, and join O D. It is self-evident, that the quadrant B A x C is a quadrant of a circle of which B A and B C are radii: and it is axiomatic, that the area of the quadrant B A x C is exactly equal to the area of an inscribed circle to the square A B C D. Let A B = 2. Then: A B^2 = 2^2 = 4 = area of the square A B C D. But, the square A B C D is divided by the diagonal A C into two equal isosceles triangles B A C and D A C: and the triangle D A C is again divided by the line D O into two equal isosceles triangles D O A and D O C; therefore, the area of the triangle D A C = 2, and the area of the triangles D O A and D O C each = 1, therefore, 2 + 1 = 3 = the sum of the areas of the triangles D A C and D O A, or D A C and D O C. Let m denote the area of that part of the isosceles triangle D A C contained by A C a diagonal of the square A B C D, and A x C the arc of the quadrant B A x C: and let n denote the area of that part of the isosceles triangle D A C outside the arc of the quadrant B A x C. The sum of m and n = 2, and $\frac{2}{2}$ = 1. What are the arithmetical values of m and n? Answer: $\frac{9}{8}$ (1), or $\frac{9}{16}$ (2), or $\frac{9}{24}$ (3) = 1·125 = m: and $\frac{7}{8}$ (1), or $\frac{7}{16}$ (2), or $\frac{7}{24}$ (3) = ·875 = n. Again: Divide A B a side of the square A B C D into two parts A E and E B, so that A E shall be to E B in the ratio or proportion of 3 to 1, and on A E describe a square. With D as centre and D A or D C as radius, describe an arc A y C: The area of the square on A E will be equal to the area contained by the arcs A x C and A y C. Is not the square of any binomial = the sum of the squares of its two terms together with twice their product? Is not A E^2 + 2 (A E · E B) + E B^2 = A B^2? The object of Euclid was to make all his Theorems of general application, and consequently, Euclid could not give the two terms of a binomial in arithmetical ratio. J. S.

Let a = 600. Then: $\frac{2}{3{\cdot}125}(a) = \frac{2 \times 600}{3{\cdot}125} = 384 = b$; and $\frac{1{\cdot}125}{2}(b) = \frac{1{\cdot}125 \times 384}{2} = 216 = c$; therefore, $a = b + c$. Well, then, whatever be the value of π, we have two ways of finding the radius of a circle from its given area. First: By dividing the area of the circle by $\frac{1}{4}(\pi)$ and thus obtaining the area of its circumscribing square; and since the diameter of every circle is equal to a side of its circumscribing square, it follows of necessity, that half the side of the square = radius of the circle. Second: Because $\pi(r^2)$ = area in every circle, it follows of necessity, that $\sqrt{\frac{\text{Area}}{\pi}}$ = radius in every circle; therefore, $\sqrt{\left(\frac{8 \text{ times area}}{8 \text{ times } \pi}\right)}$ = radius in every circle.

Now, I shall *begin* by *assuming* $\pi = 3{\cdot}125$, and *end* by *proving* that it can be nothing else.

By hypothesis, $a = 600$; therefore, $a \div \frac{\pi}{4} = 600 \div \frac{3{\cdot}125}{4} = \frac{600}{{\cdot}78125} = 768$ = area of a circumscribing square to the circle X; therefore, $\frac{1}{2}(\sqrt{768}) = \frac{\sqrt{768}}{2} = \sqrt{768 \div 2^2} = \sqrt{\frac{768}{4}} = \sqrt{192}$ = radius of the circle X; or, $\sqrt{\frac{600}{3{\cdot}125}} = \sqrt{192}$ = radius of the circle X = D B. $\frac{4}{5}$ (D B) $= \frac{4}{5}(\sqrt{192}) = \sqrt{\frac{4^2}{5^2} \times 192} = \sqrt{\frac{16}{25} \times 192} = \sqrt{6{\cdot}4 \times 192} = \sqrt{\left(\frac{8\,b}{8\,\pi}\right)} = \sqrt{\left(\frac{8 \times 384}{25}\right)} = \sqrt{\frac{3072}{25}} = \sqrt{122{\cdot}88}$ = radius of the circle Y = D E. $\frac{3}{5}$(D B) $= \frac{3}{4}$(D E) $= \sqrt{\left(\frac{8\,c}{8\,\pi}\right)} = \sqrt{\left(\frac{8 \times 216}{25}\right)} = \sqrt{\left(\frac{1728}{25}\right)} = \sqrt{69{\cdot}12}$ = radius of the circle Z = E B; therefore, $\pi(\text{D B}^2) + \pi(\text{D E}^2) + \pi(\text{E B}^2) = 1200$ = the sum of the areas of the circles X, Y, and Z. Now, $\frac{1}{2}$(D B) $= \frac{\sqrt{192}}{2} = \sqrt{192 \div 2^2} = \sqrt{\frac{192}{4}} = \sqrt{48}$ = A B the radius of the circle X′; which is also the perpendicular of the

right-angled triangle A B C. $\frac{3}{4}$(A B) = $\frac{3}{4}(\sqrt{48})$ = $\sqrt{27}$ = B C, the base of the triangle A B C; and $\frac{5}{4}$(A B) = $\frac{5}{4}(\sqrt{48})$ = $\sqrt{75}$, = A C the hypothenuse of the triangle A B C; therefore, 4 (AB^2 + BC^2 + AC^2) = 4 (48 + 27 + 75) = 4 (150) = 600 = the given area of the circle X. But the area of the circle X = 4 times the area of the circle X′; therefore, $\frac{600}{4}$ = 150 = area of the circle X′, and it follows, that $\sqrt{\frac{150}{3\cdot125}} = \sqrt{48}$ = A B. But, (AB^2 + BC^2 + AC^2) = 3·125 (AB^2) = 150; therefore, the area of the circle X = four times the sum of the areas of squares on the sides of the right-angled triangle A B C, which makes 3·125 the arithmetical value of π, and makes 8 circumferences exactly equal to 25 diameters in every circle. It will occur to you that 9 (AB^2 + BC^2 + AC^2) = the sum of the areas of the four circles.*

Now, my dear Sir, to controvert this proof of π's arithmetical value, you will have to demonstrate that the area of the circle X is not exactly equal to 4 times the area of the circle X′, which is impossible, and I feel convinced, your candour will induce you to

*I quote the following from an Article which appeared in the *Liverpool Leader* of the 11th September, 1869. That Article was from the pen of that mathematical dictator, the Rev. W. Allen Whitworth, Fellow of St. John's College, Cambridge, and formerly Professor of Mathematics in Queen's College, Liverpool. "Verily, figures *will prove anything* when they are manipulated by persons to whom figures *may mean anything*. When people attach their own ideas to arithmetical symbols, or use them without any idea at all, and work with them by processes which they accept on the faith of arithmetical treatises misinterpreted by themselves, and submit them to rules which they do not understand—rules which were established for very different cases—then, indeed, figures may seem to prove anything. But when figures are fairly used, when their meaning is strictly defined, and when the meaning, once defined, is adhered to, when the work is not done in the dark, the intermediate steps unexplained, and the result simply interpreted by a juggle, then there is no reasoning so certain as that of arithmetic, and no results so conclusive as those which figures establish." Is not this amusing, coming as it does from a "*recognised Mathematician*" who makes 8 times the sum of the series $\frac{1}{1 \times 3} + \frac{1}{5 \times 7} + \frac{1}{9 \times 11} + \frac{1}{13 \times 15}$ + &c., = 3·14159265358979323846264338327950288? Mr. Whitworth must know this to be untrue, if he ever computed the arithmetical value of the series. ! !

J. S.

make the admission, that $\frac{1}{3{\cdot}125}$ expresses the true ratio of diameter to circumference in every circle, and can be nothing else.

I am, dear Sir,
Very truly yours,
J. S.

P.S.—If the Letter you have sent to the Editor appears in the *Correspondent*, I shall make this a part of my reply. I send you by this post the short biography of Lacomme. I must ask you to return it when you have given it a perusal, as it is the only copy I possess.

The foregoing is a specimen of the "*Seaforth mince* π," which Mr. E. L. Garbett and other of your mathematical contributors, will find it very hard to digest.*

I am, Sir,
Yours very respectfully,
JAMES SMITH.

FROM "THE CORRESPONDENT," MARCH 17TH, 1866.

"BASE LINES, *MINUS* N."

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

Mr. Reddie asks you (p. 118) for "but a small space to say *all he can* in answer to such Letters" as mine. 1st.—"It is something to have elicited, after all his bounce, that he is unable to

* Neither E. L. Garbett, nor any of the contributors to the *Correspondent* furnished any proof in the columns of that Journal, that they had attempted to *digest* this letter. I could not give the diagrams in the *Correspondent*, and have therefore had to slightly alter the phraseology of the Letter, but the "*reasoning and calculation*" are the same. J. S.

tell us what was considered the nearest value of π before 3·14159 was generally received." Pray may I ask where this has been elicited?

2nd.—"And as school-books are all wrong (*ipse dixit*)..." Who is *ipse?* And who has said in the *Correspondent* that *any* school-book was wrong?

3rd.—Same time, "and Dutchmen all wretched." Who has said so, and where?

4th.—Next time, "while Rutherford and Shanks are no better." Who has said *that* before, and where?

If "such as know least of the points in dispute,"—*i.e.*, those making their first acquaintance with the *Correspondent* in No. 45, are meant to infer some other person than Mr. Reddie has said these things, here is a man saying *all he can* with a vengeance! Unable, it seems, to get through *three* lines without bearing false witness against his neighbour *four* times!

Well, after discovering that "*nobody* but Mr. Casely" is = me, he comes down upon my flat denial that "an accurate base line is essential to an accurate calculation of parallax." Possibly there may be some other reader to whom it may be necessary to point out that the word *calculation* is spelt with several more letters than the word *use*, and it is not identical. Consequently, the "calculation of" parallax may be presumed to differ from the "use of," or the "calculation *from*" parallax, of something else (namely distance), as much as the ascertaining of a thing's breadth differs from the right use of that breadth to calculate therefrom something else—say its area or its cost. Whoever can see that the calculation of the cost, say of the Holborn Viaduct, depends on knowing accurately (among other things) its length and its breadth, which are quite independent of each other, may comprehend, I suppose, that the knowledge of the distance of any inaccessible body depends on two essentials—a parallax and a base line,—but that neither of these is more essential to the other than the viaduct's breadth is necessary to ascertain its length, or length to ascertain breadth. Parallax, as Mr. Reddie quotes for us, "is the difference of direction of an object as seen from two different places," and my "barefaced contradiction of this" —(where?)—is a "base line *minus n*." But where is it contradicted? Does a knowledge of the "difference of direction" depend

on your knowing how far the "two places" are apart? Whoever says the knowledge of parallax depends *at all* upon this, ignores what he is talking about, and most certainly contradicts the above definition! But now, observe—this man who "cannot condescend to write like" me, can condescend to give your new readers, in the *first three lines* of intended "base" for their inferences, four ridiculous—no, ridiculous only to *old* readers—four scurrilous and malicious "lines *minus n*," imputing to a man, falsely, utterances of utter folly, and then apply the above periphrasis to a supposed scientific mistake of mine, which if made would have injured nobody, but which never *was* made! Instead of "Brother, let me pull out the mote," he calls my supposed mote (which all the while had no existence even as a mote) a beam, and just as he has displayed *four* real malignant beams in only *three* eyes of his own! Oh, monstrous! *Three* "lines" of his own *minus* four *n*, and *one* of mine accused falsely of lacking *one n!* Verily, "one man may steal a horse, where another must not look over the hedge." Why, friend Reddie had better have condescended to write *ever so* like his opponent, if this is as little like him as possible. I only fear it may be not so antipodally opposite as he thinks it; but I am sure I hope this last opinion is right. Nothing better can, I hope, for myself, than that he may have been right there!

And now, with washed hands, I shall let this subject alone. No more on him; no! not if I should receive a whole *Correspondent* full of mud.

A letter of Mr. James Smith without one paradox, marks p. 117 as a phenomenon indeed! Depend on it, Mr. S. is growing serious, and sees that, instead of fiddling while ships are sinking, he is really bound to assist mankind with those little calculations that he has so plainly shown that no one else in England is competent to undertake. "G. B. G." writes from benighted Cornwall the antiquated suggestion that new Tables should simply be begun as the existing ones were, from the sine of $30^\circ = \frac{1}{2}$. What else has been done? Are there any figures in page 42 that I took from Tables? * All are derived straight from $\frac{1}{2}$. I did not give the calculation of the ·707107... and ·866025... because Mr. Smith had done so, and had told us (p. 23) that these three sines were three of the "few given correctly in our Tables." If radius be called 1, can "G. B. G." make the squares of

* See Mr. E. L. Garbett's Letter, page LIX.

any angle's sine and cosine amount together to more or less? If so, and "the sine of $30°$ may be assumed $= \frac{1}{2}$," whose square is $\cdot 25$, can the square of the cosine be more or less than $\cdot 75$, and the cosine itself anything but $\sqrt{\cdot 75} = \cdot 866025\ldots$? Or can the angle, whose sine and cosine are equal, have either of their squares other than $\cdot 5$, and is not the square root of $\cdot 5 = \cdot 707107$? Has "G. B. G." then asked for anything to be done which is not done on p. 42; and if "its result on the value of π" (namely, that π is *more than* 12 *times* $\cdot 261052$) be not "inevitable and conclusive," what can be so? (Unless my fly-wheel modification of Mr. Renshaw's experiment.) Does not "G. B. G." see that with these old-word notions we shall only blunder on, like the Newtons and Aireys, into all the "mysterious" errors of pre-Smithian mathematics?

It is due to Mr. Smith to tell him that he has been misinformed about Joseph Lacomme. Whatever that worthy man may have received a medal for, it will be found no society (unless possibly a very ignorant club of mechanics) has given any person anything for discovering a ratio of circumference to diameter. If any Smith have arisen in France he had no Zadkiel, and he died, like the brave men before Agamemnon. And no mathematical Professor gave Lacomme the answer Mr. S. transcribes, for such is the force of prejudice, that there is no schoolmaster nor surveyor in Europe who would not have professed to give the area to a *square barleycorn* if required, were the reservoir as large as Liverpool, the earth, the solar system, or the Milky Way! In the first case they would even maintain they gave the circumference to a barleycorn by only using Metius "the Dutchman" his ratio, 113 : 355. I fear the high priest of quadrature will find some stupid hoax has been practised on him.

I remain, Sir,

Yours, &c.,

E. L. GARBETT.

FROM "THE CORRESPONDENT," MARCH 17TH, 1866.

SOLAR MOTION IN SPACE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

I trust that this will be my very last appearance. At length Mr. Reddie has let the cat out of the bag. This escape of poor, really poor and half-starved pussey, is a most undignified affair. "Whatever moves always to the west goes round and round, as the sun seems to do, and perhaps (I am not afraid to say) really does." I told Mr. Reddie that he had a hobby, and I foretold that it would throw him. If this be so, then have all the astronometers been blind, and blind they are to this hour. The sun goes round the earth, eh? Well, Newton and a few others, down to Sir J. Herschel, declare that the earth goes round the sun. It is naught but Prescott redivivus; and so Mr. Reddie cannot even in his dream—for it is nothing else—claim any originality. But, poor as this cat is, she scratches and tries very hard to bite. I hope I have drawn its* teeth, however. If beaten, as he is on every point, he has recourse to calling names—as many people do when they are beaten—why did he not object at first to argue with me? He knew who I was when he began. There are very few of the 75,000 purchasers of Zadkiel's Almanack who will not enjoy a good laugh at him and his very wild cat. For if he does not understand astronomy, they do.

Sir, the sun does nothing more than seem to go round the earth if he have no motion in space. For otherwise, all the astronomers, from Kepler and Newton down to our days—the brightest phalanx that we find in all our history—have been nothing more than foolish old women, their observations have been only blunders, their computations only absurdities. But if he really has motion in space, then it is entirely and utterly impossible that either body can go round the other. Mr. Reddie and his feline friend will fight in vain to prove what I can only designate as a monstrous piece of folly. As far as my system goes, he can point to no "palpable absurdity"

* Query! her?

that results from "solar motion." Because I do not consider the sun's motion "as nearly at right angles" with Virgo and Pisces, but as nearly in a line with those signs, I contend that it is not in the line with right ascension = 260°, but is equal always to 180°. Thus, Mr. Reddie is again foiled in spite of his vicious little cat. But let us come to one of his "strongest arguments" against the possibility of "solar motion" in space. He takes the rate at 65,000 miles an hour, but he omits two elements in the question; these are, the sun's distance and the sun's declination. If we make the computation, as we must do, taking the sun's distance = 91,000,000 of miles, we get the hourly motion = 95,690 miles. This is the mean motion, and this is the only motion that can possibly be true and that can agree with those elements. Now this, if doubled, as Mr. Reddie would do, will be 191,380 miles per hour. But I find that during the 174·89 days from the autumnal to the vernal equinox, the linear motion of the earth is only 143,280 miles per hour. And from the vernal to the autumnal equinox the same motion is hourly = 51,968 miles. Thus, the earth does not "hop" about as he states, let the cat scratch ever so hard. And there is a wide difference between the facts in nature and the fancies in Mr. Reddie's dreams. His claim to be orthodox in "dynamics" is ill founded, for in all his book treating on the forces in nature he never mentions traction which he cannot deny to be a force in the face of the numerous railway engines in the world. Mr. Reddie speaks as though he were some authority. He is, at any rate, far from such, and is well deserving of "ridicule" in his notions—crazy notions I will call them—of the motion of the sun round the earth. I bid him good bye, and I hope for ever.

Yours, &c.,

R. J. MORRISON.

P.S.—I beg to decline the invitation of Mr. James Smith to join in the wild whirlpool of his discussion, or to put my hand into his π. Mr. Smith's "valuable discoveries" have been made in the course of his interminable discussion, but it does not follow that I should puff him by further allusion thereto. They are not so very valuable, perhaps, after all. He had better steer clear of astronomy. Let him be right, or let him be wrong, I admire his pluck.

FROM "THE CORRESPONDENT," MARCH 24TH, 1866.

THE QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

I received a letter from your contributor "G. B. G.,' in reply to mine which appears in the *Correspondent* of this day, which he commenced by observing:—" I have studied your Letter and diagram very attentively, and still I cannot find any connection between the area of a square and the area of a circle, unless you assume a value of π. Of course it is easy enough if you do that. It appears to me that all such diagrams can only construct a set of squares which shall have to each other the same ratio that a circle has to another circle, but we know by common arithmetic that this will not give the value of either circle's area, though the areas of the squares be known. $\frac{\text{A's debts}}{\text{B's debts}} = \frac{3}{4}$ will not tell us what A and B owe. I wish you would try and get out π by any legitimate reasoning."

When, after an exchange of many letters, I found "G. B. G." arriving at this conclusion, it appeared to me worse than useless to continue the correspondence, and I wrote him the following short Letter:—

BARKELEY HOUSE, SEAFORTH,
14th March, 1866.

DEAR SIR,

Your favour of the 12th inst., together with the short account of Lacomme, are to hand. Since you have arrived at the conclusion that the reasoning in my last Letter is illegitimate, it would be folly on my part to pursue the correspondence further. My last Letter will, I hope, appear in the next number of the *Corres-*

pondent, of which I will send you a copy, and it will be for you to judge whether it is worth answering through the columns of that Journal.

I am, Dear Sir,
Very truly yours,
J. S.

I had only just posted this when I received another Letter, dated 12th inst., in consequence of which, and after considerable thought and reflection, I was induced to write to "G. B. G." as follows :—

BARKELEY HOUSE, SEAFORTH,
17th March, 1866.

DEAR SIR,

You will be aware, from my Note of the 14th inst., that it was not my intention to have troubled you with another Letter on the question at issue between us, and my resolution was arrived at from the following remark in your favour of the 12th inst.:— "*I wish you would try and get out* π *by any legitimate reasoning.*" From this observation it appeared to me indisputable, that we are not agreed upon "*axioms or fundamentals;*" and that it would be utterly useless to carry our correspondence further. I have been induced to modify my determination from a subsequent Letter of yours of the same date, but bearing the Launceston postmark of the 13th inst., which you commence by observing:—"*I was not quite accurate in saying I had fired my last shot, I only meant as to the ammunition we had so far been using. I know of only two methods ever discovered for finding the value of* π. *The one by the perimeters or areas of polygons within and without a circle; the other by expressing an arc in the terms of its tangent or sine.*" Now, my dear Sir, I deny that we can discover the value of π by either of these methods. The reason is this. The area of a circle is greater than the arithmetical mean between the areas of its inscribed and circumscribed squares; and it is axiomatic if not self-evident, that the difference between the area of a square and the area of its circumscribing circle is greater than the difference between the area of the circle and the area of its circumscribing square consequently, to whatever extent you may carry your calculations

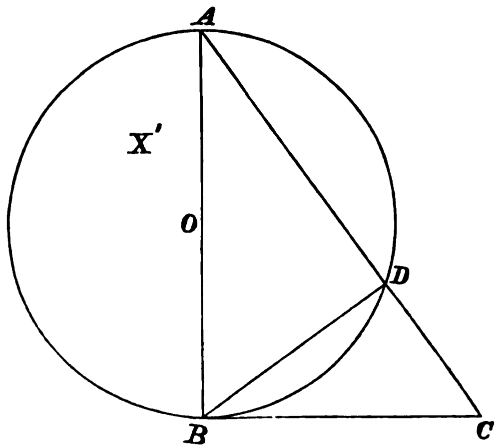
of the areas of the polygons "*within and without a circle,*" I deny the possibility of your finding the value of π by the areas of the polygons, even approximately, since the perimeters of the polygons, at every step in the operation, would enclose a larger area if in the form of the circumference of a circle. I do not deny that the arithmetical value of the logarithmic sine of an angle of $45°$ is $\sqrt{\cdot 5} = \cdot 7071068$, or that the tangent of this angle $= 1$, but for the same reasons it is utterly impossible to express an arc in terms either of its tangent or sine, and by a series of ascending powers find the value of π even approximately; and I maintain that both methods are equally fallacious.

Inscribe a polygon of 8 sides within a circle of radius 1. No one will dispute that the perimeter of the polygon is less than the circumference of the circle. If not exactly practicable, it is at any rate conceivable, that a circle of which the periphery shall be equal to the perimeter of the polygon, may be inscribed within the generating circle—that is, within a circle of radius 1. In this case, the angles of the polygon will be outside, and parts of its sides within, the derived circle. Thus, the diameter of the polygon will be greater than the diameter of the derived circle, and until we have established the value of π, it is obviously an absurdity to suppose that we can establish the ratio between the diameters of the two circles. "*Pardon me, my dear Sir,*" for I do not mean it offensively, but whether it is, or is not, within "*your comprehension,*" we are driven of necessity from these facts, to other methods of seeking for the value of π. I send you herewith two early numbers of the *Correspondent*, in which there are Letters of mine bearing on this point.

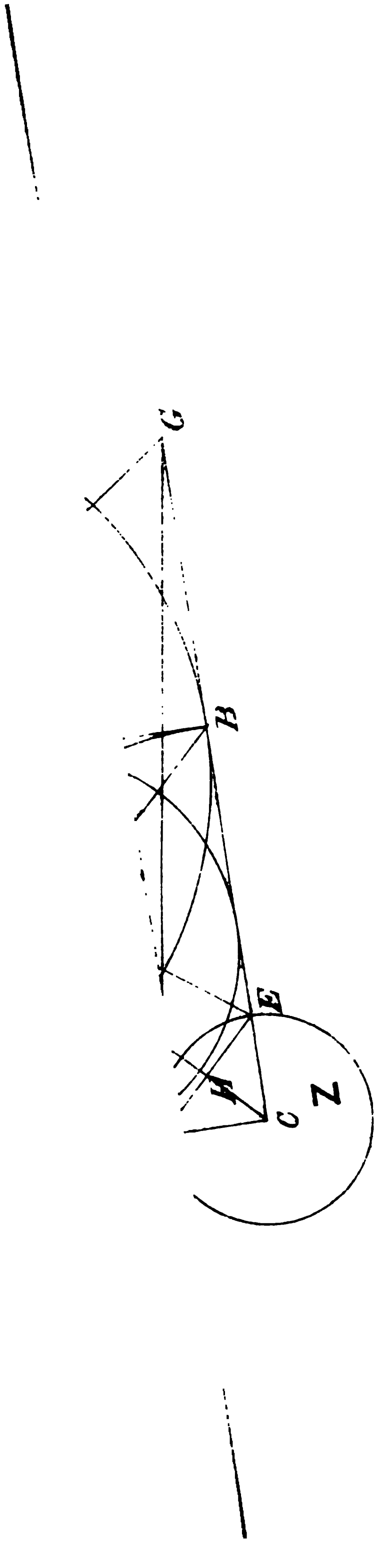
You will find my communication in this day's number of the *Correspondent* so treats the subject that I hardly think you can avoid noticing it; and I should be sorry to find you replying by such assertions and inferences as those contained in one of your Letters of the 12th inst., which, if you did, I cannot help feeling you would regret, on more mature reflection. In a kindly spirit you have been at much pains to convince me of what you suppose to be my error, for which I sincerely thank you, and I write to you again while there is a chance of preventing your doing that which might afterwards cause you pain; and I am not without hope that you may

yet be convinced of the truth of the theory, that 8 circumferences of a circle are exactly equal to 25 diameters, which makes $\frac{1}{3 \cdot 125}$ the true expression of the ratio of diameter to circumference in a circle.

With a straight line A B as diameter, or O B as radius, describe a circle *X'*. With A B as perpendicular, describe a right angled triangle A B C, making A B and B C the sides that contain the right angle, in the proportion of 4 to 3. The hypothenuse of this triangle cuts the circumference of the circle *X'* at the point D. Join DB, producing the triangles A D B and B D C. We thus obtain three similar triangles of which the sides containing the right angle are in the proportion of 4 to 3. This fact may be taken for granted as between you and me, since you have already admitted it in a previous Letter, after examining my proofs.* Describe the rectangles



* If from the angle D in the triangle A D B a straight line D M be drawn perpendicular to A B, the triangles on each side of D M will be similar to the whole triangle A D B and to each other, (Euclid: Prop. 8 : Book 6), and A B the diameter of the circle will be divided into two parts (A M and M B) in the ratio of 5 to 3 ; that is, if A B = 8, then, A M = 5, and M B = 3. Again : If from the angle D in the triangle A D B a straight line D N be drawn perpendicular to B C, and therefore parallel to A B, the triangles on each side of D N will be similar to the whole triangle B D C and to each other, (Euclid : Prop. 8: Book 6), and B C the hypothenuse of the right-angled triangle B D C will be divided into two parts, (B N and N C) in the ratio of 24 to 7 ; and it follows, that N C = $\frac{7}{24}$ (B N). Now, if A B the diameter of the circle = 8, then, B N = 3, and N C = ·875 ; and by Euclid : Prop. 2 : Book 2 : $\{$(B C × B N) + (B C × N C$\}$ =


G
B
E
H
C
Z

AB × BC, AD × DB, and BD × DC, and let F and G denote the fourth angle in the rectangles AB × BC, and AD × DB. With the diagonals of the rectangles as radii describe circles. x, y, and z. I enclose a diagram of the complete figure.

The following things are incontrovertible:—

First: $3{\cdot}125\ (AB^2) = AB^2 + BC^2 + AC^2$; $3{\cdot}125\ (AD^2) = AD^2 + DB^2 + AB^2$; and $3{\cdot}125\ (BD^2) = BD^2 + DC^2 + BC^2$.

Second: If $AB = 4$, $BC = 3$, therefore, the area of the rectangle $AB \times BC = 12$. But, the area of an inscribed regular dodecagon or 12-sided regular polygon to every circle = 6 (radius × semi-radius); therefore, the area of an inscribed regular dodecagon to the circle $X' = 6\left(\frac{AB}{2} \times \frac{AB}{4}\right) = 6\ (2 \times 1) = 12$; therefore, the area of the rectangle $AB \times BC$ is equal to the area of an inscribed regular dodecagon to the circle X′. Hence, the area of a rectangle of which the sides are in the proportion of 4 to 3, is equal to the area of an inscribed regular dodecagon to a circle of which the longer side of the rectangle is the diameter.

Third: Since A C, A B, and B C, the diagonals of the rectangles are the radii of the circles x, y, and z, it follows of necessity, that the areas of the circles are to each other as the areas of the rectangles; and since the ratios between the sides of the rectangles are known, the areas of all the rectangles are ascertainable, from a given value of one side of any of them.

Now, if $AB = 4$, $\frac{3}{4}(AB) = 3 = BC$; therefore, $4 \times 3 = 12$ = area of the rectangle $AB \times BC$. $\frac{4}{5}(AB) = 3{\cdot}2 = AD$, and $\frac{3}{5}(AB)$ or $\frac{3}{4}(AD) = 2{\cdot}4 = DB$; therefore, $3{\cdot}2 \times 2{\cdot}4 = 7{\cdot}68$ = area of the rectangle $AD \times DB$. $\frac{3}{4}(DB) = 1{\cdot}8 = DC$; therefore,

BC^2; that is, $\{(3{\cdot}875 \times 3) + (3{\cdot}875 \times {\cdot}875)\} = (11{\cdot}625 + 3{\cdot}390625) = (3 + \frac{7}{24}\,3)^2 = 15{\cdot}015625 = BC^2$; therefore, $\sqrt{15{\cdot}015625} = 3{\cdot}875 = BC$. By Euclid: Prop. 35: Book 3: and Prop. 8: Book 6: $BC = \sqrt{15}$, when A M 5, and $MB = 3$; and the root of $\sqrt{15}$ is less than 3·875. How can this be possible, if "*that indispensable instrument of science, Arithmetic*," be not "*a mockery, delusion, and a snare*?" The fault is not in Arithmetic, but in the Mathematician's application of Mathematics to Geometry! Compare these facts with the Diagram (Fig 6) on page 155, and the "*reasoning and calculation*" founded upon it, pp. 155, 156, 157, 158, 159.

J. S.

$2{\cdot}4 \times 1{\cdot}8 = 4{\cdot}32$ = area of the rectangle $DB \times DC$, and we obtain the following results:— $\frac{2}{3{\cdot}125}(AB \times BC) = \frac{2 \times 12}{3{\cdot}125} = 7{\cdot}68 = AD \times DB$. But, $\frac{3{\cdot}125 - 2}{2} = \frac{1{\cdot}125}{2}$, therefore, $\frac{1{\cdot}125}{2}(AD \times DB) = \frac{1{\cdot}125 \times 7{\cdot}68}{2}$ $= 4{\cdot}32 = DB \times DC$.

From these facts we obtain another method of finding John Bennett's quadrature, to which, *en passant,* I will call your attention $(AB^2 + BC^2 + AF^2 + FC^2 + 2AC^2) = 4AC^2 = 4 \times 5^2$ $= 100$ = area of a circumscribing square to the circle x, and $\frac{100}{2} =$ 50 = area of an inscribed square to the circle x. Now, produce CA to touch the circumference of the circle x at a point C', and produce the line AG both ways to touch the circumference of the circle x at points C' C'. Since CA and AG are at right angles, and A the centre of the circle x, the produced lines are diameters of the circle at right angles. Join the extremities of these diameters, producing an inscribed square $C C' C' C'$ to the circle x, standing upon the circle y, with its angles without, and parts of its sides within the circle. It is incontrovertible, that the area of the square $C C' C' C' = 3{\cdot}125\ (AB^2) = AB^2 + BC^2 + AC^2$; therefore, on the theory that 8 circumferences of a circle are exactly equal to 25 diameters, the square $C C' C' C'$ and the circle y are exactly equal in superficial area. You will agree with me, I think, that if "D. J. C." be a sincere enquirer after scientific truth, and a believer in John Bennett, this fact is well worthy of his attention.

Passing on ot the more immediate object of this letter, I observe:—It cannot be controverted, that if a square be circumscribed about a circle, and a regular dodecagon be inscribed within it, $\frac{4}{3}$ will express the ratio between the area of the square and the area of the dodecagon. Now, if $AC = 5$, $4(AC^2) = 100$ = area of a circumscribing square to the circle x; therefore, $\frac{3}{4}(100) = 75$ = area of a regular inscribed dodecagon to the circle x, and from this fact we obtain the following results:—$6{\cdot}25\ (AB \times BC) = 6{\cdot}25 \times 12 = 75$ = area of an inscribed regular dodecagon to the circle x; $6{\cdot}25\ (AD \times DB)$ $= 6{\cdot}25 \times 7{\cdot}68 = 48$ = area of a regular inscribed dodecagon to the circle y; and $6{\cdot}25\ (BD \times DC) = 6{\cdot}25 \times 4{\cdot}32 = 27$ = area of a regular inscribed dodecagon to the circle z. Thus, if a, b, and c denote the areas of the three dodecagons (b denoting the area of

the intermediate one), $\frac{2}{3{\cdot}125}(a) = \frac{2 \times 75}{3{\cdot}125} = 48 = b$; and $\frac{1{\cdot}125}{2}(b) = \frac{1{\cdot}125 \times 48}{2} = 27 = c$; therefore, $6{\cdot}25$ (A B × B C) $= a$; 4 (A B × B C) $= b$; and $2{\cdot}25$ (A B × B C) = c; therefore, one-fourth part of the sum of the ratios $= \frac{6{\cdot}25 + 4 + 2{\cdot}25}{4} = \frac{12{\cdot}5}{4} = 3{\cdot}125 =$ the arithmetical value of π.

Now, for the sake of argument, let it be granted that all this proves nothing as to the true value of π. At any rate, there is no getting rid of the fact that by the "*mysterious*" figures 3·125, which, to quote from that learned gentleman, Professor de Morgan, "*comes in at every window and down every chimney*," we may, from a given diameter, circumference, or area of any of the circles, find the values of every line and surface of the enclosed geometrical figure with perfect arithmetical accuracy, and that not only by direct, but by indirect methods. To prove this, it is only necessary to solve a single problem:—

From a given area of the circle x, say 600, find the radius of the circle, showing the working through the circle y.

Then: $\frac{24}{25}(600) = 576 =$ area of an inscribed regular dodecagon to the circle x. $\frac{2}{3{\cdot}125}(576) = 368{\cdot}64 =$ area of an inscribed regular dodecagon to the circle y. $\frac{3{\cdot}125}{3}(368{\cdot}64) = 384 =$ area of the circle y. $\sqrt{\frac{8 \times 384}{8 \times 3{\cdot}125}} = \sqrt{\frac{3072}{25}} = \sqrt{122{\cdot}28} =$ A B the radius of the circle y. $\frac{5}{4}(\sqrt{122{\cdot}88}) = \sqrt{\frac{5^2}{4^2} \times 122{\cdot}88} = \sqrt{1{\cdot}5625 \times 122{\cdot}88} = \sqrt{192}$; or, $\sqrt{\frac{8 \times 600}{8 \times 3{\cdot}125}} = \sqrt{\frac{4800}{25}} = \sqrt{192} =$ A C the radius of the circle x; and $3{\cdot}125\,(\sqrt{192}^2) = 3{\cdot}125 \times 192 = 600 =$ the given area of the circle x.

To give another problem. Let the area of the circle y be any given quantity—say 384. Find the area of the circle x, showing the working through the circle λ'.

Then: The diameter of the circle y = twice the diameter of the circle X′; therefore, the area of the circle y = 4 times the area of the circle X′; therefore, $\frac{384}{4} = 96$ = area of the circle X′. $\frac{3}{3 \cdot 125}(96) = 92 \cdot 16$ = area of the rectangle A B × B C. $4(92 \cdot 16) = 368 \cdot 64$ = area of an inscribed regular dodecagon to the circle y. $\frac{3 \cdot 125}{3}(368 \cdot 64) = 384$ = the given area of the circle y; and $\frac{3 \cdot 125}{2}(384) = 600$ = the area of the circle x.

With these facts before you, I should be sorry to insinuate that you are incompetent to find the arithmetical values of every other line and surface in the enclosed diagram; and I cannot help thinking, that when you shall have given this Letter, in connexion with my communication to this day's number of the *Correspondent*, your careful attention, you will be wishful to withdraw the following notion that was floating in your mind at the time you penned your Letter of the 5th inst.: "*In all your Letters the point I fail to see is the equation or connexion between triangles and curvilinear figures.*"

In conclusion, I may observe:—Since you cannot fail to perceive that π is involved in a very peculiar way, and that these problems can only be solved by a value of π which makes 8 circumferences of a circle exactly equal to 25 diameters, I still "*feel convinced, that your candour will induce you to make the admission that,* $\frac{1}{3 \cdot 125}$ *expresses the true ratio of diameter to circumference in a circle, and can be nothing else.*"

I am, dear Sir,

Very truly yours,

J. S.

What reply "G. B. G." will make to this remains to be seen; but I suspect he will have something to say in your next number in reply to Mr. E. L. Garbett. Poor Mr. Garbett! he appears to me to be quite a Paganinni in his way—a genuine fiddler on one string. If he really is what he pretends to be, let him grapple with argu-

ment, and guard against buoying himself up with the "*fancy*" that he can stifle truth by "*fiddling*" on that one stringed instrument, ridicule.

Captain Morrison begs to decline an invitation to put his hand into my π. Who ever gave him any such invitation? Not I, certainly. Without invitation he did put his hand into my π, and in trying to get it out again makes as pretty a display of inconsistency and absurdity as ever fell to the lot of poor human nature.

I am, Sir,

Yours very respectfully,

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
17th March, 1866.

FROM "THE CORRESPONDENT," MARCH 24TH, 1866.

THE QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

It is always painful to have to write with seeming disrespect of those who have acted towards you with an apparent intention of courtesy, and therefore I, in the outset, tender my apologies to Mr. William Peters for the remarks which I feel constrained to make on a pamphlet which he has sent to me on "the difference between square and superficial measurement," which I see by the outside has been described as "a very clever work" by the *Cambridge Independent Press*, from which I conclude that the Editor of that paper knew at most two of the celebrated "Three Rs.;" he *may* have learnt reading and riting, but he certainly *can*

have known nothing of rithmetic. Mr. Peters attacks vigorously the notion that there are 144 square inches in a foot square, *because* $12 \times 12 = 144$, and in that he is right: 144 is *called* the square of 12, *because* it can be proved by independent measurement that there are 144 square inches in a square foot, 144 square miles in a square block each of whose sides is 12 miles long, and so forth. He then proposes as the unit of superficial measure a circle whose diameter = the linear unit. Now, as the units of measure, space, and capacity, are perfectly arbitrary, there would be nothing *illogical* in assuming a unit inch and a unit sphere instead of a square and a cube, but the change would be extremely *inconvenient*, because you can get accurately at the area of *any* rectilinear figure of whatever shape, and the volume of *any* solid bounded by plain rectilinear faces by means of the square unit and the cubical unit, whereas the unit circle would be useless except for circular areas, the unit sphere except for spherical solids, and these are of much less frequent recurrence. Let Mr. Peters try to express the area of an acre in "circular feet," and he will see the difficulty at once. Another difficulty would arise from the want of a simple expression for the relation *of idea* between the different units; of course, there is no true *comparison* between a line and an area, and between either and a solid, but at present there is a very simple relationship between the units, which would be lost were Mr. Peters' units to be adopted. For instance, taking an inch as the unit, a piece of tape an inch long would be called $= 1$, a piece of paper an inch square, not $= 1$, but $= \frac{4}{\pi}$, and a piece of wood an inch cube, not $= 1$, but $= \frac{6}{\pi}$.

Mr. Peters next proceeds to "square the circle." If the gentleman in question had not been so anxious to emulate our old friend Dogberry, as actually to print, not only his "demonstration," but the answer to it, and his answer to that answer, and send them to me in type, I would scarcely have had the hard-heartedness to tell anyone about it, but as he has done so I presume he will be rather obliged to you than otherwise if you will kindly give his ideas publicity in your columns. Mr Peters' problem is the inverse of the ordinary one. It is:—"To construct a circle equal to a given square," and he proceeds to do so by quadrisecting the sides and drawing a circle through 8 points of quadrisection (not being points

of bisection) of the sides. I must really trouble you with his demonstration. "The square," he says truly, "is divided into 16 squares. Let, therefore, the given square $= a = 16$, and the circle $= z = ?$

§ 1. $z = 16 + 4y - 4x =$, also, $12 + 4m + 4y$; and, as each $= z$.

§ 2. $16 + 4y - 4x = 12 + 4m + 4y$, and $4 - 4x = 4m$. Therefore,

§ 3. $z = 16 + 4y - 4x = 12 + 4 - 4x + 4y$,
and therefore, as

$$\left.\begin{array}{l} z = 16 + 4y - 4x \\ z = 12 + 4 - 4x + 4y \end{array}\right\} \text{The unknown quantities cancel each other, and}$$

$2z = 32$. And as $32 \div 2 = 16$, $z = 16 = a$. Q. E. D."

It was pointed out to Mr. Peters, as he admits, that his "demonstration" proved merely that $o = o$ and gave us no help as to the value of z, but he is perfectly impervious to the objection, and replies :—"As the xs and ys and ms vanish from the two equivalents of z, by their acting and reacting on each other (which they do, and the zs along with them), I must be at perfect liberty to restore z to its place, and write

$$z = 16 = (12 + 4 =) \; 16."$$

It would be an insult to your readers to suppose that this self-*exposé* needed any supplement from me.

One word more on this pamphlet. *If* Mr. Peters' "demonstration" be right, it follows of necessary consequence, that $\pi = \frac{16}{5} = 3\frac{1}{5}$, as any of your readers will see by drawing the figure.

As Mr. Peters professes, in his preface, great admiration for Mr. James Smith, I must leave those two worthies to fight out their conflicting values of π. I had hoped that I had done with Mr. Smith and his pentagon, but as in his Letter in the *Correspondent*, No. 39, he attempts, at last, a sort of proof of the only point in which I ever took issue with him; I am constrained to notice it. The question concerning this pentagon does not, as I will show, depend upon any value of π whatever. Mr. Smith says truly that $4x \times 2\pi = 24\frac{\pi}{3}(x)$, but I totally deny that this latter expression, either really or "apparently" = the circumference of the circle *for*

any value of π whatever. If it were, it would follow, from the definition of π, that $4x = r$, which I will proceed to show is not true.*

Mr. Smith admits that Sin. $30^\circ = \frac{1}{2}$, and he will, I suppose, admit that for any angle whatever, sin. $2\theta = 2 \sin\theta \cos\theta$ is a fundamental equation not depending upon π. I will also assume

* Describe a regular hexagon, a regular dodecagon, and a regular 24-sided polygon within a circle. It is self-evident, that the sides of the hexagon, dodecagon, and 24-sided polygon will be chords to arcs of the circle; and it is *admitted* that the sides of the hexagon = radius of the circle, and that 6 (radius) = perimeter of the hexagon. Let a denote a side of the hexagon, let b and c denote sides of the dodecagon and 24-sided polygon, and let x denote the circumference of the circle. Then: 6 times $\frac{\pi}{3}$ (a), 12 times $\frac{\pi}{3}$ (b), and 24 times $\frac{\pi}{3}$ (c) are equal, and all $= x$. Hence: $a \times 2\pi = 24\,\frac{\pi}{3}\,(c)$, and this equation $= x$, whatever be the value of π. This "*apparently*" makes one side of a pentagon equal to the other four sides, when one side of the pentagon is also a side of a regular inscribed hexagon to a circle, and the other four sides, sides of a regular inscribed 24-sided polygon to the circle. This is a paradox in Mathematics, or rather a "*Curiosity of Mathematics*" as applied to Geometry. Mr. Miller might, and would have learned the *cause* of this, if he had read my Letters in the *Correspondent* with ordinary care. The *cause* is thus explained. If the circumference of a circle be divided into any number of equal arcs, and if from one of these arcs $\frac{1}{25}$th part be deducted, and the remainder multiplied by the number of arcs, the product is equal to the perimeter of a regular hexagon inscribed in the circle. Hence: In all polygons of more than four sides inscribed in a circle, the ratio between the sides of the polygons and their subtending arcs is as 3 to π. These facts are independent of "*the circumference of the circle for any value of π whatever.*" Let A B denote a side of a regular hexagon inscribed in a circle, and let A C B denote its subtending arc. Conceive a straight line D E equal in length to a side of the hexagon, to rest on the line A B, and the extremity E of the line D E to be fixed at B an extremity of the line A B. Conceive the line D E to revolve in the direction of C and B until the line D E is at right angles with the line A B. The line D E will then be wholly outside the circle. But, in the course of the revolution of the line D E, round the arc A C B, it is self-evident, that as the part of the line D E within the circle diminishes, its subtending arc diminishes in the same ratio or proportion. I must here refer the reader to the diagram (Fig. 2) on page 132, and the "*reasoning and calculation*" founded upon it, pp. 131, 132, 133.

J. S.

that the 13th Proposition of the 2nd Book applies to every triangle, whether two of its angles happen to be in the circumference of a circle or not, and therefore in the isosceles triangle formed by joining the two ends of one of Mr. Smith's short sides to the centre one, $x^2 = 2r^2 - 2ry$, where y is the intercept between the centre and the foot of the perpendicular, or in other words $= r \cos. 15°$. Starting from this, the rest of the work is shortly as follows (perhaps those of your readers who do not care to follow figures will kindly skip half a dozen lines and take my word for the result):—*

$$\frac{2^2}{r} = 2(1 - \cos. 15°.)$$

But calling $15° = \theta$.

$$\cos.^2\theta + \sin.^2\theta = 1.$$

$$2 \sin, \theta \cos. \theta = \sin. 30° = \tfrac{1}{2}$$

whence $\cos. \theta + \sin. \theta = \dfrac{\sqrt{3}}{\sqrt{2}}$

$$\cos. \theta - \sin. \theta = \frac{1}{\sqrt{2}}$$

$$2 \cos. \theta = \frac{1 + \sqrt{3}}{\sqrt{2}}$$

$$\frac{x^2}{r^2} = \frac{\sqrt{2} - (\sqrt{3} + 1)}{\sqrt{2}}$$

But $2\sqrt{2}$ is rather more than 2·819

* Mr. Miller observes :—" Mr. Smith admits that Sin. $30° = \frac{1}{2}$." *Granted!* Why do I make this admission? Because I *know* that an angle of 30° is the lesser of the acute angles in a right-angled triangle, of which the hypothenuse and perpendicular are in the ratio or proportion of 2 to 1. (Is not an angle of 30° equal to half of any one of the three angles in an equilateral triangle?) Now, if in a right-angled triangle, the hypothenuse be to the perpendicular in the ratio or proportion of 4 to 1, then, $\frac{1}{4} = {\cdot}25$ is the sine of the lesser of the acute angles. But I do not admit—as Mr. Miller assumes—that "for any angle whatever $2\theta = 2 \operatorname{Sin} \theta \operatorname{Cos} \theta$," nor can Mr. Miller demonstrate this equation. Mr. Miller *assumes* r Cos. 15° as a starting point, and observes :—" Starting from this, the rest of the work is shortly as follows, (perhaps those of your readers who do not care to follow figures, will kindly skip half a dozen lines and take my word for the result). The sentence in brackets is extremely mathematician like, but no "*reasoning geometrical investigator*" will be bamboozled into a blind acquiesence in this very modest request of Mr. Alex. Edw. Miller. J. S.

$1 + \sqrt{3}$ is rather less than 2·733.

whence $\frac{x^2}{r^2} > \frac{\cdot096}{1\cdot414} > \cdot0671$.

$\therefore \frac{x}{r} > \cdot256 : 4x > (r)\ 1\cdot024$,

and, therefore, much more than r. Q. E. D. I have in every instance taken the side of the approximations which is most in favour of Mr. Smith, and therefore have brought out an answer more favourable to him than the exact truth, but still sufficient for his refutation. All this, I beg Mr. Smith particularly to observe, is without any use of any Tables, and without any reference to the value of π. The fallacy in Mr. Smith's attempted proof in the Letter which first drew my attention to him, is, that he assumes that because equal chords subtend equal arcs, unequal chords subtend proportional arcs, which is not true. I repeat, therefore, that it "never entered into my head to discuss the value of π with Mr. Smith or any one else." I never wrote on the general question of circle quadrature, but merely to expose a particular transparent fallacy. If Mr. Smith likes to extend his mathematical studies, and find out a finite arithmetical value for π also, I shall never think it worth my while to discuss the matter with him. How this can be said to be "wriggling out of" anything I cannot see.

The length of this letter has deprived me of the opportunity of referring to the moon this week, but I hope to do so in my next.

Yours very truly,

ALEX. EDW. MILLER.

LINCOLN'S INN, *Feb.* 6.

FROM "THE CORRESPONDENT," MARCH 24TH, 1866.

PARALLAX AND PLAIN SPEAKING.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

Mr. Garbett asks some questions in his last letter, to which, therefore, I seem challenged to reply; otherwise I should have preferred to say no more, taking for granted that your readers, who care for the truth involved in this controversy, will have by them at least all the numbers of the *Correspondent* for this year. I might be in evil case had I to appeal to a *Public Opinion* under the control of a literary Casely.

Mere quibbling and *unprovoked* personal attacks, in a paper like the *Correspondent*, ought especially to be scouted. It might be for the interest of some persons to damage it; and we then might be driven once more to anonymous journalism exclusively, whenever we wanted to think or write independently. Let people only ask themselves—What is the object of this or that Letter? Is it to attack persons or things? Does it suggest to us anything new, or justify anything old? Is it meant merely to annoy the promulgator of unpopular views, or does it add to our stock of knowledge, help us out of any difficulty, or furnish any food for thought? If not, is it not, then, merely literary *Garb(ett)age?*

To "friend" Garbett's first question I reply:—If he shows *any* knowledge, it is as to the value of π, and the usual methods of demonstration. Still, he has *not* told us what value of π was in vogue before 3·14159 was accepted. I drew my own conclusion, and remember that the person who challenged me on this point describes himself, in *Correspondent* No. 44, as "like Mr. Casely." Question 2: "Who is he?" In the preceding sentence in my Letter, where I say "*ipse dixit*" (p. 118), the word "his" occurs, following "Mr. Garbett" in the penultimate sentence. "*Ipse*" is, therefore, *grammatically*, "he." As to "school-books being all wrong, Dutchmen all wretched, and Rutherford and Shanks no better," I refer to No. 44 and the Letter of "E. L. Garbett." That he did not write so

plainly as that, is true. Some people *can't* look straight, or speak or write plain. But that I fairly gave the only intelligible *drift* of the expressions really used, I can only again appeal to your readers. I am not, of course, concerned to deny that all are "*base lines — n* ;" I treated them as "rhodomontade."

"All I can" was a misprint for "all I care;" also "fire" for "pie," and × for ÷, on pp. 118-119. I *could* have said a great deal more *had I cared*. (See Mr. Reddie's Letter, page cix.)

There is one other question unblushingly asked:—"Where is it contradicted that parallax is the difference of direction of an object as seen from two different places?" I answer: In *Correspondent*, No. 44. Not *totidem verbis*, it may be. Catch a literary Casely at plain speaking if you can!—but virtually, and indeed, in *Correspondent*, No. 44, in E. L. Garbett's Letter. In flatly contradicting that "an accurate base line is essential to the accurate calculation of *parallax*," the former is *as flatly* contradicted, unless a quibble is intended. "Parallax" being the "difference," described by Professor Airy, from which "distance" is *calculated*, has become a synonyme for "distance" itself; *and is so used by all Astronomers*. I added another quotation (p. 118) from Professor Airy, expressly to show this to the uninitiated. But I don't believe Mr. Garbett was ignorant of this usage of the word, and I don't believe he thought me ignorant of it; and, if I am right in these impressions, what can be thought of what he dared to write on the subject? Nay, suppose I am wrong, then he has described himself and his compound ignorance in No. 44; and even then, what can we think of all he has written? What *could* be its object? I appeal to the "public opinion" of *all your readers*.

As regards Captain Morrison, *alias* "Zadkiel," besides the public compliments he has paid me in your columns—which I have always repudiated, coming from such a quarter—I have some private Letters of his, which I *could* cite to his confusion. I am quite content, however, to accept his abuse, and so to have it plainly seen that he and I *differ* totally in *all* things. I suppose the "75,000 purchasers" make his Almanac *pay*, and therefore the *wise* "Zadkiel" is quite content, even if he does *not* know that the West cannot *possibly* be a point, or single direction, in space! A "philosopher" who can compare the *falling* of bodies in the air, or the motions of

the planets in space, to "traction," *on the solid earth*, by "railway engines," is, in my opinion, the fit instructor of the "75,000" *dupes* who buy "Zadkiel's Almanac," and "understand astronomy" *as he does!*

Your faithful servant,

J. REDDIE.

HAMMERSMITH, *20th March*, 1866.

P.S.—To prove that Captain Morrison's "discoveries" in *Astronomy*, *Astrology*, and *Geography* are all on a par, I append an advertisement cut out of to-day's *Standard*, *without comment*:—

"GREAT VOLCANIC ERUPTION at SOLFERINO.—This event was foretold in ZADKIEL'S ALMANAC, 1865, from the Eclipse of the Sun in October last. The 75th Thousand of 'Zadkiel's Almanac' for 1866 still on sale."

FROM "THE CORRESPONDENT," MARCH 24th, 1866.

π AND PARALLAX.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

Surely Mr. Garbett must now be convinced that I, as *amicus curiæ*, dealt more leniently with him than "friend Reddie."

I now venture to suggest to my learned brother that he will condescend to tell us whether or not $\pi = 3{\cdot}125$ was ever promulgated by any one before, what he delights to call "the Seaforth mince-pie" was concocted; and, if so, when, and by whom? This should interest Mr. Smith, and I am really curious to know.

Next, will he kindly inform us (ignoramuses compared with him) what is the exact blunder about parallax he meant to charge against "Terra"* and Mr. Reddie? Especially he might enlighten us on the

* "Terra" was a synonyme of J. S.

main point, as I understand the question, which was raised by the latter. It was quite clear to me—and I find it in all the "authorities," and I think a boy of twelve might understand it, when he has learnt that 2 rad. = diam.—that if the earth revolved round the sun at a mean distance of 95 million miles, and if the sun were at rest, as Copernicus, Galileo, Kepler, and Newton taught, then the base line extending between the earth's two places at six months' interval must be, as Professor Kelland said, exactly 190 million miles long. But if the sun is moving in space, and the earth apparently still going round it, and at the same mean distance of 95 million miles, I confess I do not understand how a gentleman who knows the value of π, or any other man, can possibly believe the earth, in that case, to be still only 190 million miles distance from where it was at intervals of six months.

Let Mr. Garbett join issue on this point first; for it seems to me almost like a question of fact, which the jury of your readers can decide upon. Let us settle the facts first, and we can argue upon them afterwards. This is good for physical laws as well as "law" *par excellence.*

Or, I give Mr. Garbett another opportunity of showing that he has something to say to some purpose, and can really instruct us. Will he venture to say that the calculations of "parallax, or distance," will be *all the same*, whether we have one base line or another? Or, will he tell us *what his plea was on this subject?* and why he began and ended with abusing the other side?

I must say, I think he would be quite right not to reply to "a whole *Correspondent*-full of mud." His opponents, I fancy, will think so too, and be silent. But *I* have thrown no mud. I fear *he* cannot truly say as much. And if these questions of mine drop him into the mire, I think he must now begin to feel that the pit has been one of his own rash digging. Still, I do not say he may not creep out and escape. He *is* very clever, in a way, and, as he says himself, "something like Mr. Caseley." *Mais, nous verrons!*

X.

March 19*th*, 1866.

FROM "THE CORRESPONDENT," MARCH 31st, 1866.

THE QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

Your contributor, "G. B. G.," and I, after a long correspondence as to the solution of the problem—What is the true ratio of diameter to circumference in a circle?—have, by mutual consent, agreed to make a pause for a time. True, there is a good reason for this, since we have just discovered that we are not agreed upon "*axioms or fundamentals.*" Strange that we should not have made the discovery sooner! Your readers would observe from my communication to your last number, that "G. B. G." "*cannot find any connection between the area of a square and the area of a circle,*" without an assumption. This appears to me extraordinary indeed, since it is admitted by every Geometer and Mathematician that the area of a square multiplied by π = area of a circle of which the side of the square is the radius, no matter what the arithmetical value of π may be. I venture to ask you to favour me by giving insertion to my last letter to "G. B. G.," with two others written in the early part of our correspondence, which I think will be striking and interesting to such of your geometrical readers as are sincere and earnest enquirers after scientific truth. My last letter ran as follows:—

"Barkeley House, Seaforth,
20th March, 1866.

"DEAR SIR,

"Many thanks for your very kind letter of the 17th inst. I heartily reciprocate the sentiment to which you have given utterance, that if other opponents arise, "*all I can hope is the arguments may be conducted in the tone that has, at least, marked all our correspondence.*" Before receiving your Letter I had posted one to the Editor of the *Correspondent*, which I hope will appear in the next number of that Journal, of which I will send you a copy. You

will find some slight corrections and alterations in comparing it with my last Letter to you, but none in the least degree affecting the arguments. I am sure you cannot fail to perceive that I have connected a series of circles with squares, right-angled triangles, unequal-sided rectangles, and equal-sided polygons, in a way to which the "*mysterious*" $\pi = 3·14159265...$, or any other but that which makes 8 circumferences of a circle = 25 diameters, is utterly incompetent.

"You say:—'*I confess I do reckon your method of argument illegitimate*,' and you subsequently observe:—'*It would be quite legitimate if you went to work thus.* I assert $\pi = 3·125$, for if not, let $\pi = (3·125 \pm x)$ x being any quantity added or subtracted.' Well, then, I will comply with your suggestion; but, before doing so, I must call your attention to some indisputable certainties, and one striking probability.

"$\frac{\pi}{300}$ is the algebraical expression for the arithmetical value of an arc subtending an angle of 36′. (This fact you have admitted in one of your Letters.) 24 times 36′ = 14° 24′, and 25 times 14° 24′ = 360°. But, 24 × 25 = 600, and 600 times the circular measure of an angle of 36′ = 2π; therefore, 300 times the circular measure of an angle of 36′ = π, whatever be the value of π. So far you can have no doubt or difficulty. Now, $24^2 \times 36' = 576 \times 36'$ = 24 times 14° 24′ = 345° 36′, and $\frac{3·125}{3}$ (345° 36′) = 360°. But, $\frac{345^\circ\ 36'}{6}$ = 57° 36′, which, multiplied by 2 (3·125) = 360°; therefore, on the theory that 8 circumferences of a circle are exactly equal to 25 diameters, 57° 36′ = radius, and 345° 36′ represents the value of the perimeter of a regular inscribed hexagon to a circle of circumference = 360°. This, it is true, involves an assumption, and I only mention it as a "*striking probability*" in favour of the truth of the theory, and it must just be taken for what it is worth. This, however, should be considered and taken in connexion with my Letter in the 40th number of the *Correspondent*. (See *Letter*, page lx.)

"Now, let x denote 30°, let y denote the sine of an angle of 30° = $\frac{1}{2}$ = ·5, and let $\frac{n \text{ times } x}{180} = m$. Then: If $n = 3·125$, 300 $(m - y) = 2n$. For example: $\frac{n \text{ times } x}{180} = \frac{3·125 \times 30^\circ}{180}$ = ·5208333

with 3 to infinity = m; and $m - y$ = ·5208333 — ·5 = ·0208333... ; therefore, 300 (·0208333) = 6·24999 with 9 to infinity, = $2n$ approximately, and would obviously = $2n$ = 6·25, if we could pick up the &c.'s. This makes ·5208333 with 3 to infinity, the circular measure of an angle of 30° = $\frac{n}{6} = \frac{3·125}{6}$, and makes 3·125 the true arithmetical value of π. There is no method or contrivance by which we can get rid of the fact that $\frac{\pi}{6}$ = the circular measure of an angle of 30°, whatever be the value of π.

"Well, then, I assert that $n = \pi$, if π denote the circumference of a circle of diameter unity; for if not, let n be greater than 3·125, say 3·1416. Then: $\frac{n \text{ times } x}{180} = \frac{3·1416 \times 30°}{180}$ = ·5236 = m; and $m - y$ = ·5236 — ·5 = ·0236; therefore, 300 ($m - y$) = 300 (·0236) = 7·08, and is greater than $2n$. Again: Let n be less than 3·125, say 3·12. Then: $\frac{n \text{ times } x}{180} = \frac{3·12 \times 30°}{180}$ = ·52 = m, and $m - y$ = ·52 — ·5 = ·02; therefore, 300 ($m - y$) = 300 (·02) = 6, and is less than $2n$. But to come to still closer quarters. Let n be as little greater as we please than 3·125, say 3·126. Then: $\frac{n \text{ times } x}{180} = \frac{3·126 \times 30°}{180}$ = ·520999 with 9 to infinity = m, and $m - y$ = ·520999 — ·5 = ·020999; therefore, 300 ($m - y$) = 300 (·020999) = 6·2997, and is considerably greater than $2n$. Again: Let n be as little less than 3·125 as we please, say 3·124. Then: $\frac{n \text{ times } x}{180} = \frac{3·124 \times 30°}{180}$ = ·520666 with 6 to infinity = m, and $m - y$ = ·520666 — ·5 = ·020666; therefore, 300 ($m - y$) = 300 (·020666) = 6·1998, and is considerably less than $2n$. Now, to get rid of this proof you must demonstrate that 300 times the circular measure of an angle of 36′ is not exactly equal to π, which is impossible; and if you take this in connexion with the geometrical proofs established by means of my last two diagrams, I hardly think you can fail to arrive at the conclusion, that the arithmetical value of π must be 3·125, and can be nothing else.

"But, further: By the same formula, if x denote 15° and y denote half the sine of an angle of 30° = $\frac{·5}{2}$ = ¼ = ·25; then,

$600\,(m - y) = 2\,n$. Again: If x denote $7^{\circ}\,30'$, and y denote one-fourth part of the sine of an angle of $30^{\circ} = \frac{\cdot 5}{4} = \frac{1}{8} = \cdot 125$, then $1200\,(m - y) = 2\,n$, and so on we might proceed, *ad infinitum*. In these arguments and proofs there is no reference whatever to any Tables.

"I had written so far when your favour of yesterday's date came into my hands, from which I have got the impression that I have given you more trouble than is agreeable, which I sincerely regret. You say:—'*And now I desire a little rest*,' and probably a truce will be of advantage to both of us. In making a pause at a point when neither has succeeded in convincing the other, it is, at any rate, pleasant to reflect that our long conflict has been conducted without one word of intentional discourtesy on either side. You close your Letter by observing:—'*It is hard to hold that* $\pi = 3{\cdot}125$, *when I have proved, at least to my own satisfaction, that it must be greater*.' I also find it equally hard to hold that π is greater than $3{\cdot}125$, when I have proved, '*at least to my own satisfaction*,' that to maintain this notion would be to make Geometry and Mathematics inconsistent with each other, and thus render one or both a mere 'mockery, delusion, and a snare.'

"In taking leave of you, at least for a time, I beg to thank you for the labour you have expended upon me, and wishing you the enjoyment of health and happiness,

"Believe me, my dear Sir,

"Very sincerely yours,

"J. S."

At this stage of our correspondence "G. B. G." would appear to have forgotten that in the early part of it I furnished him with a proof by the method which he admits to be legitimate, as will be seen from the following Letters:—

"BARKELEY HOUSE, SEAFORTH,
30*th January*, 1866.

"DEAR SIR,

"I was called up to London last week very unexpectedly on public business, and thus prevented from sooner acknowledging the receipt of your two last Letters. I send you a copy of last week's *Correspondent*, in which you will observe a

Letter from Mr. E. L. Garbett, who adopts the same objection that you have done, but works it out in another form. I have sent a Letter in reply, which I hope will appear in the *Correspondent* of next Saturday, and to this I beg to refer you as an answer to your argument, and to which I may add the following:—

"An angle of $14° \ 24' = \frac{360°}{25}$, and an angle of $15° = \frac{360°}{24}$, whatever be the arithmetical value of π. Hence: If a denote an angle of $14° \ 24'$ and b its subtending arc; and if c denote an angle of $15°$ and d its subtending arc, then, since the angles at the centre of a circle are to each other as their subtending arcs, $a : c : b : d$. But, $24 : 25 :: 14° \ 24' : 15°$; therefore, $3 : 3{\cdot}125 :: 14° \ 24' : 15°$; and assuming $\pi = 3{\cdot}1416$, I think you will find that this would make the arc subtending an angle of $15° = 15° + {\cdot}07968$, which is impossible, and therefore absurd. With many thanks for your very kind communications.

"I remain, dear Sir,

"Very truly yours,

"J. S."

"BARKELEY HOUSE, SEAFORTH,
2nd February, 1866.

"Dear Sir,

"I am in receipt of your favours of the 30th and 31st ult., which I have no doubt were posted before my short Letter of the former date could reach you. I had thought of bringing out the whole subject through the columns of the *Correspondent*, but observing the great pains you have taken to convince me of the fallacy of the theory that 8 circumferences of a circle = 25 diameters, I feel it would be ungenerous on my part to withhold from you, even for a week, the complete answer I can give to your argument.

"I take the following, which is substantially the same as your own diagram, and any Geometer will readily construct the diagram from the following description. (Let F C B A O represent a quadrant, O the right angle, O C and O B radii, so that the arc C A shall subtend an angle of 45°, and the arc B A an angle of 30°. The arc C B will subtend an angle of 15°. From B and C let fall the perpendiculars B N and C M, and draw the straight lines B E and

C D parallel to O A. Then, B N will be the sine and O N which is equal to B E the cosine of the angle B O A; and C M will be the sine and O N which is equal to C D and C M the cosine of the angle C O A. Join C B and B A.)

"Now, I at once admit most frankly that every calculation in your Letter is strictly correct, and in harmony with those of Mr. Garbett; but you both fail to observe that there are certain other facts in connexion with this geometrical figure, a knowledge of which is essential to the settlement of the question at issue between us, and to which I now beg to call your attention.

"If radius O A = 1, C O B is an angle of 15°, and B O A an angle of 30°, and, O A = 2 B N; therefore, 2 B N = 1 is the chord of an arc of 60° to a circle of radius 1. But, B N = ·5 is the natural sine of the angle B O A, and O N = B E = $\sqrt{\cdot 75}$; therefore, ·866025 is the natural cosine of the angle B O A; therefore, O A — O N = 1 — ·866025 = ·133975 = N A, the natural versed sine of the angle BOA; and it follows from the celebrated 47th proposition of the first book of Euclid, that $\sqrt{BN^2 + NA^2} = \sqrt{\cdot 5^2 + \cdot 133975^2} = \cdot 517637\ldots$ = the chord B A subtending the angle B O A; that is, = the hypothenuse of the right-angled triangle B N A. Now, the angles at the centre of a circle are to each other as their subtending arcs; therefore, the angle C O B : the angle B O A : : the arc C B : the arc B A; and since straight lines bisecting the angles C O B and B O A would also bisect their subtending chords and arcs; by parity of reasoning, the chord B A subtending the angle B O A should be in the same ratio or proportion to the arc B A as the chord C B to the arc C B, or the chord 2 B N to the arc 2 B A. But this is not a fact; for, in an angle of 60° to a circle of radius 1, the subtending chord is to its subtending arc in the ratio or proportion of 1 to 60°, while the chord subtending the angle B O A, an angle of 30°, is to its subtending arc in the ratio or proportion of ·517637... to 30°. Thus, a straight line bisecting the angle B O A and produced to its subtending arc B A would bisect the arc at a point, say *p*; but a straight line drawn from *p* parallel to O N the cosine of the angle B O A, would not bisect either the sine of the angle or its subtending chord. Hence the utter inapplicability of the 47th Euclid (directly) to the measurement of the circumference of a circle.

"If a denote an angle at the centre of a circle, and b its subtending arc, and c denote another angle at the centre of the circle, and d its subtending arc, you will admit that $a : c :: b : d$.

"Now, if $b = \frac{360°}{25} = 14° \ 24'$, $d = \frac{360°}{24} = 15°$. Take the fraction, or ratio, $\frac{25}{24}$, and let $\frac{x}{y}$ denote an equivalent ratio. It is admitted, that we may divide the two terms of a ratio by the same number without altering the ratio itself. Divide the two terms of the ratio $\frac{25}{24}$ by 8, and we obtain the equivalent ratio $\frac{3{\cdot}125}{3}$. For this substitute $\frac{x}{y}$. Again: Divide the two terms of the ratio $\frac{25}{24}$ by 25, and we obtain the equivalent ratio $\frac{1}{{\cdot}96}$. For this substitute $\frac{x}{y}$. By analogy or proportion, $3{\cdot}125 : 3 :: 1 : {\cdot}96$, and it follows, whether $\frac{x}{y}$ denote the ratio $\frac{3{\cdot}125}{3}$ or $\frac{1}{{\cdot}96}$, that, by analogy or proportion, $x : y :: 15° : 14° \ 24'$: and, by alternation, $y : x :: 14° \ 24' : 15°$. Hence: x is to y as the circumference of a circle to the perimeter of its inscribed regular hexagon: and the perimeter of a regular hexagon to a circle of diameter unity is a known quantity $= 3$. Hence: $\frac{\pi}{3}$ expresses the ratio between the circumference of every circle and the perimeter of its inscribed regular hexagon, whatever the value of π may be. These facts demonstrate the true arithmetical value of π to be $3{\cdot}125$, "*for if not*," let π be greater than $3{\cdot}125$, say $3{\cdot}1416$. Then: $3 : \pi :: 14° \ 24' : 15° + {\cdot}07968$, and is greater than 15°, whichis impossible, and therefore absurd. Again: Let π be less than $3{\cdot}125$, say $3{\cdot}12$. Then: $3 : \pi :: 14° \ 24' : 14° + {\cdot}976$, and is less than 15°, which is impossible, and therefore absurd.

"Hence, the arithmetical value of π cannot be either greater or less than $3{\cdot}125$, which makes 8 circumferences = 25 diameters in every circle. Therefore, $\frac{1}{3{\cdot}125}$ expresses the true ratio between the diameter and circumference of a circle. Hoping I may have made the demonstration intelligible and convincing,

"I remain, dear Sir,

"Very truly yours,

" "J. S."

I am quite prepared to enter the lists with any of your geometrical contributors who evince an honest desire after the search of scientific truth, and give some sort of evidence that they have not arrived at that stage in a Mathematician's career when prejudice usurps the seat of reason, and deprives her of her legitimate sway. Opponents of the E. L. Garbett stamp I shall not for the future condescend to notice.

In my Letter in your last number there is a printer's mistake of considerable importance. The equation, $\frac{2}{3 \cdot 125}$ (576) = 368·64 = area, &c., which comes in at the 18th line of the second column, page 141, should have been followed by the equation, $\frac{3 \cdot 125}{3}$ (368·64) $= \frac{2}{3 \cdot 125}$ (600) = 384 = area of the circle y. This equation was correctly inserted in the MSS., but by some oversight has been omitted. This equation forms an important link in the chain of proof in the solution of my first problem, and without it the proof is imperfect.

This morning's post brought me a reply from "G. B. G." to my Letter of the 20th inst., and I will give your readers the benefit of his objections:—"I follow out and concur with your equation, $300(m - y) = 2n$, if so be that $\pi = 3 \cdot 125$; for $\frac{n x}{180} = m$, that is, $\frac{n}{6} = m$, and $300 (m - y) = 300 \left(\frac{n}{6} - \frac{1}{2}\right) = 50 (n - 3)$. Now, if we like to put $50 (n - 3) = 2n$, $48n = 150$, $n = 3\frac{1}{8} = 3 \cdot 125$. But I can't see that $50 (n - 3)$ does equal $2n$. Since n is really your π, the above equation amounts to saying, 300 (arc 30° — sin. 30°) = circumference. Now, I know no property of the circle which gives this equation, and I deny its correctness."

To this I reply:—The theory that makes 300 (arc 30° — sin. 30°) to a circle of radius 1 = circumference of the circle, also makes 600 times the circular measure of an angle of 36′ = the circumference of a circle of radius 1. The latter fact is admitted by the Editor of the *Athenæum*, or, I might say, by Professor De Morgan himself. (See *Athenæum*, 5th August, 1865: Article, "Our Library Table.")

Now, if "G. B. G." can produce any other value of π but that which makes 8 circumferences of a circle exactly equal to 25 diameters, and by means of it get the same result, I admit he will have the best of the argument.

I am, Sir,

Very respectfully yours,

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
26th March, 1866.

FROM "THE CORRESPONDENT," MARCH 31ST, 1866.

π AND PARALLAX.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

Under the above heading, "X." calls on me to tell him "whether or not π = 3·125 was ever promulgated by any one before . . . and if so, when and by whom." This, he says, "would interest Mr. Smith." I thought both "D. J. C." and Mr. Smith himself had given this information. If not, as it does not in the least interest me, I really must decline attempting to ascertain it.

"X." will find the rest of his letter answered at p. 58, first column, third paragraph, last sentence.

I remain, Sir,

Yours obediently,

E. L. GARBETT.

27th March, 1866.

P.S.—Mr. Smith, I see, also asks me to "grapple with argument;" but he has not grappled with the question whether a straight line or the twenty-fourth of a circle between the same points is the shorter. I am ready to grapple with any one who first answers this.

FROM "THE CORRESPONDENT," APRIL 7TH, 1866.

THE QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

The game between your contributor, "G. B. G.," and me has become extremely interesting and important. The following Letters give the last moves; and, by careful observers, it will be seen that check-mate must speedily follow:—

J. S. *to* G. B. G.

BARKELEY HOUSE, SEAFORTH,
27th March, 1866.

DEAR SIR,

I am suffering from an attack of gout and confined to the house, and, you may readily suppose, not in the best frame of mind for careful mathematical analysis or close argumentation. The argument in your favour of the 24th inst., however, appears to me to be fatal to your theory, and that it will not be a difficult matter to convince you of this, and with that object I am induced to make an effort, and write you a short Letter.

You observe:—"Now, if we like to put $50\ (n - 3) = 2\ n$, $48\ n = 150$, $n = 3\frac{1}{8} = 3{\cdot}125$. But I can't see that $50\ (n - 3)$ does equal $2\ n$."

Now, you will admit that the arithmetical value of π is greater than 3 and less than $3{\cdot}2$; consequently, the value of π must be represented by the integer 3, plus certain decimals of less arithmetical value than $\cdot 2$. You will also admit that $\frac{3}{\pi}$ expresses the ratio between the perimeter of every regular hexagon and the circumference of its circumscribing circle; and that the perimeter of a regular inscribed hexagon to a circle of radius $1 = 6$ times radius $= 6$.

Well, then, the perimeter of a regular inscribed hexagon to a circle of diameter unity = 6 times radius = 3. Now, let n denote $(3 + \frac{1}{24} 3)$. Then: $25 (\frac{1}{24} 3) = n$; therefore, $50 (n - 3) = 2 n$, and $n = 3{\cdot}125$. No other fractional or decimal quantity added to the integer 3 but its one twenty-fourth part can produce this result.

From these facts we may obtain several equations, on the theory that $n = \pi$; that is, that n = the circumference of a circle of diameter unity. For, 300 (arc 30° — sin. 30°) to a circle of radius 1 = 600 (the circular measure of an angle of 36′) $= \frac{n}{3} (6) = 2 n$. Thus, all these expressions are equal to the circumference of a circle of radius 1 = $50 (n - 3) = 6{\cdot}25$; and every other value of π but that which makes 8 circumferences of a circle equal to 25 diameters is utterly incompetent to produce this result.

You can certainly have no doubt or difficulty as to the fact that $50 (n - 3) = 2 n = 6{\cdot}25$; and it appears to me it would be absurd to suppose, that it will not be equally plain to you, that the arithmetical value of π must be 3·125, and can be nothing else.

Geometry is incompetent, *per se*, to teach us the value of π, but the value of π, once ascertained by the aid of Mathematics, Geometry steps in to prove its correctness in a thousand ways.

I am, dear Sir,

Yours very truly,

J. S.

G. B. G. *to* J. S.

LAUNCESTON,

29th March, 1866.

DEAR SIR,

If my Letters were of an irritating character I would not address you when suffering from gout; I hope, however, this will find you relieved from the attack.

To put $50(\pi - 3) = 2 \pi$, is the same as $300 \left(\frac{\pi}{6} - \frac{1}{2}\right) = 2 \pi$. 300 (arc 30° — sin. 30°) = circumference, and I say there is no property of the circle that leads to this equation. To assert such equality is, in fact, to assume what we are seeking as unknown, namely, the value of π.

I allow that π lies between 3 and 3·2, as its value is 3·14159... Also, $\frac{3}{\pi}$ is the ratio of perimeter of hexagon to the circumference of its circumscribing circle. No doubt on this matter. But I utterly deny your right to take for granted that $50(\pi - 3) = 2\pi$. Your "theory" gets out all the relations you state; and very pretty they are, but it is only a "theory." You have never proved, ***without assuming it***, that $\pi = 3{\cdot}125$.

It is true and inevitable that no other value of π will give your results, but that does not prove such results correct. So you see my objection. To prove $\pi = 3{\cdot}125$ you ***frame an equation without the least warrant***, which would hold if π ***did*** equal 3·125. Where do you learn from any property of a circle that (arc 30° — sin. 30°) = $\frac{1}{300}$ circumference ?

I should prefer this equation :—$44(\pi - 3) = 2\pi$; or, $44\pi - 2\pi = 132$. $42\pi = 132$. $\pi = \frac{132}{42} = \frac{22}{7}$, which was the value (nearly) deduced by Archimedes. This is the same as saying that $264\left(\frac{\pi}{6} - \frac{1}{2}\right) = 2\pi$, or (arc 30° — sin. 30°) = $\frac{1}{264}$th part of circumference. Now, why is not my equation as good as yours ?* There has been no attempt to prove either. Each is constructed to suit a ***previously fixed*** value of π, and no reasoning leads to either.

Why not try and find π without making ***any*** assumption of its value till you have ***proved*** such assumption ? For example : The circumference of a circle is greater than the perimeter of a hexagon within it ; therefore, π is greater than 3. Use this and welcome, but don't assume $\pi = \frac{25}{8}$, which has ***not*** been proved any more than the $\frac{22}{7}$ of Archimedes. This, to my mind, is the flaw in your ingenious writings. You keep on saying, "On the assumption that $\pi = 3{\cdot}125$" certain results follow, and ***they do follow***. You add, the same will not follow if π be not 3·125, which is also perfectly true. But all this helps us not one bit towards finding what π really is in

* In this Letter Mr. Gibbons puts the question :—"Now, why is not my equation as good as yours?" Answer ! Because 264 (arc 30° — sin 30°) does not equal circumference, whether the arithmetical value of the symbol π be *finite* and *determinate*, or an *indeterminate* and never ending series ! ! Let Mr. Gibbons prove his assumption independently ! J. S.

value. I would ask again:—Why is the equation 300 (arc 30° — sin. 30°) = circumference, better than 264 (arc 30° — sin. 30°) = circumference, unless you prove either of them independently?

Yours very truly,

G. B. G.

J. S. *to* G. B. G.

BARKELEY HOUSE, SEAFORTH,
2nd April, 1866.

DEAR SIR,

I am obliged to confess that your favour of the 29th March, which has had my careful attention, has somewhat surprised me. Your reasoning, however plausible, is extremely fallacious; and, whether successful or unsuccessful, I will make an attempt to show you why.

You charge me with assuming a "theory" which makes $\frac{25}{8}$ express the ratio of circumference to diameter in a circle; or otherwise stated, makes 300 (arc 30° — sin. 30) to a circle of radius 1 = 600 (the circular measure of an angle of 36'), and makes both = 2π. You propose an alternative "theory," which you say you prefer, namely, that of $\frac{22}{7}$, as the ratio of circumference to diameter; and I admit, as you say, that this was the "value (nearly) deduced by Archimedes". You then observe:—"This is the same as saying that $264\left(\frac{\pi}{6}-\frac{1}{2}\right) = 2\pi$, or (arc 30° — sin. 30°) = $\frac{1}{264}$th part of circumference." You then put the question:—"Why is not my assumption as good as yours?"*

Either of these assumptions establishes a very peculiar property of a circle of which the diameter is unity, which I suspect was never dreamed of in the mathematical philosophy of our scientific authorities, and to which I shall now direct your especial attention.

Let the diameter of a circle be unity, and let p denote the perimeter of a regular hexagon within it = 6 (radius) = 3. Let n denote the circumference of the circle, and m the difference between n and p. Then, n must be greater than p, that is greater than 3. Now, if $n = \frac{25}{8} = 3{\cdot}125$, it follows of necessity that m =

* I have given the reason in the Foot note on page clviii. J. S.

$(3{\cdot}125 - 3) = {\cdot}125$; therefore, $8\ (m) = 1$ = diameter of the circle, and $25\ (m) = 3{\cdot}125$ = circumference. But, if $n = \frac{22}{7} = 3{\cdot}142857142857$ with the recurring decimal $\cdot142857$ to infinity, then, $m = (3{\cdot}142857142857 - 3) = {\cdot}142857142857...$; therefore, $7\ (m) = {\cdot}999999999999$, with 9 to infinity, and would obviously be 1 if we could pick up the &c.'s ; and $22\ (m) = 3{\cdot}142857142854 = n$ to the last decimal figure, and were we to repeat the recurring decimal a thousand times, would still equal n to the last decimal figure.

Thus, either and both these assumptions establish the following very peculiar and unique property of a circle of diameter unity. *The difference between the circumference of the circle and the perimeter of its inscribed regular hexagon, multiplied by the two terms of the fraction which expresses the ratio of circumference to diameter, gives the circumference and diameter.**

This establishes an exact relation between the diameter and circumference of a circle, by means of the relations existing between the perimeters of rectilinear figures and the peripheries of curvilinear figures ; or in other words, by a comparison of a certain straight line with a curved line, and which, I have reason to believe, never before came within the observation of any Mathematician. If I am wrong in this conclusion you can readily prove it, and may reprove me for my ignorance.

I am not well enough to proceed further to-day, but when I hear from you in reply to this, I shall be prepared to furnish you with a geometrical and mathematical proof that will resolve into a gross absurdity all efforts on your part to controvert the "theory" that 8 circumferences of a circle = 25 diameters, which makes $\frac{25}{8} = 3{\cdot}125$, the true arithmetical value of π; and establishes $\frac{1}{3{\cdot}125}$ as the true expression of the ratio of diameter to circumference in a circle.

I am, Sir,

Yours very truly,

J. S.

* From this fact, it follows of necessity, that whether we assume a ratio of diameter to circumference in a circle, or find a ratio of diameter to circumference by some sort of calculation, we must demonstrate the value of π which the ratio gives, by establishing its inseparable connection with the areas of rectilinear figures within or about a circle. I have proved that *exact* and *definite* relations exist between the area of any circle, and the area of its *inscribed* and *circumscribed* squares.

A few more moves will finish our game, and establish an answer to the long vexed question—What is the true ratio of diameter to circumference in a circle?

I am, Sir,

Yours very respectfully,

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
2nd April, 1866.

P.S.—In my Letter in your last number there are several printer's mistakes, but they are not of such a character as to deceive a Mathematician, and consequently I do not consider it necessary to call especial attention to them.

FROM "THE CORRESPONDENT," APRIL 7TH, 1866.

QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

After a long and very friendly private correspondence with Mr. Smith on the value of π, our controversy is closed, for the reason he states, namely, that I could not see how he connected his squares and circles, unless he *assumed* the very thing in question,—that is, the value of π.

I do not employ Euclid to determine this value. Euclid only enables me to find, by Trigonometry, the length of a *straight* line; he never deals with the length of a *circular* arc. If the equal sides of an isosceles triangle be circle unity, and the vertical angle 15°, the base A B will be the chord of an arc of 15° in a circle whose radius is 1. It will be one side of a regular polygon of 24 sides.

Mr. Miller shows, in your number for March 24, that—

$2 \text{ Cos. } 15^\circ = \frac{1 + \sqrt{3}}{\sqrt{2}}$ which, for convenience of division, I will express as $\frac{\sqrt{2} + \sqrt{6}}{2} = 1{\cdot}9311851\ldots$

But, $AB^2 = 2 - 2 \text{ Cos. } 15°$ by Euclid 2-13.
$= \cdot 068149...$
Or, $AB = \cdot 26105...$ the *chord* of $15°$.

But arc of $15° = \frac{\pi}{12}$ and the arc is *greater* than its chord; therefore,

$\frac{\pi}{12}$ is greater than $\cdot 26105$.

That is, π is greater than $3\cdot 13260$.*

G. B. GIBBONS.

LAUNCESTON, CORNWALL,
26th March, 1866.

FROM "THE CORRESPONDENT," APRIL 7TH, 1866.

CIRCLE SQUARING.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

The courteous observations of Mr. J. Smith have induced me to overcome my extreme repugnance to put my fingers into his *Pi;* and I think I really must break the crust and expose the emptiness of the dish. This will be done without any reference to sines or tangents, but merely by the use of right lines. The circumstances that apply to one circle apply to all circles, as we have been told by Mr. J. Smith many times. Let us, therefore, take a circle having the radius equal to 60, when the diameter will be = 120. This, multiplied by $3\cdot 125$, will be 375. Now, I shall show that the side of a regular polygon of 24 sides, inscribed in this circle, will

* This Letter is no answer to mine in the *Correspondent* of March 17, 1866 (see page cxiii) in which I have pointed out the fallacy that lies at the root of the reasoning of Mr. Gibbons, Mr. Miller and Mr. Garbett. The fact is, if their reasoning be logical reasoning on indisputable geometrical data, a right-angled triangle, of which the hypothenuse and perpendicular are in the ratio or proportion of 4 to 1, could be a geometrical impossibility. What next? I must again refer the reader to the diagram on page 155, and the "*reasoning and calculation*" founded on it! J. S.

measure 15·66312, which, multiplied by 24 = 375·91488. And as it is *impossible* that the circle itself, within which such polygon is inscribed, can measure *less* than the polygon does, it follows that no kind of argument can refute this fact, and that Mr. J. Smith's measure of the circumference is fallacious, erroneous, and preposterous.

In the first place, let us take an angle of 60 degrees in the circle, and draw a line from one extremity of each radius to the other. We then have a triangle, the measure of each of the three lines of which is 60. Let these lines be marked S in the centre, A at the extremity of one radius, and X at the extremity of the other. Let us next bisect this angle and bring down another radius from S, forming two angles of 30 degrees each. Let us affix the letter C at the end of this radius, and the letter B at the point where it intersects the line A X. Join A C, and we have now three right-angled triangles, of which we shall use two only.

To find the length of S B we have to consider that A B is = 30, and A S = 60. Therefore, $AS^2 = 3600 - AB^2 = 900 = SB^2 = 2700$. Then the $\sqrt{SB} = 51·96175$. Take this from C S = 60, and we have B C = 8·03825. We now have to consider another right-angled triangle, A B C, of which we know two of the sides, viz., A B = 30, and B C = 8·03825 ; and we have to find the side A C. Then we have $AB^2 = 900$, and $BC^2 = 64·613463 = AC^2 = 964·613463$, of which the $\sqrt{} = AC = 31·05825$. This, then, is the side which subtends 30 degrees, or one-twelfth of the circle.

Draw, now, another circle having the same radius, and let A C be divided by a line from S, which will measure an angle of 15 degrees. Then we shall have A C bisected = ab = 15·529125, and, as before, we find sb thus:—We say $as^2 - ab^2 = bs^2 = 241·15372326$, &c., and the $\sqrt{bs^2} = bs = 57·95557$. Then, $sc = 60 - bs = bc = 2·04443$. And the square of this, added to the square of A B = 245·33341729, &c. And the square root of this number presents us with the length of the side ac = 15·66312, which is the measure of the side of the polygon, which is the 24th part of the circle. Then, 24 × 15·66312 = 375·91488. And this *exceeds* the measure of the circle given by Mr. James Smith, which is only 375. Hence his theory is proved, beyond all doubt, to be utterly and entirely false and foolish.

It is obvious that the angle s, being $= 15°$, the other two angles of the isosceles triangle $a s c$ are together $= 165°$, and that $a = 82°\ 30'$. Then, we might come at $a c$ by merely saying, "As sine $a = 82°\ 30'$ is to 60, so is sine $s = 15°$ to $a c = 15{\cdot}66311$." But Mr. Smith repudiates the sines, and I have, therefore, worked it out by the squares of the sides only. And unless he deny that the square of the hypothenuse is equal to the sum of the squares of the other two sides, escape he cannot the dire consequence, and he must be driven from the field with obliquy.

Yours, &c.,
R. J. MORRISON.

P.S.—If we pursue the investigation we shall find that the side of a polygon of 48 sides will measure about 7·8485, and this multiplied by 48, will give for the length of the periphery* of the polygon about 376·73. Yet Mr. J. Smith gives us only 375 for the length of the circumference within which it is inscribed!

FROM "THE CORRESPONDENT," APRIL 14TH, 1866.

QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

Such of your readers as take an interest in the "Circle" controversy would observe that in my letter of the 2nd inst. to your contributor "G. B. G.," I promised to furnish him with such a proof of the theory I hold on this important subject as shall resolve into a gross absurdity all efforts on his part to controvert it. In fulfilment of my promise, I wrote him as follows:—

* Query? Perimeter!

BARKELEY HOUSE, SEAFORTH,
3rd April, 1866.

DEAR SIR,

Without waiting your reply to my letter of yesterday, I may proceed to furnish you with an unanswerable proof of the "theory" that 8 circumferences of a circle are exactly equal to 25 diameters. I have more than once told you that you look at the question at issue too much from a mathematical point of view, and overlook the geometry that is involved in it. How happens it that I have never been able to induce you to examine the properties of a right-angled triangle, of which the hypothenuse and perpendicular are in the ratio or proportion of 5 to 3? Be the reason what it may, I hardly think you will venture to assert that what is geometrically true can be mathematically false, or that what is mathematically true can involve geometrical absurdity.

I beg to enclose a diagram, of which the following may be taken as the method of construction :—

Let A A′ be a straight line of indefinite length. With A as centre and A O part of A A′ as interval, describe the circle X. Produce O A to B, making A B = 2 (O A), and with A as centre and A B as interval, describe the circle Y. On A B describe the square A B C D, and draw the diagonals A C and D B. With C as centre and C E or C F equal to $\frac{1}{4}$ (C B) or $\frac{1}{4}$ (C D) as interval, describe the circle Z, and join E F, A E, and A F. With C as centre and C B as interval, describe the arc of the quadrant C B D. From A the centre of the circle X draw a straight line at right-angles to the line A F to meet the line C B produced at the point G, and join G F. In the square A B C D inscribe the circle X′. (*See Diagram* E.)

E F the base of the isosceles triangle A E F is bisected at the point H. I cannot pretend within the limits of a Letter to exhaust the properties of this remarkable geometrical figure, but I shall direct your attention to some interesting geometrical facts in connection with it, which you will find to be incontrovertible.

First: Because C E is a radius of the circle Z, and because A O equal to 2 (C E) is the radius of the circle X, and because A B = 2 (A O) equal to 4 (C E) is the radius of the circle Y; 4 (area of the circle Z) = area of the circle X; 4 (area of the circle X) = area of the circle Y; therefore, 16 (area of the circle Z) = area of the

circle Y. It is obvious that the same relations will hold with regard to circumscribing squares to the three circles.

Second: Because the area of the circle Y = 4 (area of the circle X), the area of the circle X = area of the quadrant A D B. But A O the radius of the circle X = $\frac{1}{2}$ (A B), and A B is a side of the square A B C D; therefore, the circles X and X′ are equal; and it follows, that the area of the circles X and X′ = area of the quadrants A D B or C D B.

Third: Because C E = $\frac{1}{4}$ (C B), B E = $\frac{3}{4}$ (C B), and because C F = $\frac{1}{4}$ (C D), D F = $\frac{3}{4}$ (C D). But, C B, C D, A B, and A D are equal; therefore, the triangles A B E and A D F are similar and equal right-angled triangles, having the sides containing the right angle in the ratio or proportion of 4 to 3, by construction.

Fourth: Because G F is the diagonal of a square on A F, A G = A F. But, A D = A B; therefore, B G = B E = D F; therefore, A B G, A D F, and A B E, are similar and equal right-angled triangles.

Fifth: Because B G = B E, and B C = A B, G E = A B + B E, or, A D + D F; and C E or C F = A B — B E, or, A D — D F, and C is a right angle; therefore, G C^2 + E C^2 = G F^2 = (A B^2 + B E^2 + A E^2), or, (A D^2 + D F^2 + A F^2), or, (A B^2 + B G^2 + A G^2); therefore, G F^2 = {4 (the rectangle B C × B E) + 4 ($\frac{1}{2}$ the rectangle C E × C F)}, and the rectangle B C × B E = 4 (the rectangle B E × C E).

Sixth: Because the circumferences of circles are to each other as their radii, the circumference of the circle Y = 2 (circumference of circle X). But, the arc of the quadrant A D B = $\frac{1}{4}$ (circumference of circle Y); therefore, the arc of the quadrant A D B = semi-circumference of the circle X. But, the circles X and X′ are equal; therefore, the arcs of the quadrants A D B and C D B are equal, and both = semi-circumference of the circle X′; therefore, the sum of the arcs of the quadrants A D B and C B D = circumference of the circle X′.

I think I have gone far enough in the purely geometrical direction to serve my purpose. You may probably ask me—*Cui bono?* This, you may say, may all be admitted; but, after all, these are simply so many geometrical facts, and afford no clue whatever to the value of π. And, perhaps, you may be inclined to tell me,

"they have no more to do with the value of π than the price of butcher's meat." *Nous verrons!*

I shall now make two assumptions, but they shall be such as I hardly think you will venture to quarrel with.

First: I shall assume it to be admitted, that the area of a circle is equal to the area of a rectangle, of which the longer side = semi-circumference of the circle, and the shorter side = radius.

Second: I shall assume it to be admitted that π times A B^2 = area of the circle; and it follows of necessity, that $\frac{\pi \, A \, B^2}{4}$ = (semi-circumference of circle X′ × radius), whatever be the value of π.

Now, so far the reasoning is purely geometrical, and, it may be admitted, affords no clue, *per se*, to the value of π. But, by the aid of Mathematics—that is to say, by giving an arithmetical value to some line or surface in our geometrical figure—we may not only get a clue to the value of π, but we may demonstrate its true and exact value, beyond the possibility of dispute or cavil. Well, then, I shall take the line A B = 4, and work out my proofs by incontrovertible facts, and you will find that the facts I shall bring under your notice are far from exhausting the properties of our geometrical figure.

First fact: The isosceles triangle A E F and the right-angled triangle G C F contain the same superficial area. For example: G C = A B + B E = 7, and C F = A B — B E = 1; therefore, half the rectangle G C × C F = $\frac{7 \times 1}{2}$ = 3·5 = area of the triangle G C F. But A E and A F, the legs of the isosceles triangle A E F = $\sqrt{A\,B^2 + B\,E^2}$, or, $\sqrt{A\,D^2 + D\,F^2}$ = 5; and E F the base of the triangle = $\sqrt{C\,E^2 + C\,F^2} = \sqrt{2}$. Now, E F the base of the triangle = A E F is bisected by A C a diagonal of the square A B C D at the point H; therefore, H E and H F = $\frac{1}{2}(\sqrt{2}) = \sqrt{\cdot 5}$; therefore, $\sqrt{A\,E^2 - H\,E^2}$, or $\sqrt{A\,F^2 - H\,F^2} = \sqrt{5^2 - \sqrt{\cdot 5}^2}$ $= \sqrt{25 - \cdot 5} = \sqrt{24 \cdot 5}$ = A H; therefore, the rectangle A H × H E, or the rectangle A H × H F = $\sqrt{24 \cdot 5} \times \sqrt{\cdot 5} = \sqrt{24 \cdot 5 \times \cdot 5} =$ $\sqrt{12 \cdot 25}$ = 3·5 = area of the isosceles triangle A E F; therefore, the isosceles triangle A E F and the right-angled triangle G C F contain the same superficial area. Q. E. D.

Again: It is obvious from a mere inspection of the diagram,

that (area of the square A B C D — area of the isosceles triangle A E F) = the sum of the areas of the triangles A B E, A D F, and F C E. From this, four facts follow of necessity. First: ($\frac{1}{2}$ the rectangle A B $\times$ B E + $\frac{1}{2}$ the rectangle A D $\times$ D F + $\frac{1}{2}$ the rectangle F C $\times$ C E) = $\frac{AB^2 + BE^2 + AE^2}{4}$, or, $\frac{AD^2 + DF^2 + AF^2}{4}$; that is to say, $(6 + 6 + \cdot5) = \left(\frac{4^2 + 3^2 + 5^2}{4}\right) = \left(\frac{16 + 9 + 25}{4}\right) = \frac{50}{4}$ = 12·5. Second: The sum of the areas of the triangles A B E, A D F, and F C E = $\frac{3\cdot125\ (AB^2)}{4}$; that is to say, $\frac{3\cdot125\ (4^2)}{4} = \frac{50}{4}$ = 12·5 = the sum of the areas of the triangles A B E, A D F, and F C E. Third: 4 (the sum of the areas of the triangles A B E, A D F, and F C E) = 3·125 (AB^2) = (AB^2 + BE^2 + AE^2), or, (AD^2 + DF^2 + AF^2), or, (AB^2 + BG^2 + AG^2). Fourth: 3·125 (AB^2) + 4 (area of the isosceles triangle A E F), or, 3·125 (AB^2) + 4 (area of right-angled triangle G C F) = {3.125 (4^2) + 4 (3·5) } = 50 + 14 = 64 = area of a circumscribing square to the circle Y. This fixes 3·125 as the value of π; for, if not (and this you have admitted to be legitimate reasoning), let π be greater than 3·125, say 3·126. Then: {3·126 (AB^2) + 4 (area of the triangle A E F, or G C F)} = (50·016 + 14) = 64·016, and is greater than the area of a circumscribing square to the circle Y, which is impossible, and therefore absurd. Or, let the value of π be less than 3.125, say 3·124. Then: {3·124 (AB^2) + 4 (area of the triangle A E F, or G C F)} = (49·984 + 14) = 63·984, and is less than the area of a circumscribing square to the circle Y, which is impossible, and therefore absurd.

Now, my dear Sir, I do not ask your opinion as to the value of π. I tell you without the slightest hesitation that π = 3·125, and can be nothing else. If you can't see it, I can't help it, but the fact remains notwithstanding.

The following facts, among others, follow of necessity:—The area of a square on the line G F = area of the circle Y. The area of a square on the line A E = area of a circumscribing circle to the square A B C D. The area of the circle X′ = area of the isosceles triangle A F G. The area of that part of the square A B C D contained by the arcs of the quadrants A D B and C D B

$= BE^2$ or DF^2; and the area of those parts of the square A B C D outside the arcs of the quadrants = $AB^2 - BE^2$, or $AD^2 - DF^2$, and the sum of the separate parts = area of the square A B C D.

Thus, π times A B = $3\cdot125 \times 4 = 12\cdot5$ = semi-circumference of the circle Y; and A B the radius of the circle = 4, therefore, semi-circumference $\times$ radius $= 12\cdot5 \times 4 = 3\cdot125\ (AB^2) = 50$ = area of the circle Y, and is exactly equal to the area of a square on G F the hypothenuse of the right-angled triangle G C F.

Now, my dear Sir, if I am wrong in my theory that 8 circumferences of a circle = 25 diameters, these cannot be facts; and if not, you can readily prove it, and expose my ignorance and folly. If you find this to be impossible, as most assuredly you will, I cannot help thinking your candour will induce you to admit that you are checkmated, and that the true arithmetical value of π must be $3\cdot125$, and can be nothing else.

I am, dear Sir,

Very sincerely yours,

J. S.

I have written another letter to "G. B. G.," in which I work out the same results by a method based on the admission of a recognised mathematical and geometrical authority, but this I must reserve for another communication.

I must leave your readers to form their own opinions and draw their own inferences on the wisdom displayed by "Zadkiel," in his Letter in your last number. The prophet is a bold man, and will probably thrust his hand into the foregoing dish; if so, I tell him he will not get it out again without burning his fingers. Let him try!

I am, Sir,

Yours very respectfully.

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,
9th April, 1866.

FROM "THE CORRESPONDENT," APRIL 21ST, 1866.

THE QUADRATURE OF THE CIRCLE.

TO THE EDITOR OF THE CORRESPONDENT.

SIR,

My Letter of the 3rd inst., to your contributor, "G. B. G.," which appears in your publication of last Saturday, was followed up by another, which ran as follows :—

BARKELEY HOUSE, SEAFORTH,
7th April, 1866.

MY DEAP SIR,

I did not wait your reply to my Letter of the 2nd, and posted one to you before receiving your favour of the 4th instant, furnishing the promised proof of the truth of the theory that 8 circumferences of a circle = 25 diameters ; of such a character as will render all your efforts to prove the value of π greater than $\frac{25}{8}$ = 3·125 a gross absurdity. (I inadvertently omitted to acknowledge the receipt of your favour of the 2nd inst., which had reached me when I wrote you. I may say, however, it contains nothing new). In your Letter of the 4th inst. you observe :—"Your Letters are always courteous, and I am never troubled by receiving them, but I should have ceased, for the present, our correspondence (our controvery I consider already over) had not your Letter, received this morning, stated that you await my reply." As regards the words in brackets, I beg to differ from you, as I have not "*fired my last shot*," or yet made any serious impression on my "*ammunition*."

I send you herewith numbers 34 and 35 of the *Correspondent*. In my Letter in the latter I give a quotation from a popular scientific publication, "*Orr's Circle of the Mathematical Sciences*," in which the writer observes,—"It is plain, therefore, that there is nothing visionary or absurd in the search after this square (a square exactly equal in surface to a given circle) as if such a thing had no existence :" and he gives the reason, by adopting the idea of a continuously expanding square arrested in its progress at that

DIAGRAM F.

particular stage of its expansion when it has exactly attained the magnitude of the circle. In other words, he admits ;—"If this individual square could be isolated and exhibited, the geometrical quadrature of the circle would be solved." (*En passant*—if Mathematicians would only take the trouble to reflect, they could not fail to perceive that it is both "visionary and absurd" to suppose we can ever discover the geometrical without finding the numerical quadrature of the circle, since a knowledge of the latter is essential to the proof of the former. Thus they necessarily stand or fall together.)

You will observe that in my Letter of the 23rd December, I have made the following assertion : "We may work out geometrically and mathematically this idea of the expanding square, arrest its progress at that particular stage of its expansion when it has exactly attained the magnitude of the circle, and thus isolate and exhibit the individual square." Several causes have, hitherto, combined to prevent me bringing out the proof through the columns of the *Correspondent*, and I now beg to bring it under your notice.

The following may be taken as the method of construction of the enclosed diagram. On the straight line A B describe the square A B C D, draw the diagonal A C, and with A as centre and A B as interval, describe the circle. From A the centre of the circle draw a straight line to meet any point E on the line B C, producing a right-angled triangle A B E. On A E describe the square A E F G, draw the diagonal G E, and join G D. On G E describe a square G E H I. (*See Diagram* F.)

Now, A C a diagonal of the square A B C D = the side of an inscribed square to the circle, and 2 (A B) = the side of a circumscribing square to the circle. The area of a square on the former is obviously less, and the area of a square on the latter is obviously greater than the area of the circle.

Again: We may obviously make A E the side of the square A E F G as little greater as we please than A B the side of the square A B C D. Or, we may make G E as little greater as we please than A C the diagonal of the square A B C D. We may obviously make A E as little less as we please than A C the diagonal of the square A B C D. Or, we may make G E as little less as we please than the side of a circumscribing

square to the circle. Thus, we may make B E the base of the triangle A B E as small as we please, or as nearly equal to the length of the line B C as we please.

Again: On whatever point in the line B C the extremity of the line A E may be made to rest, A B = B C, and, it follows of necessity, that A B + B E = G C; and A B — B E = C E; therefore, the sum of the areas of squares on G C and C E = the sum of the areas of squares on A B, B E, and A E.

Now, conceive us to have made B E as small as we please to start with (it may be represented mathematically by a decimal quantity terminating with the figure 1 with as many ciphers before it as we please), and to have increased its value by infinitesimally small steps until we should have made B E as nearly equal to the line B C as possible. In this way it is conceivable that the extremity of the line A E may be made to rest on trillions of points on the line B C in its passage from B towards C. Now, to take up the idea of the writer from whom I have quoted. Supposing the operation I have described to be performed (and this is conceivable), the square on A E would be gradually expanded from a square as little larger as possible than the square on A B to a square as little smaller as possible than an inscribed square to the circle; and the square on G E would be gradually expanded from a square as little larger as possible than an inscribed square to the circle to a square as little smaller as possible than a circumscribing square to the circle. Thus, as all intermediate magnitudes between the inscribed and circumscribed squares to the circle would have been reached and passed through by the square on G E, at one stage of its progress it must have exactly attained the magnitude of the circle.

The question then arises—Can the square on G E be isolated and exhibited at that particular stage of its progress when it has exactly attained the magnitude of the circle? This question I answer in the affirmative. It occurs when the extremity of the line A E, in its passage from B towards C, has passed over exactly $\frac{3}{4}$th parts of the line B C. Now, having made the line B E in the diagram = $\frac{3}{4}$(B C), by construction, the square G E H I is isolated and exhibited, and is exactly equal in superficial area to the circle.

The following are the inevitable results:—A E + B E = G C,

and $GC + CE = 2AB$; therefore $(GC + CE)^2$ = area of a circumscribing square to the circle. $GC^2 + CE^2 = GE^2 = AB^2 + BE^2 + AE^2 = 3{\cdot}125\ (AB^2)$ = area of the circle. $GC^2 - CE^2 = 6\ (AB \times \frac{1}{2} AB)$ = area of an inscribed regular dodecagon to the circle. Thus, the area of the square A E F G = area of a circumscribing circle to the square A B C D, and the area of the right-angled isosceles triangles G A E or G F E = area of an inscribed circle to the square A B C D.

Now, my dear Sir, I do not ask your opinion as to the value of π, but it will be to me strange indeed, if, after you shall have given this proof your careful consideration in connexion with that given in my last Letter, you can have any further doubt on your mind as to the ratio of diameter to circumference in a circle.

I find the Letter you spoke of in your favour of the 3rd instant as having sent to the *Correspondent* last week, appears in this day's number, and also one from Captain Morrison, which I think will rather surprise you after having read his review of my last work in his "Almanac for 1866." The argument in his Letter and yours are at bottom the same, and is neither more nor less than the one which has been so oft repeated, and is, in fact, the only plausible one that can be, or ever has been, brought against the theory that 8 circumferences of a circle are exactly equal to 25 diameters, and, I admit, would be unanswerable if you could prove that any figure inscribed within a circle could be made to enclose as large an area as the circle; or, that any rectilinear figure can be made to enclose as large an area as a circle of which the circumference is equal to the perimeter of such rectilinear figure.

Now, it will be admitted that one of the orthodox methods of finding the value of π is by means of the areas of inscribed regular sided polygons to a circle of radius 1, and I admit that other methods adopted by Mathematicians give the same result. What appears to me strange is, that Mathematicians cannot perceive that whether we start from the four-sided or six-sided inscribed polygon to a circle of radius 1 to find the value of π by means of areas, a circle equal in circumference to the perimeters of the polygons would enclose a larger area than the polygons, and is fatal to any such method for finding the value of π. In fact, it is neither more nor less than an attempt to measure (directly) a curved line by

means of straight lines, which is a gross absurdity. The 47th proposition of the first book of Euclid is only applicable (directly) to finding the values of straight lines, but by a proper application of it (indirectly) may be brought into play to prove, by means of areas or superficies, the circumference of a circle of diameter unity, that is, the true arithmetical value of π. The circumference of a circle equal to the perimeter of an inscribed square to a circle of radius $4 = \sqrt{32}$. The area of this circle $= 2{\cdot}56 = 4\,({\cdot}8^2)$, and if to this be added the difference between the areas of the 4 and 8-sided inscribed polygons $= {\cdot}8284271$ we obtain the arithmetical quantity $3{\cdot}3884271$. The circumference of a circle equal to the perimeter of an inscribed regular polygon of 6 sides to a circle of radius $1 = 6\,(\text{radius}) = 6$. The area of this circle $= 2{\cdot}88 = 3$ times $\frac{24}{25}$ (unity); and if to this be added the difference between the areas of the 6 and 12-sided inscribed polygons $= {\cdot}401925$, we obtain the arithmetical quantity $3{\cdot}281925$. You may tell me I have assumed my value of π to get these results. Be it so. Well, then, take your own value of π; you will find that you cannot alter the following fact:—In either case we obtain an arithmetical quantity greatly in excess of the area of the circle, and you will find this argument carried to its legitimate conclusion in my Letter in the 40th number of the *Correspondent.*

Referring to your Letter in this day's *Correspondent*, I beg to call your attention to a statement made in your favour to me of the 2nd inst:—" Geometry shows that A B = $\cdot$26105. The arc A C B = $\frac{\pi}{12}$, and the arc is greater than its chord. " Now, my dear Sir, will you be good enough to tell me how you prove $\frac{\pi}{12}$, greater than $\cdot$26105 without the aid of mathematics?

I am, my dear Sir,

Yours very truly,

J. S.

It appears to me that the following will be the best method of bringing this long vexed question to a practical and final issue: Let Mathematicians furnish a proof of the value of π by a method into which no application of the 47th proposition of the first book of

Euclid shall enter in any way or shape. I tell them without hesitation, if they can't do it, I can.

I am, Sir,

Yours very respectfully,

JAMES SMITH.

BARKELEY HOUSE, SEAFORTH,

16*th April*, 1866.

APPENDIX F.

FROM "ZADKIEL'S ALMANAC," 1866.

REVIEW.

The Quadrature of the Circle Geometrically and Mathematically Demonstrated. By JAMES SMITH, ESQ. Simpkin Marshall & Co.

THIS work has created a loud controversy and has aroused the bile of many of our ablest mathematical critics, who have, according to their wont, indulged in all kinds of vulgar abuse and insult, to excuse themselves from any investigation of its contents. Mr. Smith, the author, is well versed in the art of arithmetic, and displays not a little knowledge of geometry. The gist of Mr. Smith's theory is, that $\frac{25}{8} = 3 \cdot 125$ is the true arithmetical value of the circumference of a circle, of which the diameter is unity: in other words, that 8 circumferences of a circle are equal to 25 diameters. He brings a vast amount of geometry and arithmetic to prove this fact, of which we will presently give a few specimens. While his adversaries, among whom rank the Editor of the *Athenæum*, Dr. Whewell, Professor De Morgan, &c., are mostly content to revile and insult the author. Instead of quietly disproving his positions, they assert that the circumference of a circle is not 3·125, but is 3·14159265, &c., and that it cannot be expressed arithmetically. Well, if the critics were honest, they would have demolished Mr. Smith's pretentions long ago, if possible; but this they find no easy

task. He offers them the following nuts, which they will find hard to digest—

He says that $\frac{360}{25} = 14{\cdot}4 = 14^\circ\ 24'$ is the value of an angle, (one 25th part of the circumference) and the circular measure of this angle is $\frac{14^\circ\ 24' \times \pi}{180 \times 60} = \frac{864' + 3{\cdot}125}{180 \times 60} = \frac{2700}{10800} = {\cdot}25$. The circular measure of an angle of 90° is $\frac{90 \times \pi}{180} = 1{\cdot}5625$.

But $1{\cdot}5625$ is the value of the quadrant of a circle of radius one; and $\frac{1{\cdot}5625}{{\cdot}25} = 6{\cdot}25$, or the circumference of a circle ot radius one, and is equal to $2\,\pi$. He goes on to shew, that on the orthodox theory, the circular measure of the angle (one 25th part of the circumference) $= \frac{864' \times 3{\cdot}1416}{180 \times 60} = \frac{{\cdot}27143424}{10800} = {\cdot}251328$; and the circular measure of an angle of $90^\circ = \frac{90^\circ \times 3{\cdot}1416}{180} = 1{\cdot}5708$; which is equal to the quadrant of a circle of radius one, on the hypothesis that $\pi = 3{\cdot}1416$. But then, $\frac{1{\cdot}5708}{{\cdot}251328} = 6{\cdot}25$, and is *not* equal to $2\,\pi$; but *is* equal to the circumference of a circle of radius one, on his, Mr. Smith's theory. And this fact, he argues, establishes his theory that $\frac{25}{8} = 3{\cdot}125$; which he gives for the true arithmetical value of the circumference of a circle, of whlch the diameter is unity. But he adds that "the number of degrees contained in an angle at the centre of a circle, subtended by an arc equal to the radius, is $\frac{180}{\pi}$; and on the theory that 8 circumferences of a circle are exactly equal to 25 diameters, $= \frac{180}{3{\cdot}125} = 57{\cdot}6 = 57^\circ\ 36'$; and the circular measure of an angle of $57^\circ\ 36'$ is $\frac{57^\circ\ 36' \times \pi}{180 \times 60} =$
$\frac{345{\cdot}6 \times 3{\cdot}125}{180 \times 60} = \frac{10800}{10800} = 1, =$ radius."

Now, in the name of common decency, what is there in these facts and arguments on the part of Mr. Smith that he should be overwhelmed with abuse and ribald vulgarity? Is it because he, the discoverer of these important properties of the circle, is a self-educa-

ted man, while his defeated adversaries were educated at Cambridge? This must be the true mathematical sore place. It only proves that France, having no old Universities, is better off and produces better Mathematicians than Cambridge can. And it is a potent argument for the pulling down of these ancient monopolies, that England may enjoy an equal field for the natural talents and abilities of all her gifted sons. At present these vile old monopolies beat down and destroy the claims of such natural-born nobles as the writer of this very valuable and important work. We cordially recommend Mr. Smith's work to the notice of our readers.

LIVERPOOL: A. & D. RUSSELL, PRINTERS, 30, MOORFIELDS, LIVERPOOL.

ERRATA.

ADDRESS TO THE READER.

Page xii.—Fifteenth line from top : for B C *read* A C.

" xlii.—Last paragraph : for inhability *read* inability.

CORRESPONDENCE.

Page 7.—Second line from bottom : after cosines *insert* of the angles.

" 15.—Thirteenth line from top : for given A B = 16° 16′ *read* given B A C = 16° 16′.

" 28.—Fourth line from bottom : for O H *read* C H.

" 107.—Seventh line from bottom : for 14·4 = radius, *read* 14·4 = chord B C = $\frac{\text{radius}}{4}$.

" 111.—Footnote : for Aug. 12 *read* Aug. 16.

" 114.—Ninth line from top : for area of circle F D C when A B = 3, *read* area of circle when A B = 3.

" 117.—Ninth line from top : for Sin. 30° *read* Sin. 90°.

" 118.—Twentieth line from bottom : for Mathematician *read* Mathematicians.

" 142.—Sixth line from bottom : for

$$\left\{(AF^2 + EC^2) + 2(BF \times FC)\right\} = AC^2.$$

read $\left\{(AF^2 + FC^2) + 2(BF \times FC)\right\} = AC^2.$

" 192.—Top line : for whereas *read* whenever.

" 280.—Eighth and thirteenth lines from bottom : for Appendix D *read* Appendix E.